Probabilistic Logic Programming

ALP Summer School on Computational Logic 2014

Angelika Kimmig
angelika.kimmig@cs.kuleuven.be

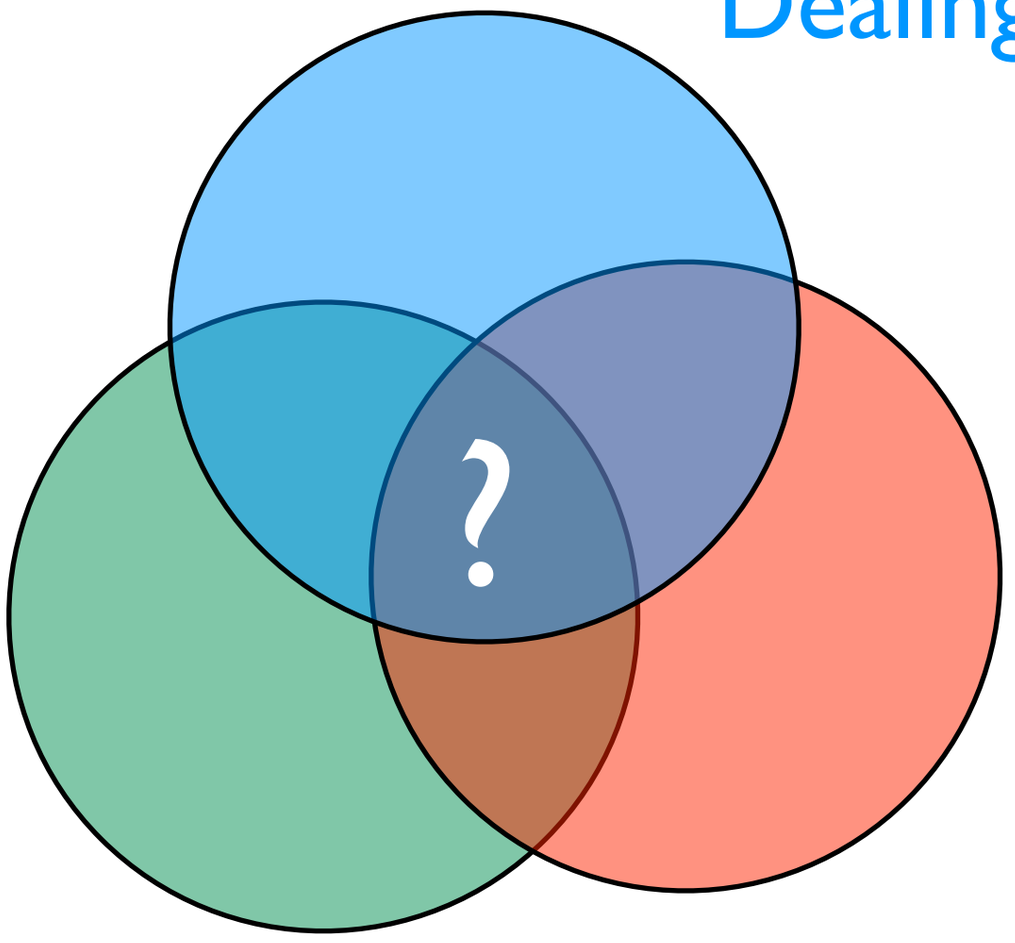
Probabilistic Logic Programming

Thanks to Luc De Raedt & many others working on PLP and especially ProbLog!

A key question in AI:

Dealing with uncertainty

Reasoning with relational data



Learning

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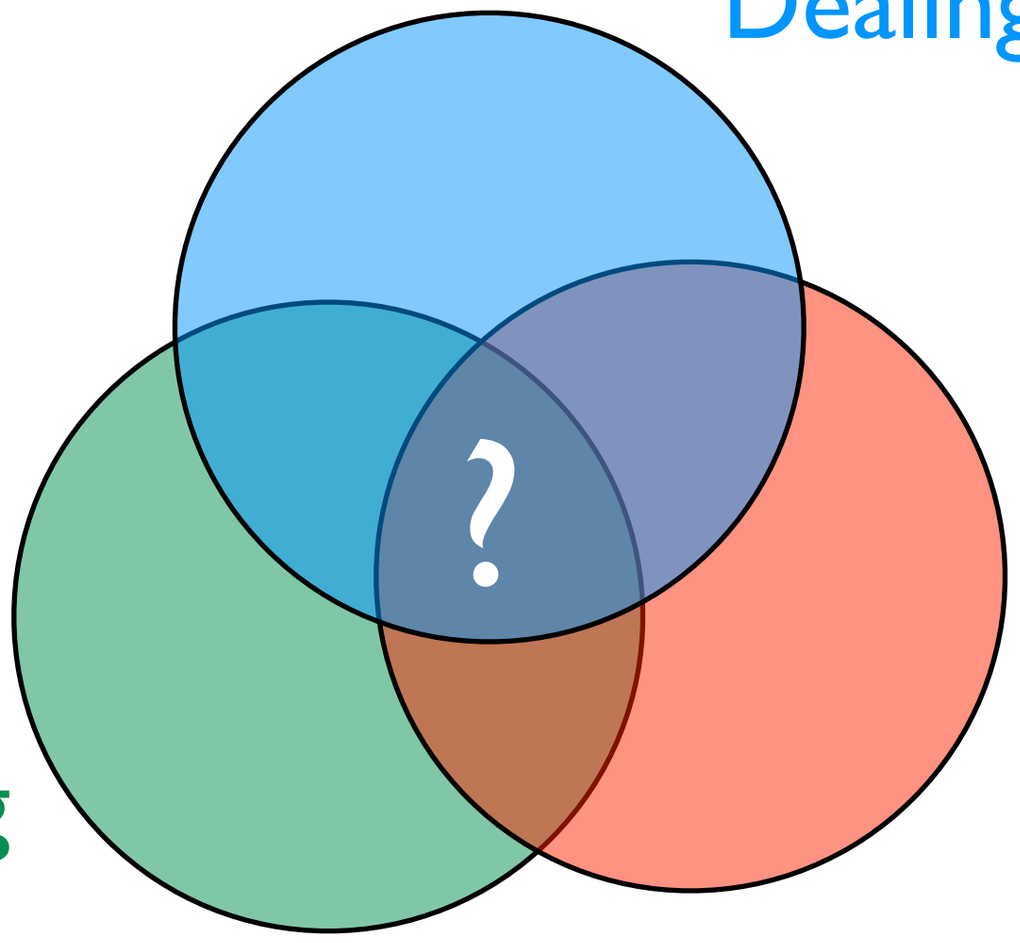
Dealing with uncertainty

Reasoning with relational data

- logic
- databases
- programming
- ...



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Dealing with uncertainty

- probability theory
- graphical models
- ...

Reasoning with relational data

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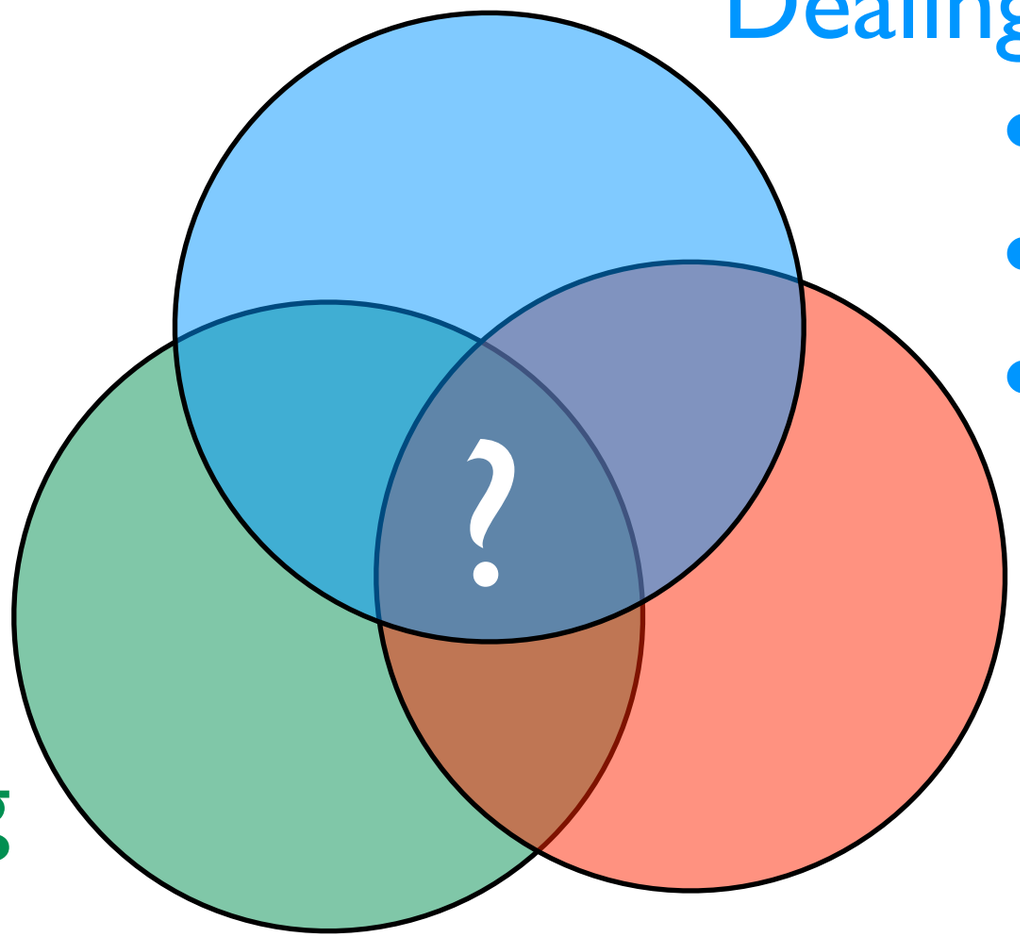

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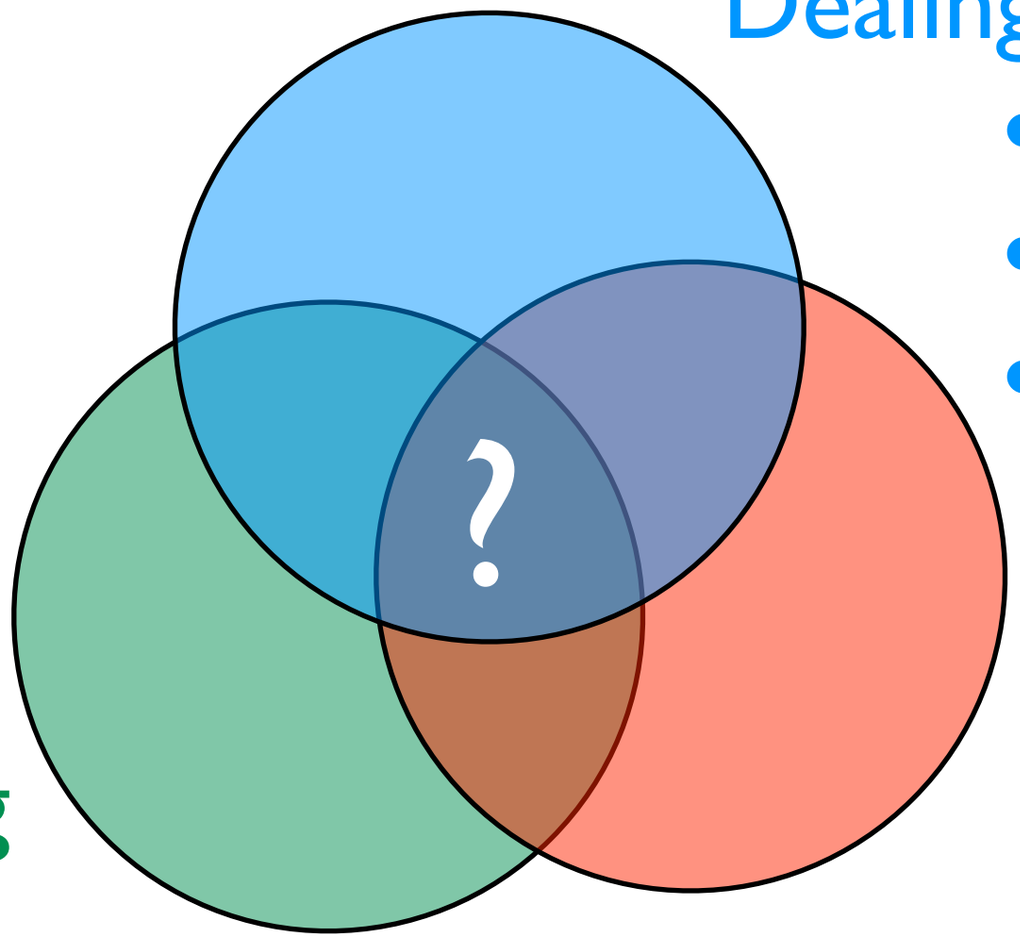

Learning

- parameters
- structure

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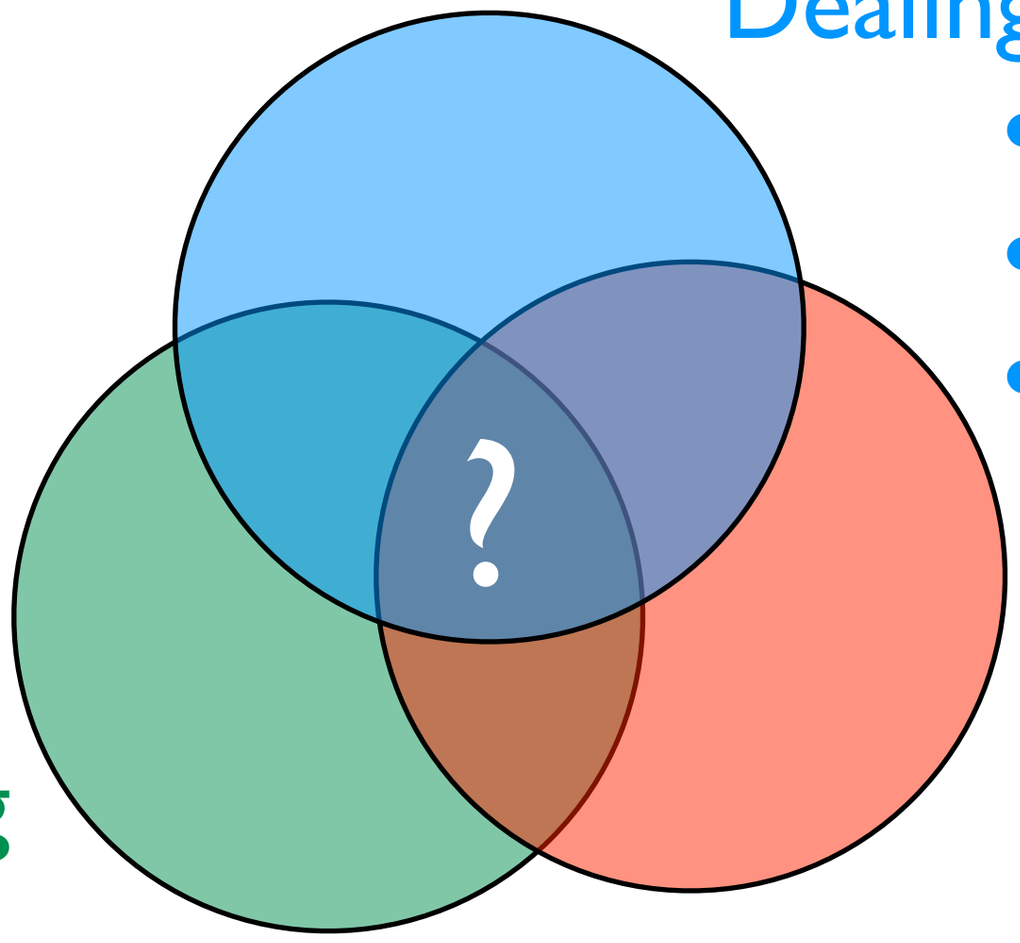

Dealing with uncertainty

- probability theory
- graphical models
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Learning

- parameters
- structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

Example: Information Extraction

Recently-Learned Facts twitter Refresh

instance	iteration	date learned	confidence
kelly_andrews is a female	826	29-mar-2014	98.7
investment_next_year is an economic sector	829	10-apr-2014	95.3
shibenik is a geopolitical entity that is an organization	829	10-apr-2014	97.2
quality_web_design_work is a character trait	826	29-mar-2014	91.0
mercedes_benz_cls_by_carlsson is an automobile manufacturer	829	10-apr-2014	95.2
social_work is an academic program at the university rutgers_university	827	02-apr-2014	93.8
dante wrote the book the_divine_comedy	826	29-mar-2014	93.8
willie_aames was born in the city los_angeles	831	16-apr-2014	100.0
kitt_peak is a mountain in the state or province arizona	831	16-apr-2014	96.9
greenwich is a park in the city london	831	16-apr-2014	100.0

NELL: http://rtw.ml.cmu.edu/rtw/

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instances for many different relations

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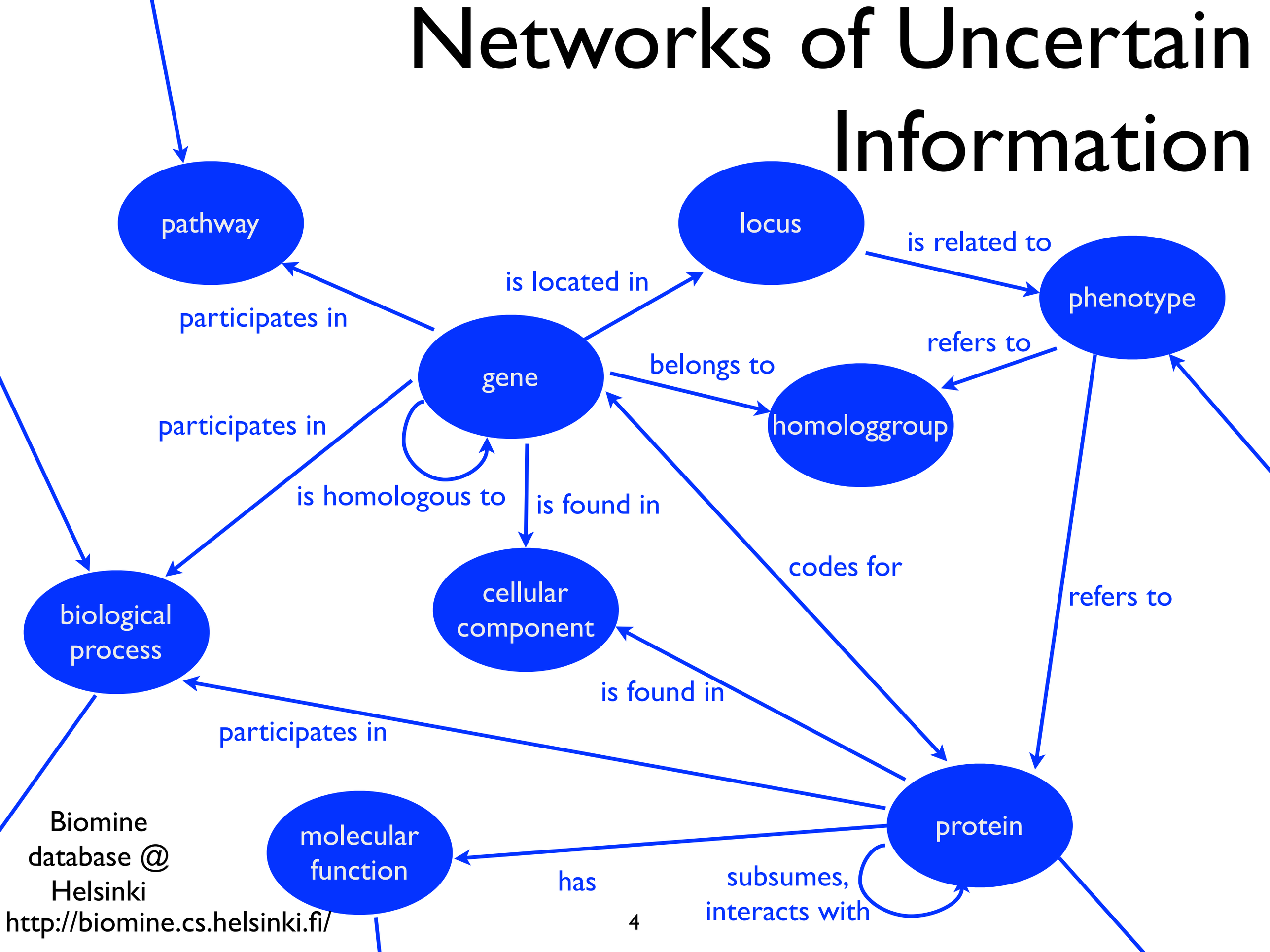
instances for many different relations

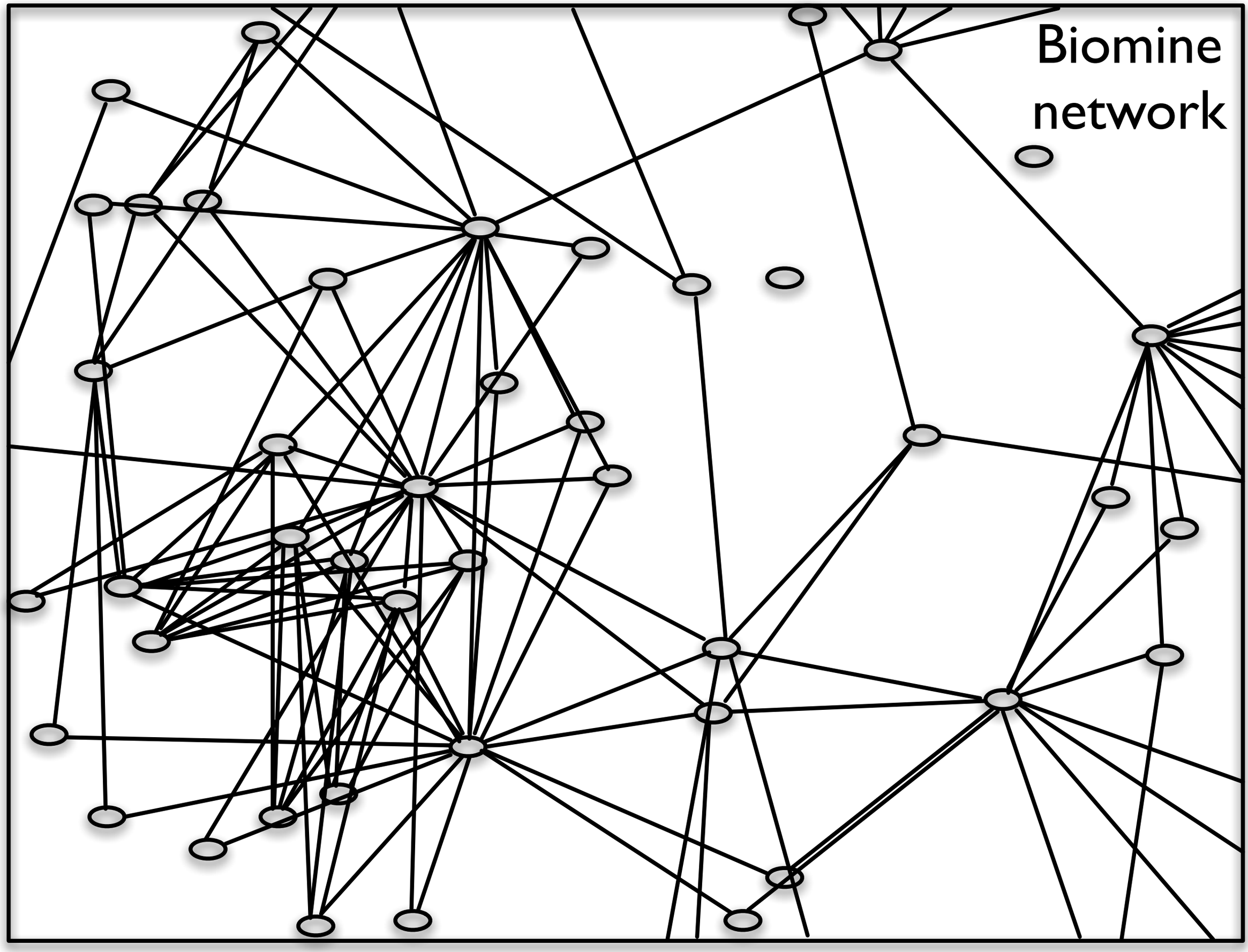
degree of certainty

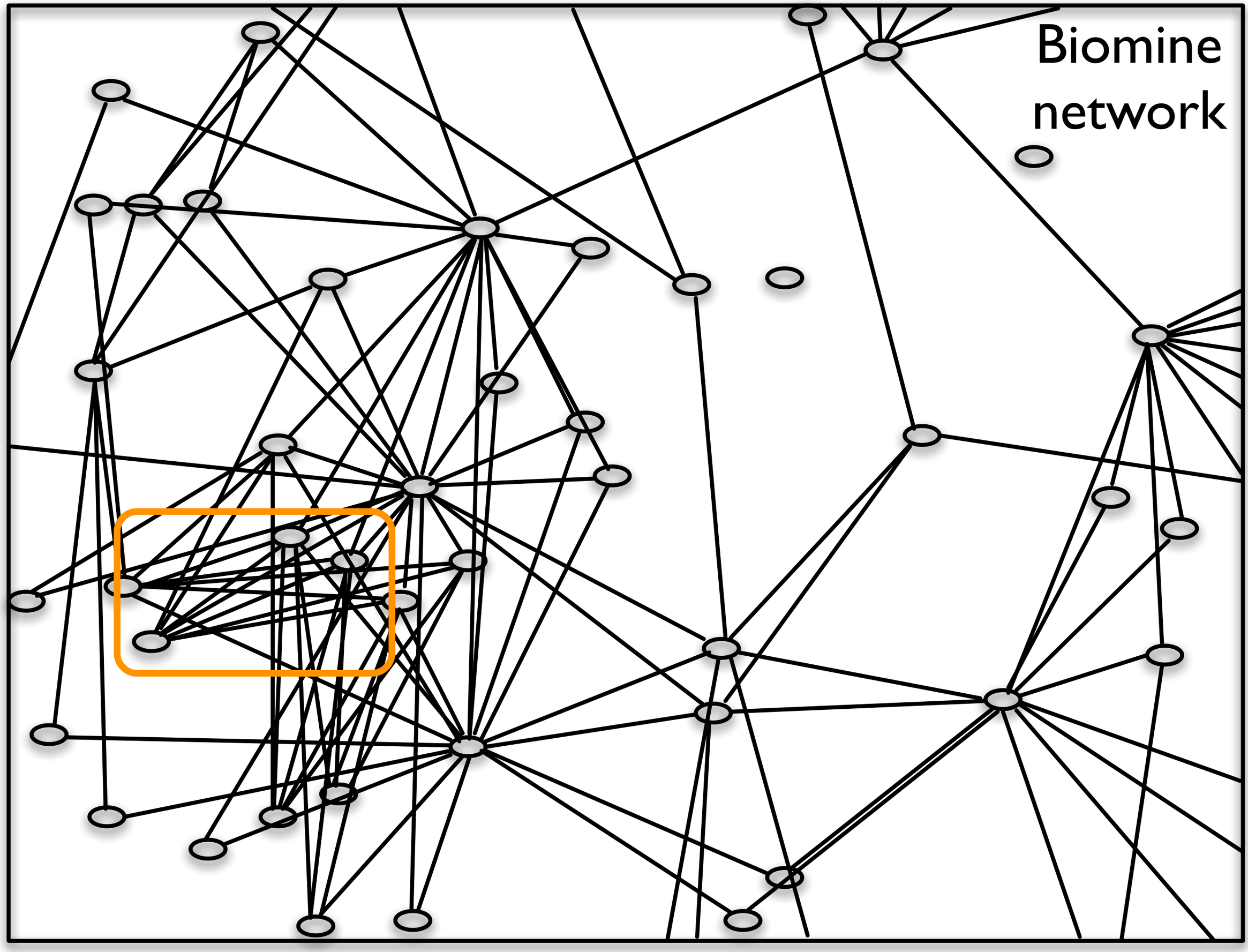
NELL: http://rtw.ml.cmu.edu/rtw/

Networks of Uncertain Information



Biomine database @ Helsinki
http://biomine.cs.helsinki.fi/


Biomine
network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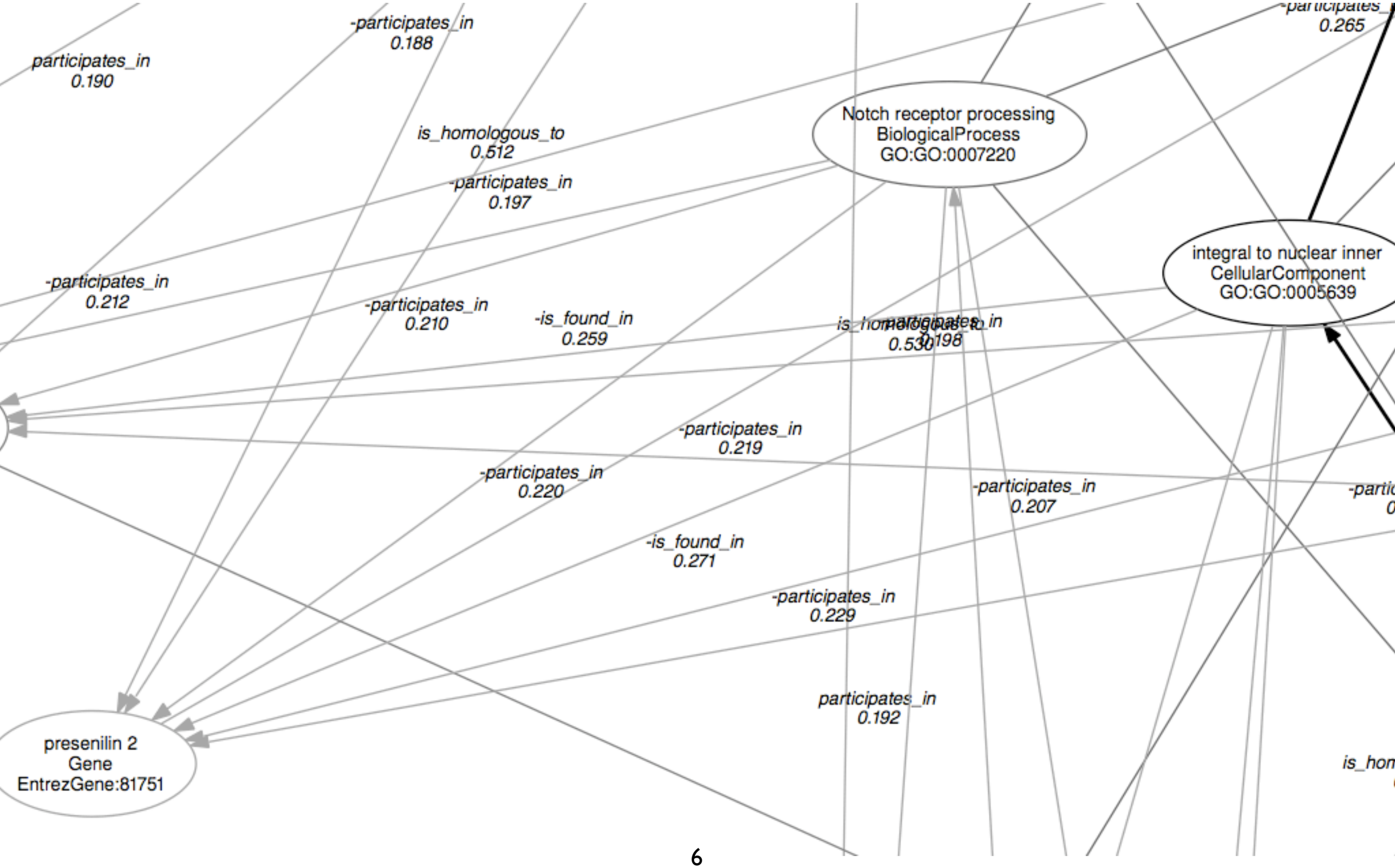
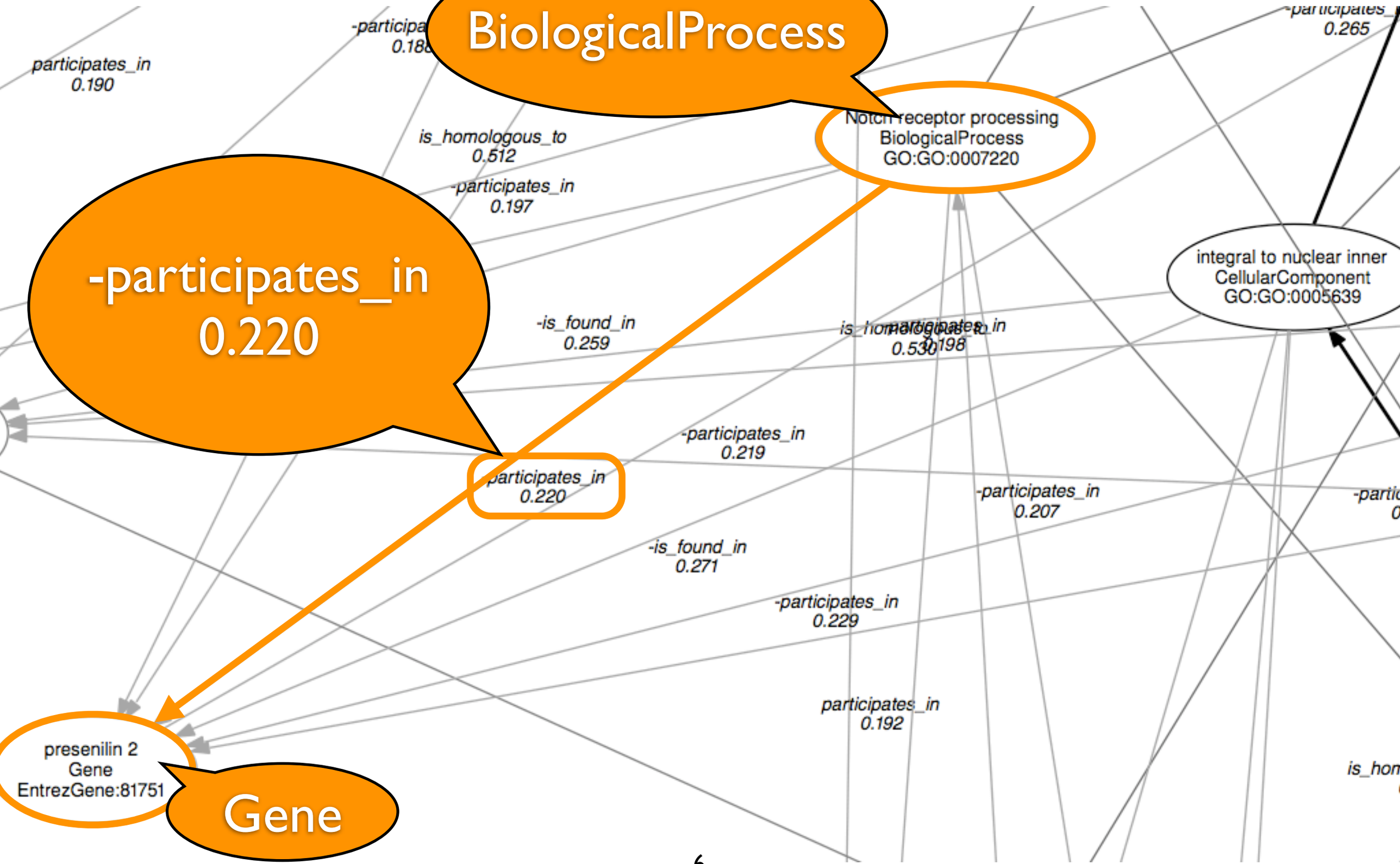


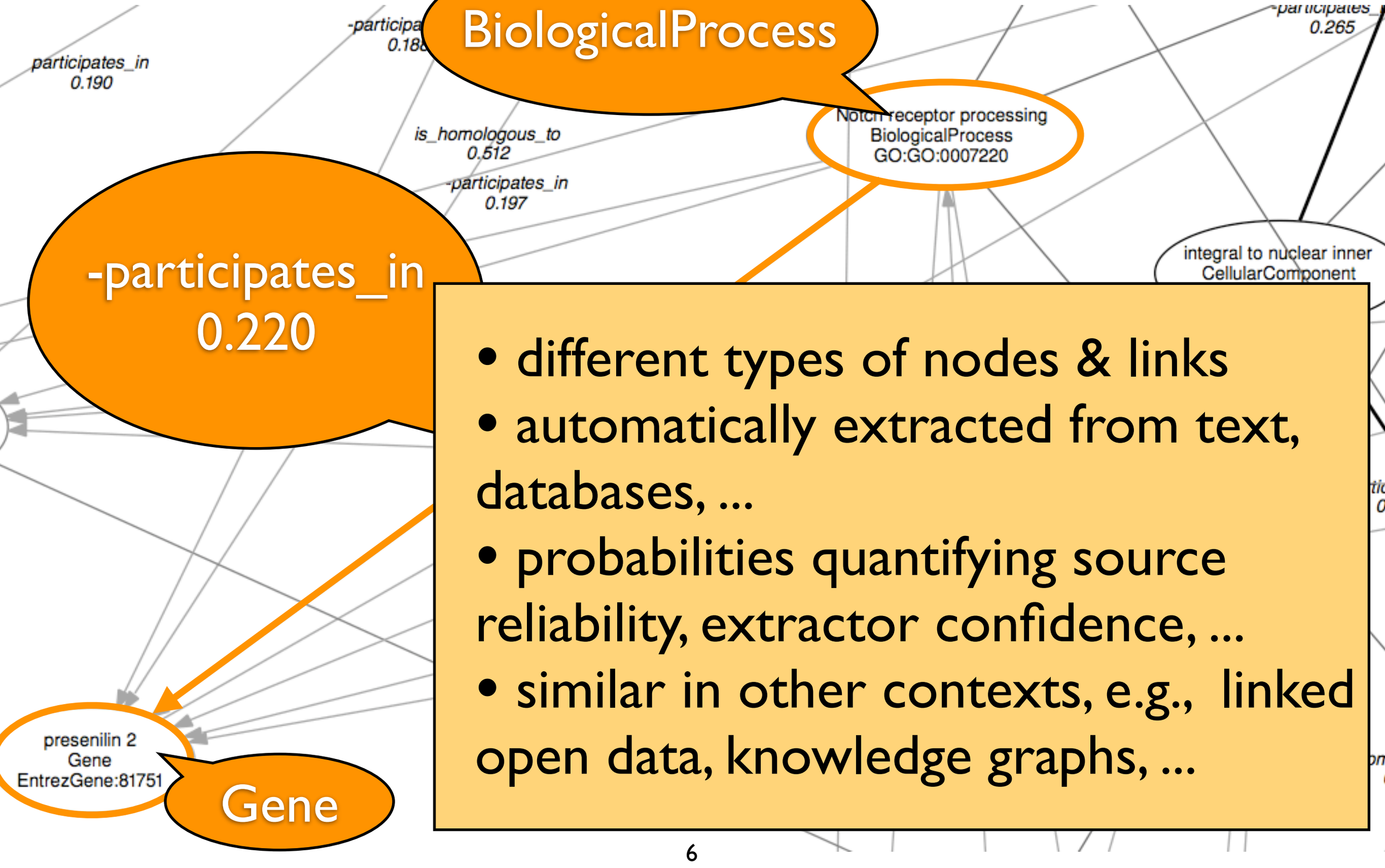
Notch receptor processing
BiologicalProcess
GO:GO:0007220

presenilin 2
Gene
EntrezGene:81751

Biomine Network



Biomine Network

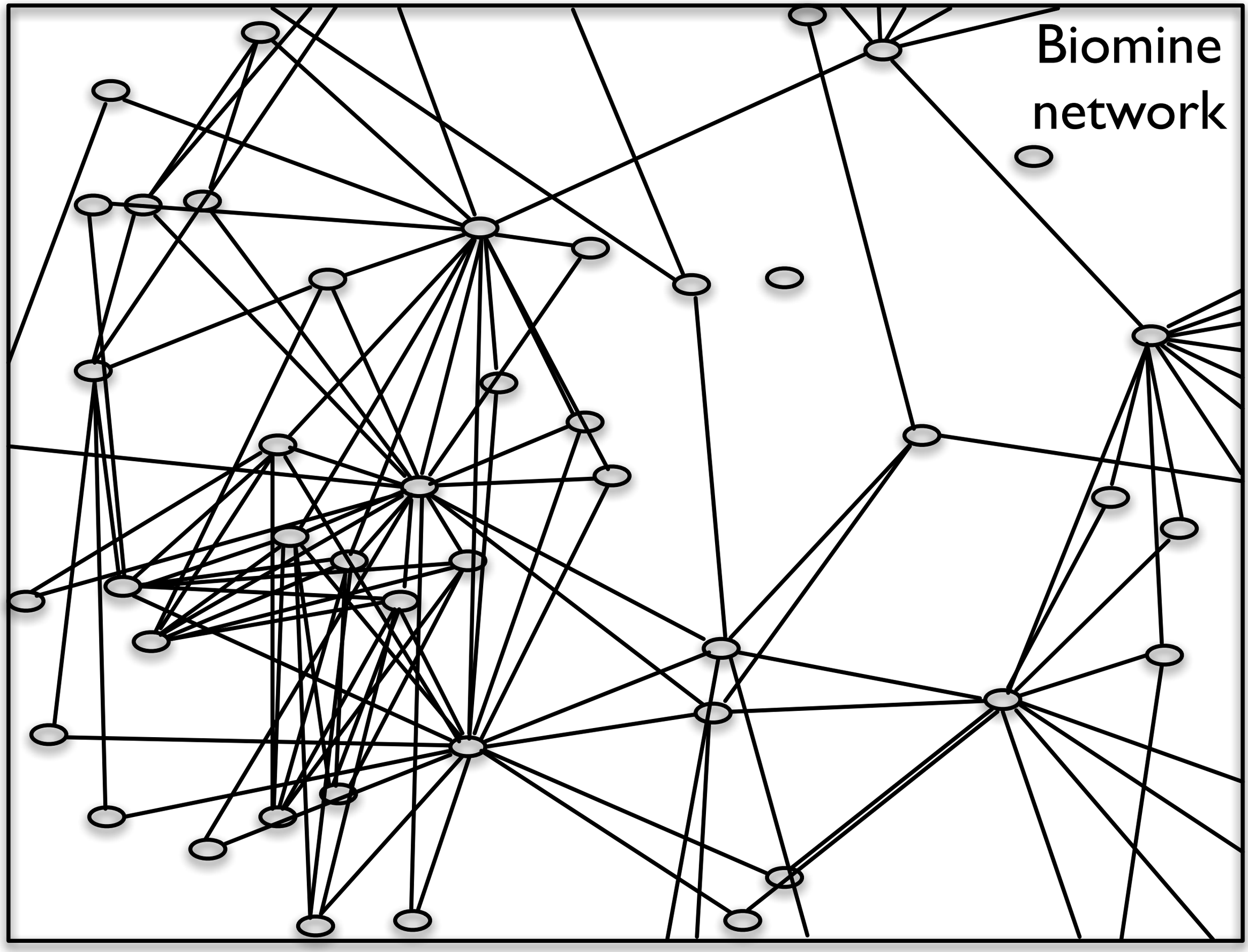


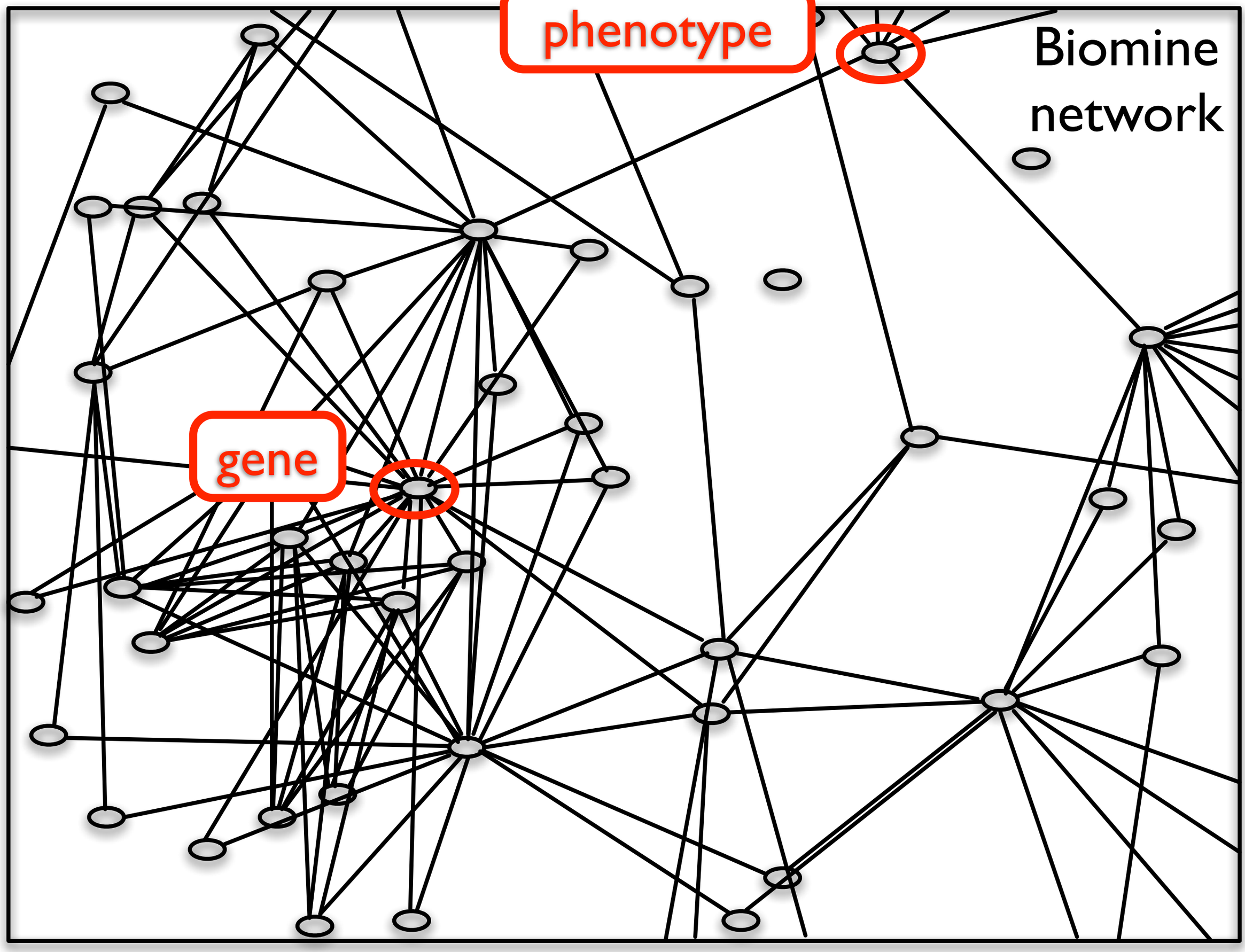
BiologicalProcess

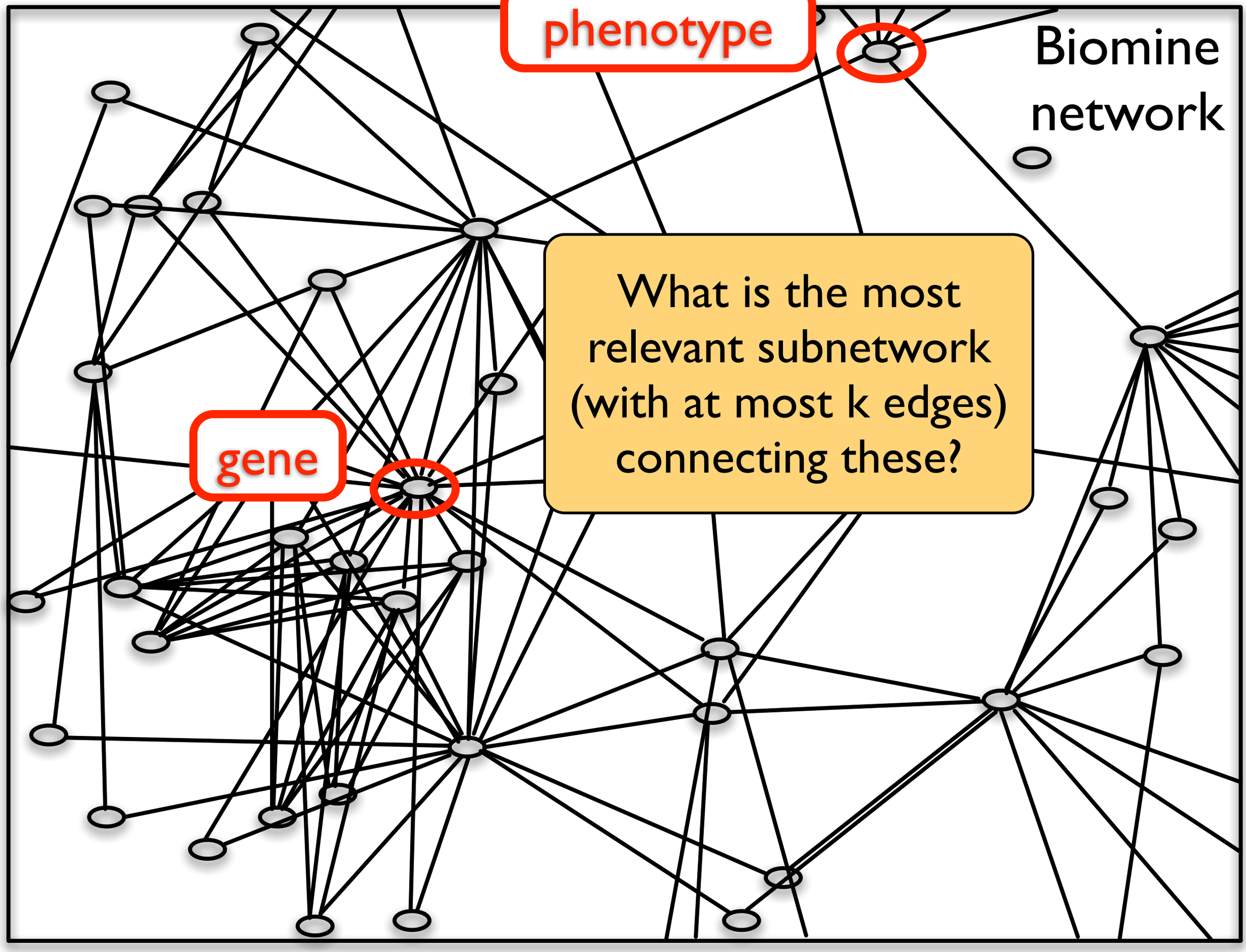
-participates_in
0.220

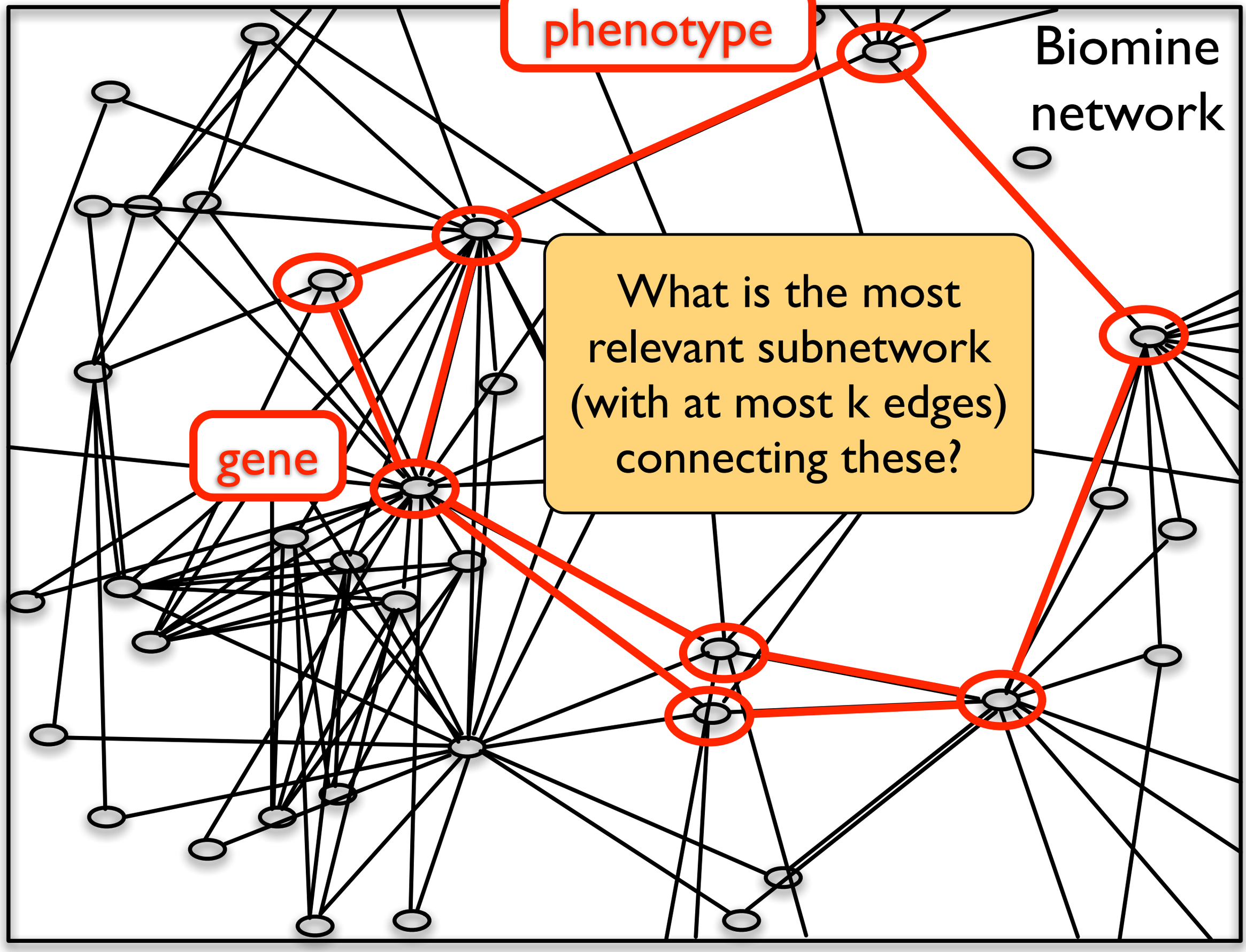
Gene

- different types of nodes & links
- automatically extracted from text, databases, ...
- probabilities quantifying source reliability, extractor confidence, ...
- similar in other contexts, e.g., linked open data, knowledge graphs, ...


Biomine
network


phenotype
Biomine
network
gene


phenotype
Biomine network
gene
What is the most relevant subnetwork (with at most k edges) connecting these?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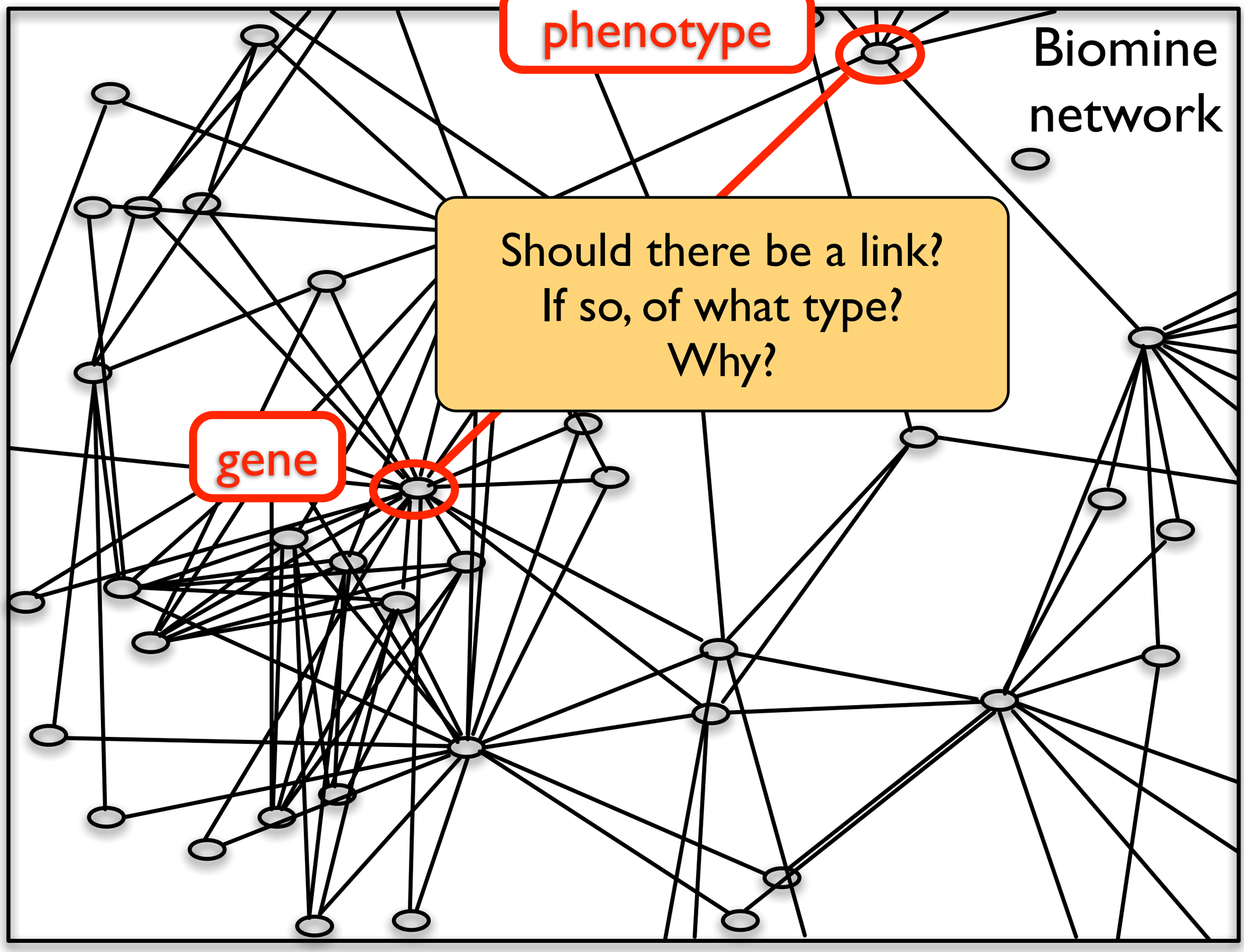
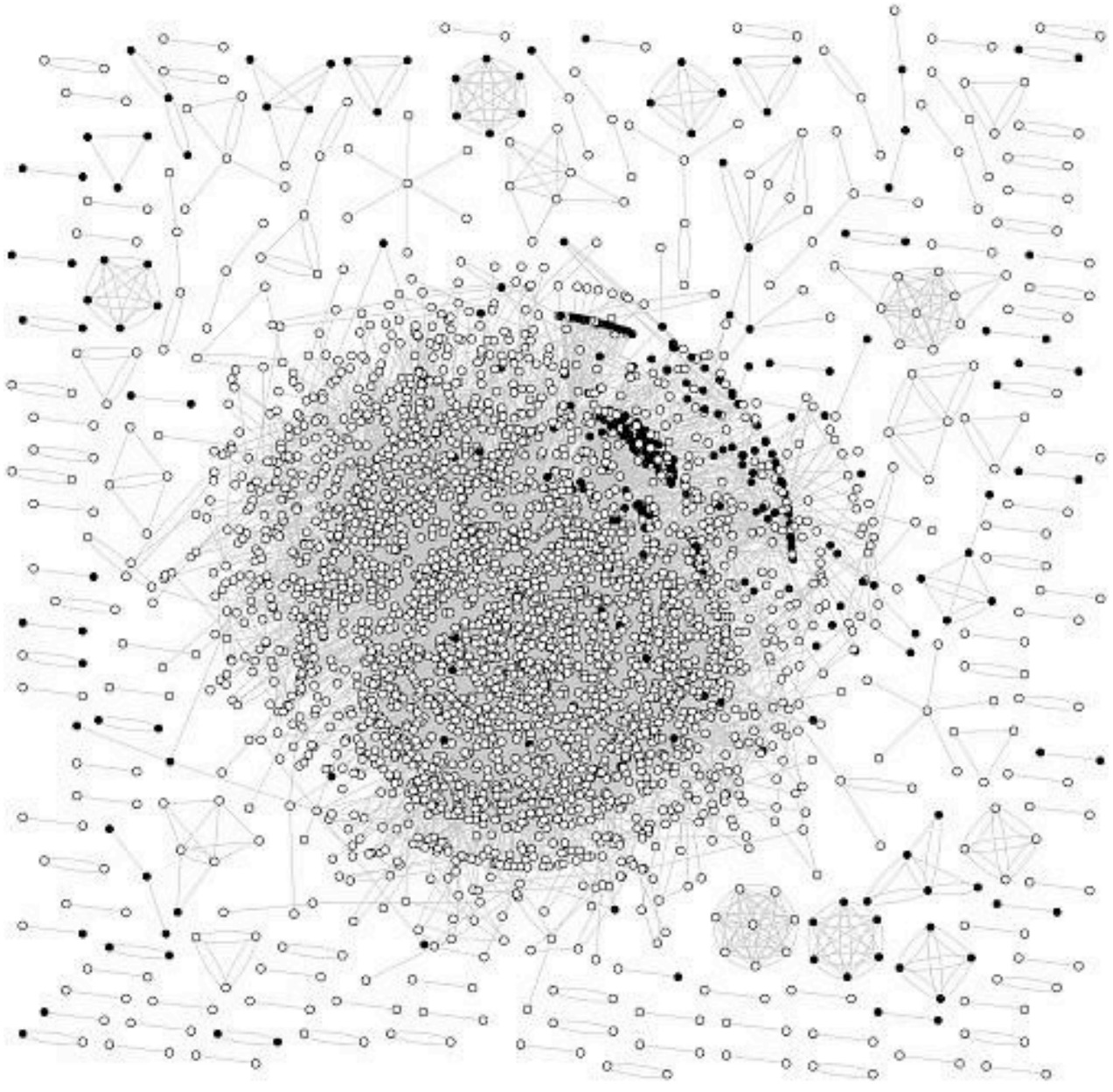

phenotype
Biomine network
Should there be a link?
If so, of what type?
Why?
gene

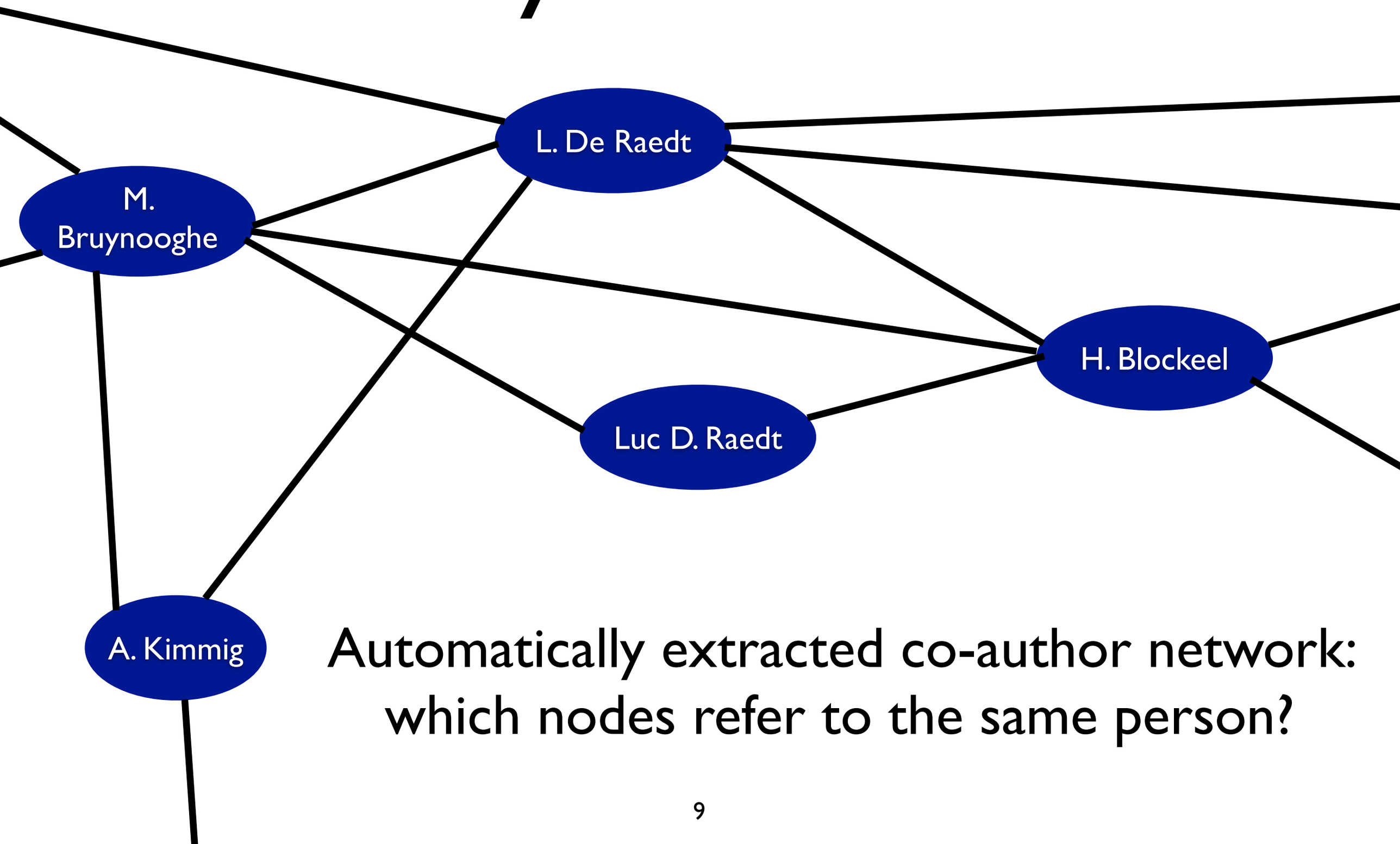
Node Classification



Can we predict the type of a node given information on its neighbors?

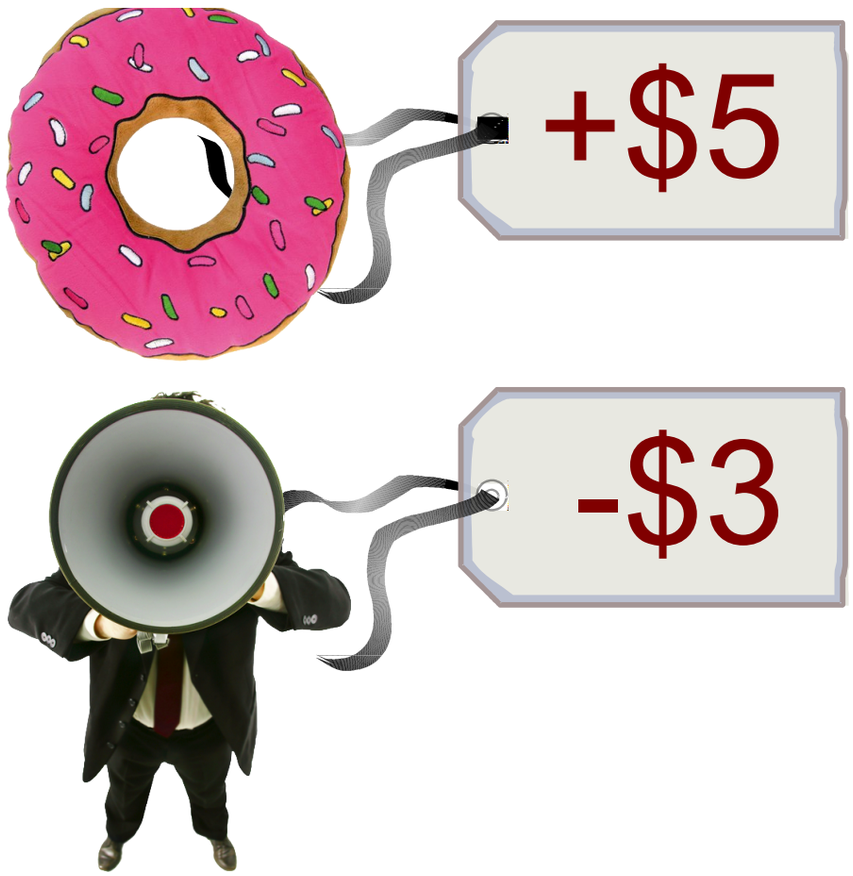
e.g., the type of a webpage given its links and the words on the page?

Entity Resolution

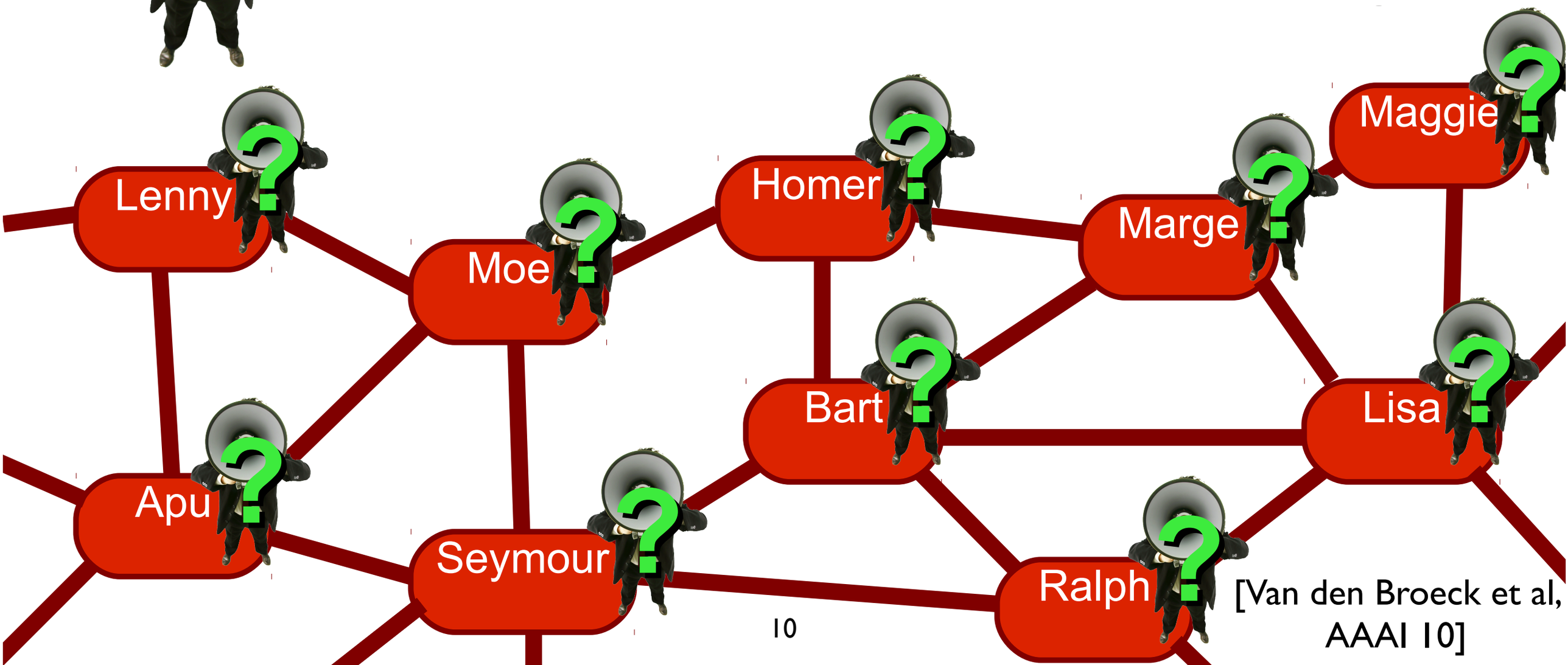


Automatically extracted co-author network: which nodes refer to the same person?

Viral Marketing

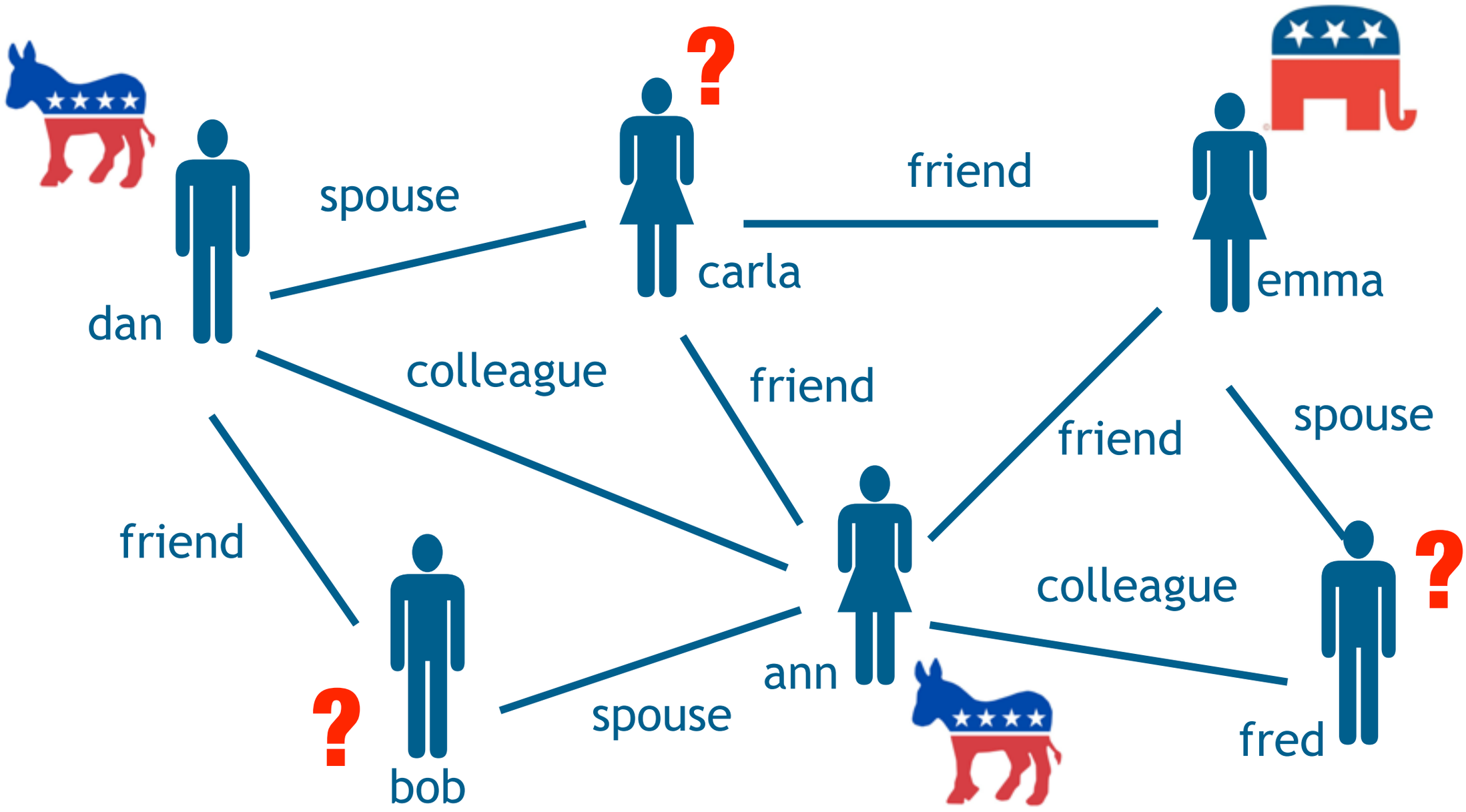


Which advertising strategy maximizes expected profit?



[Van den Broeck et al, AAAI 10]

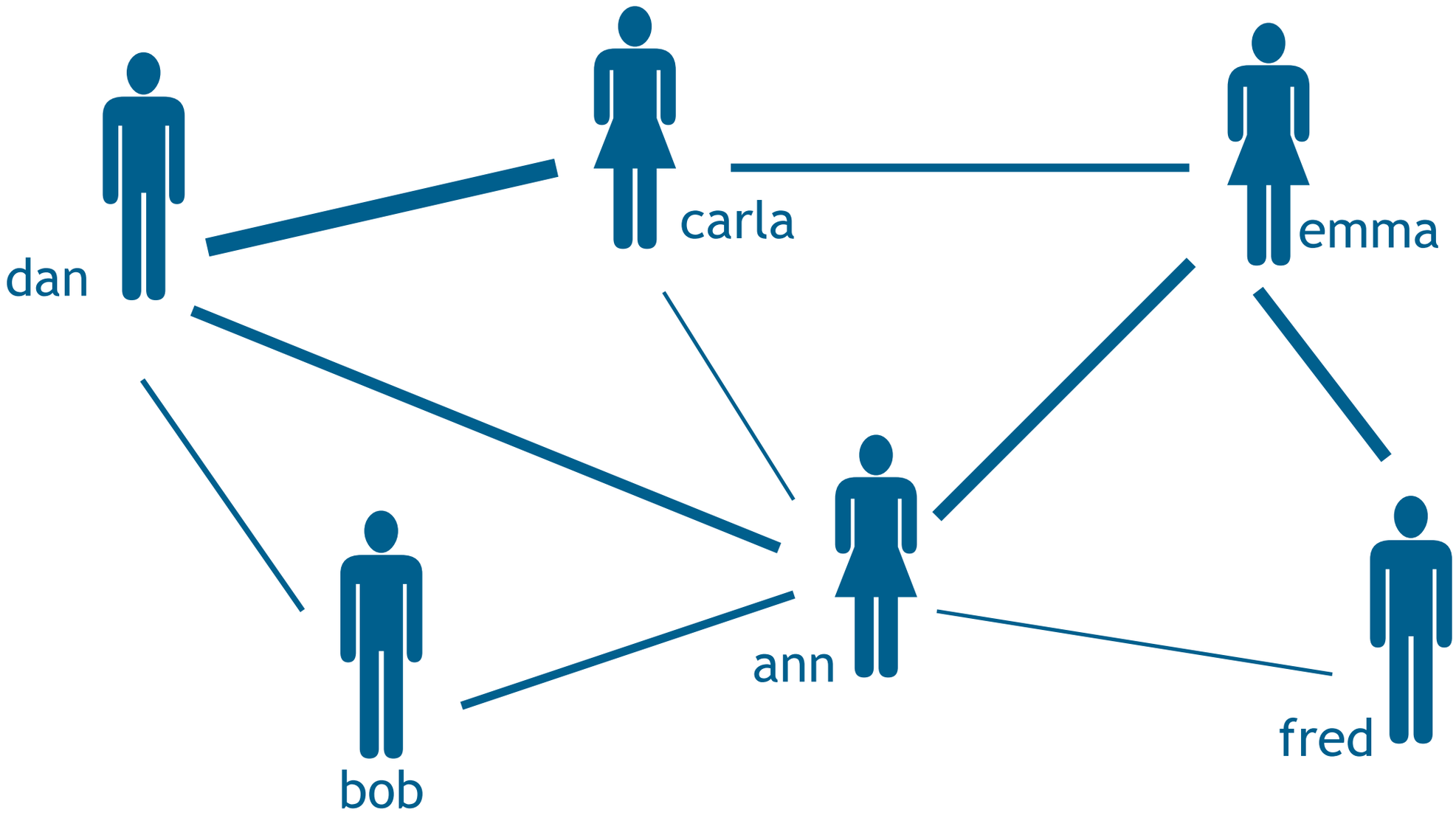
Voter Opinion Modeling



Can we predict preferences?

[Bach et al, NIPS 12]

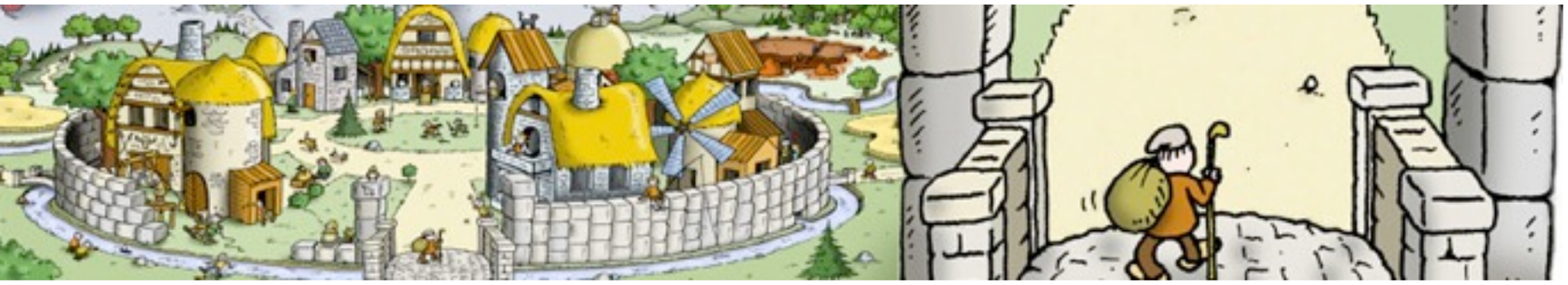
Social Trust



Can we predict strength of ties?

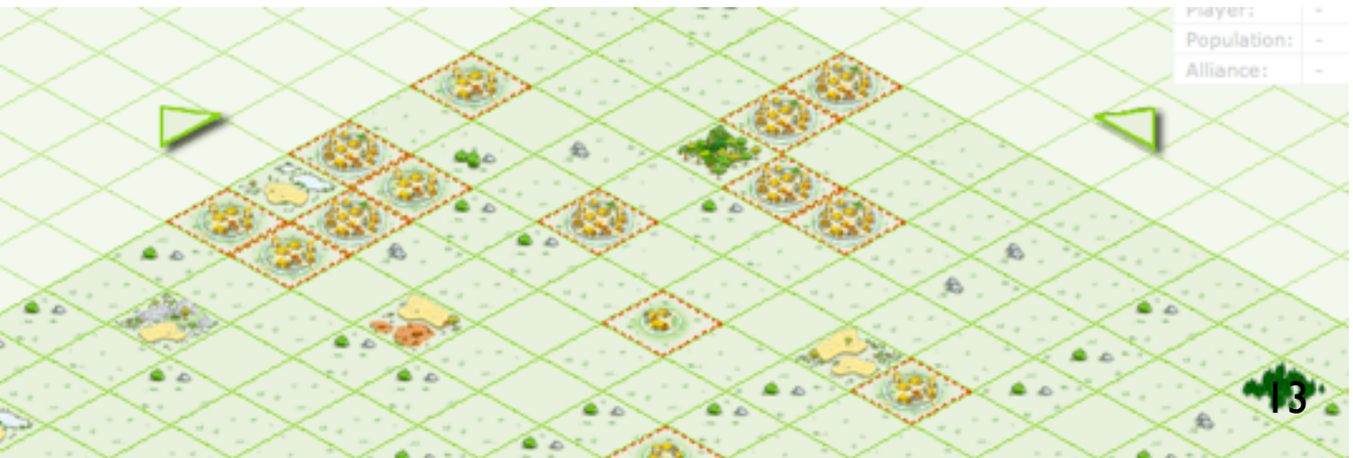
[Huang et al, SBP 13]

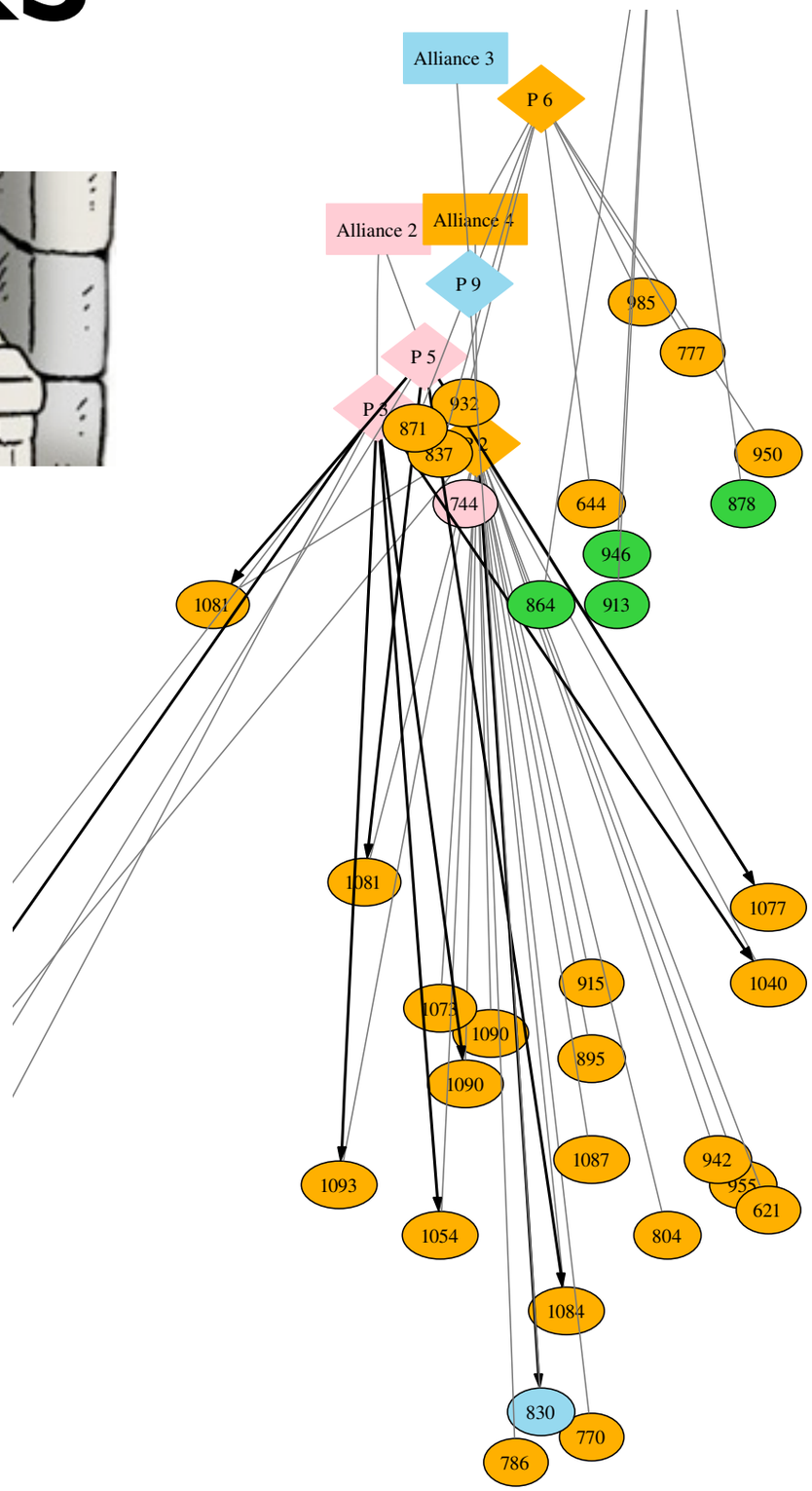
Dynamic networks



Travian: A massively multiplayer real-time strategy game

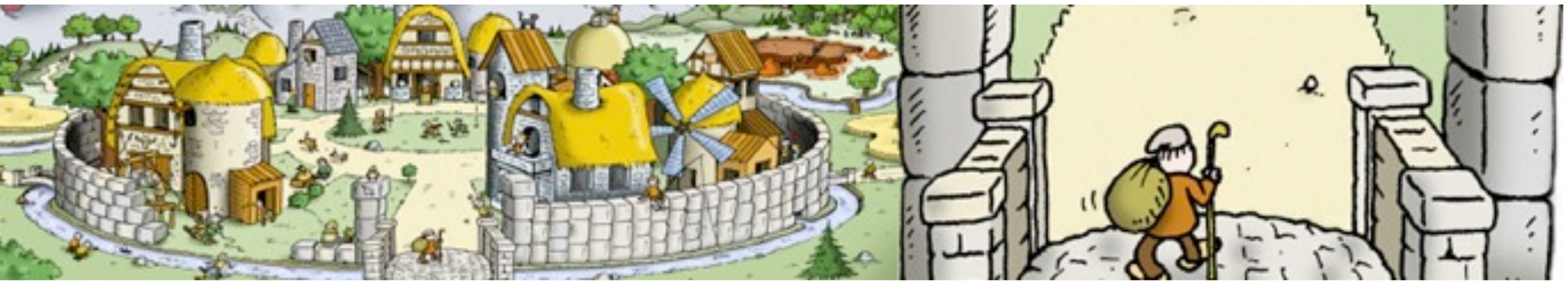
Can we build a model
of this world ?
Can we use it for playing
better ?





[Thon et al, MLJ 11]

Dynamic networks



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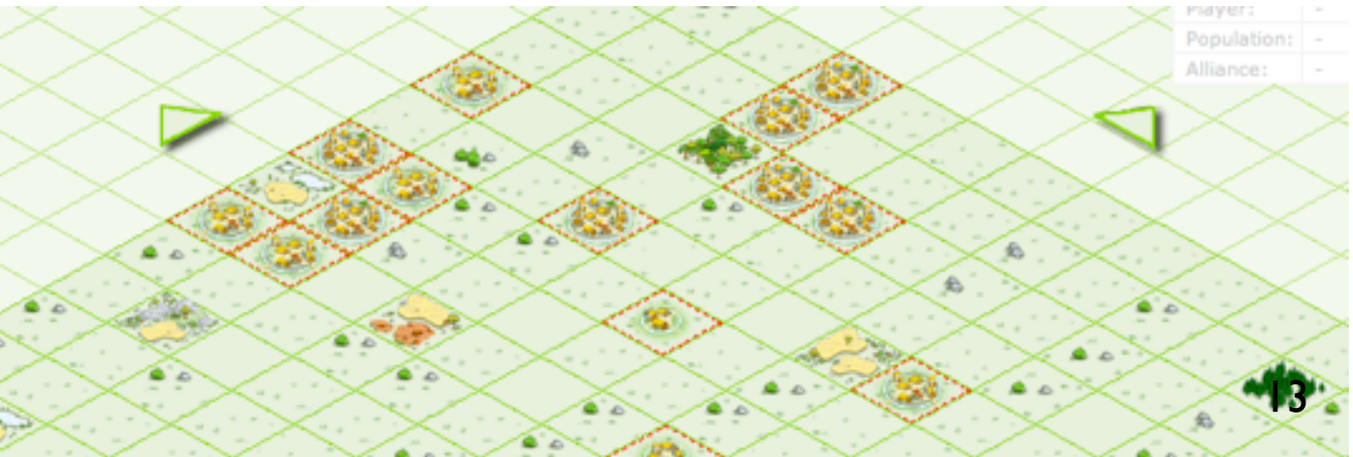
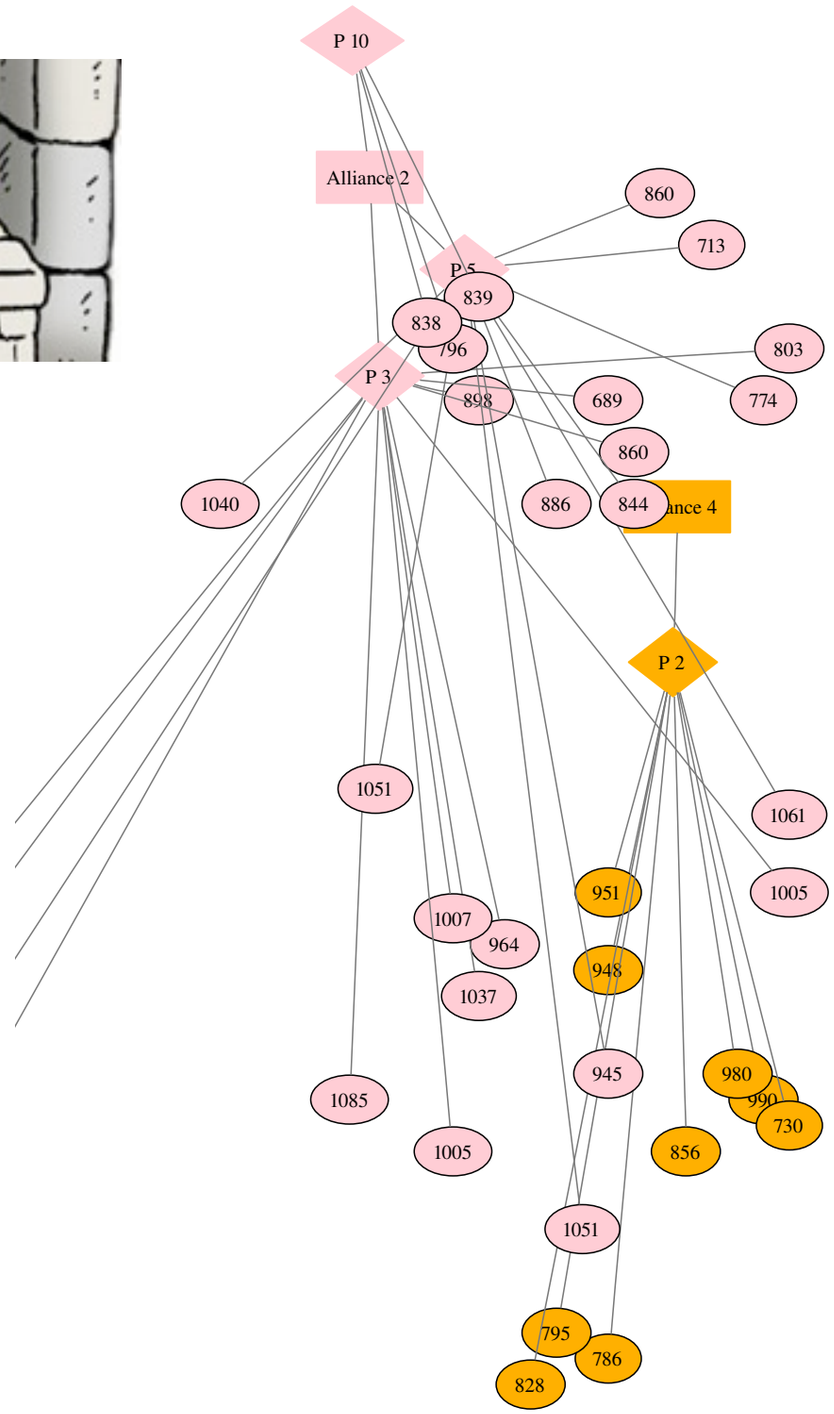
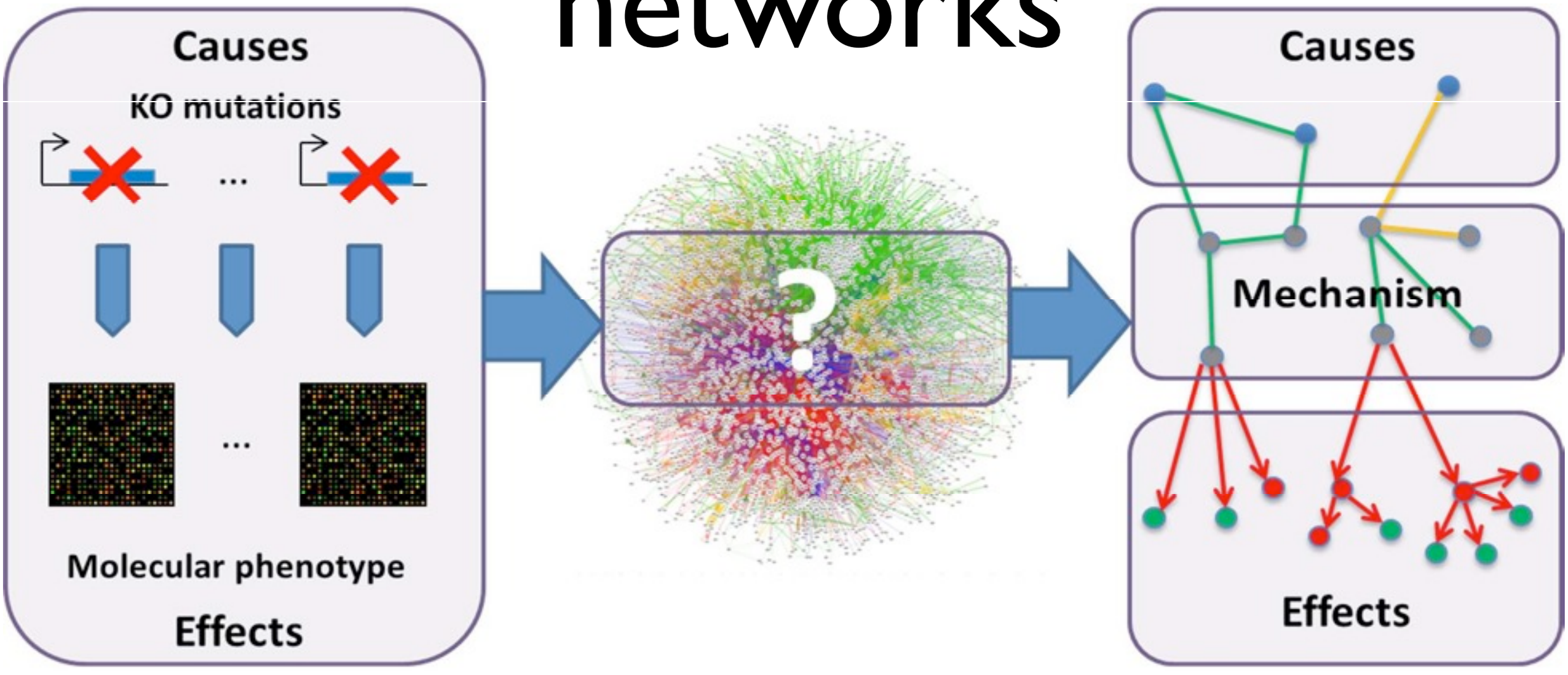




[Thon et al, MLJ 11]

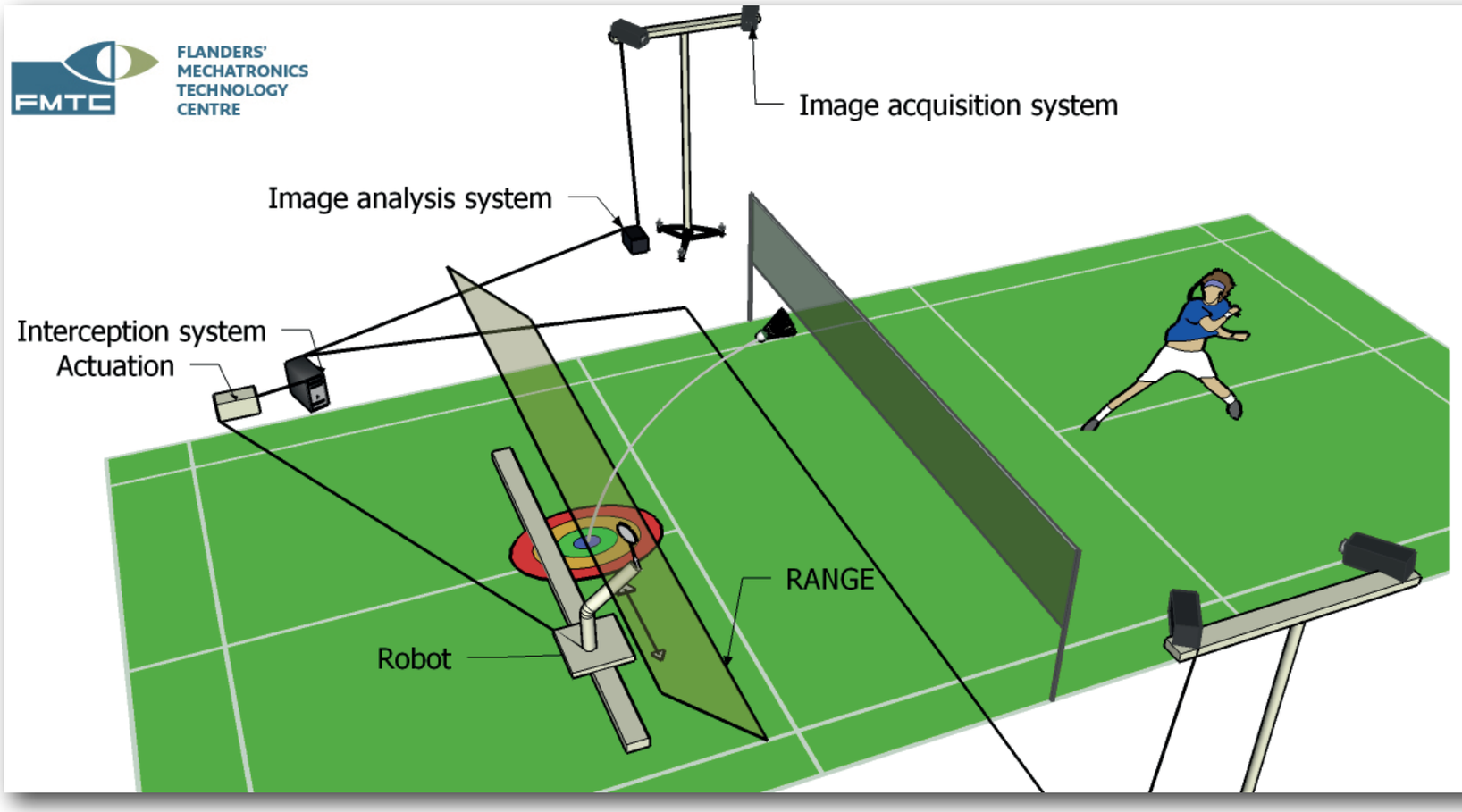
Molecular interaction networks

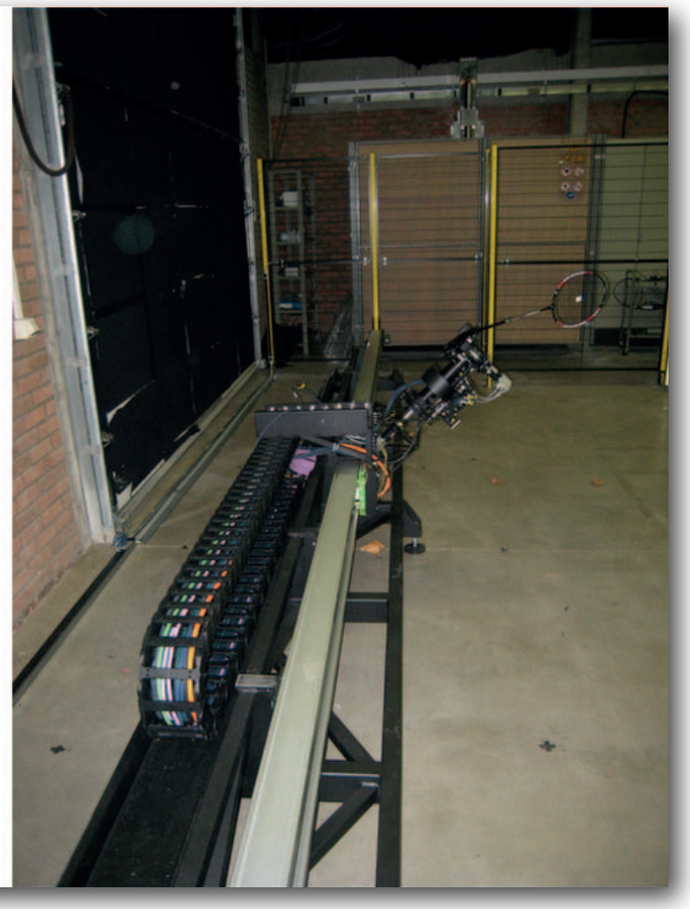


Can we find the mechanism connecting causes to effects?

[De Maeyer et al, Molecular Biosystems 13]

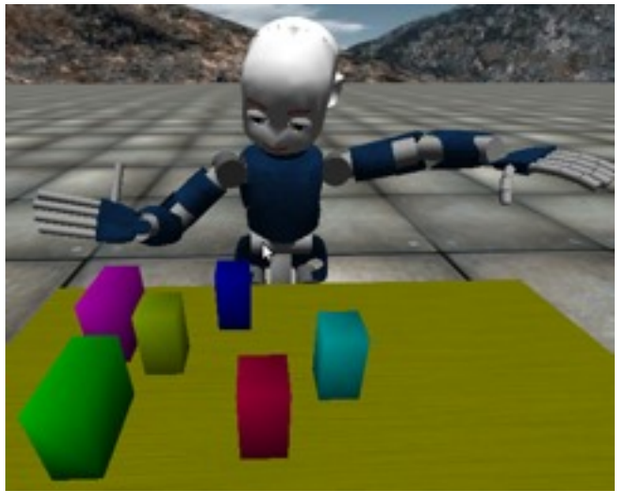
Diagnosing machine failures



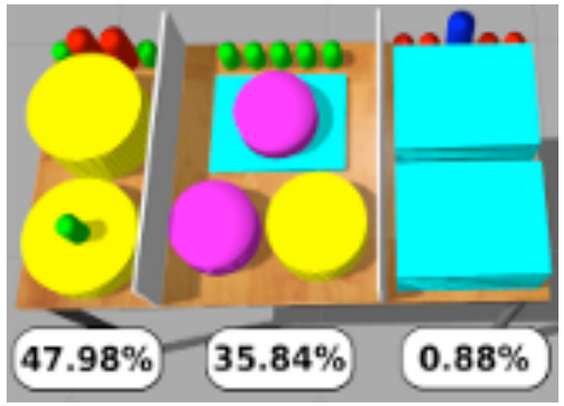


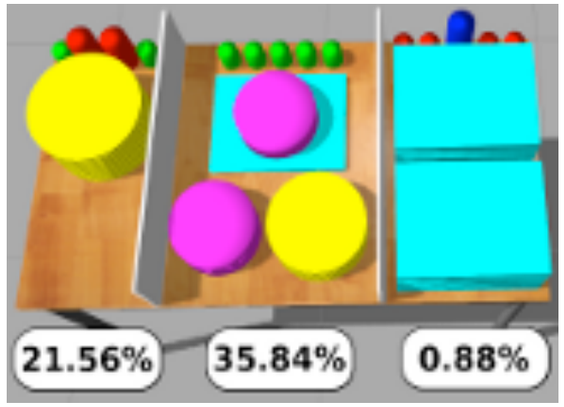
Can we build a model of the robot's working and use it to find causes of failures?

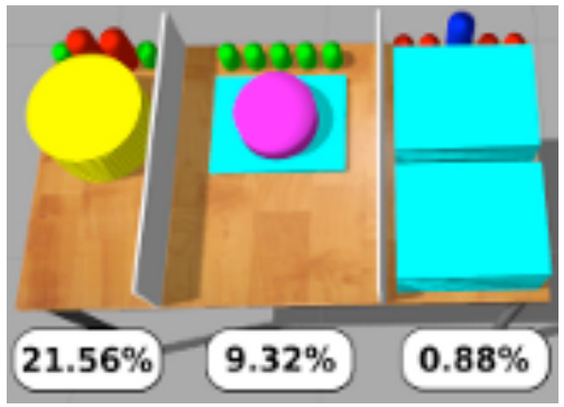
[Schramm, Meert and Driessens]



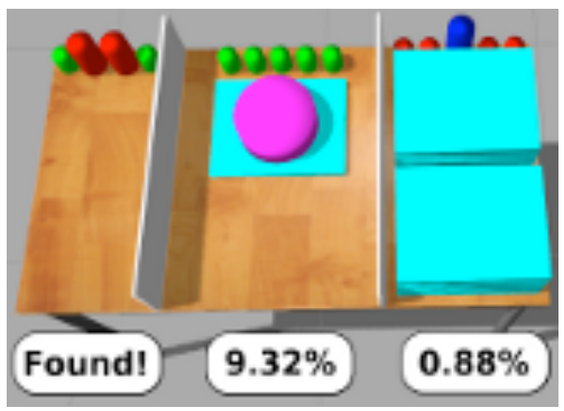
Robotics





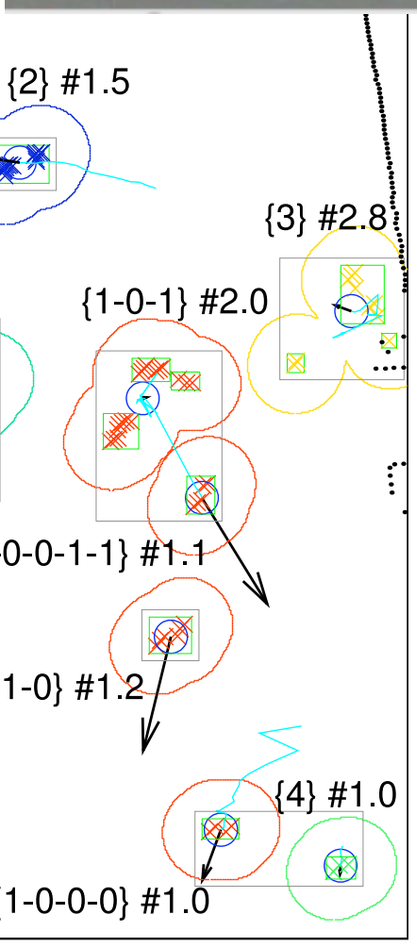


- How to achieve a specific configuration of objects on the shelf?
- Where's the orange mug?
- Where's something to serve soup in?



[Moldovan et al, ICRA 12, 14]

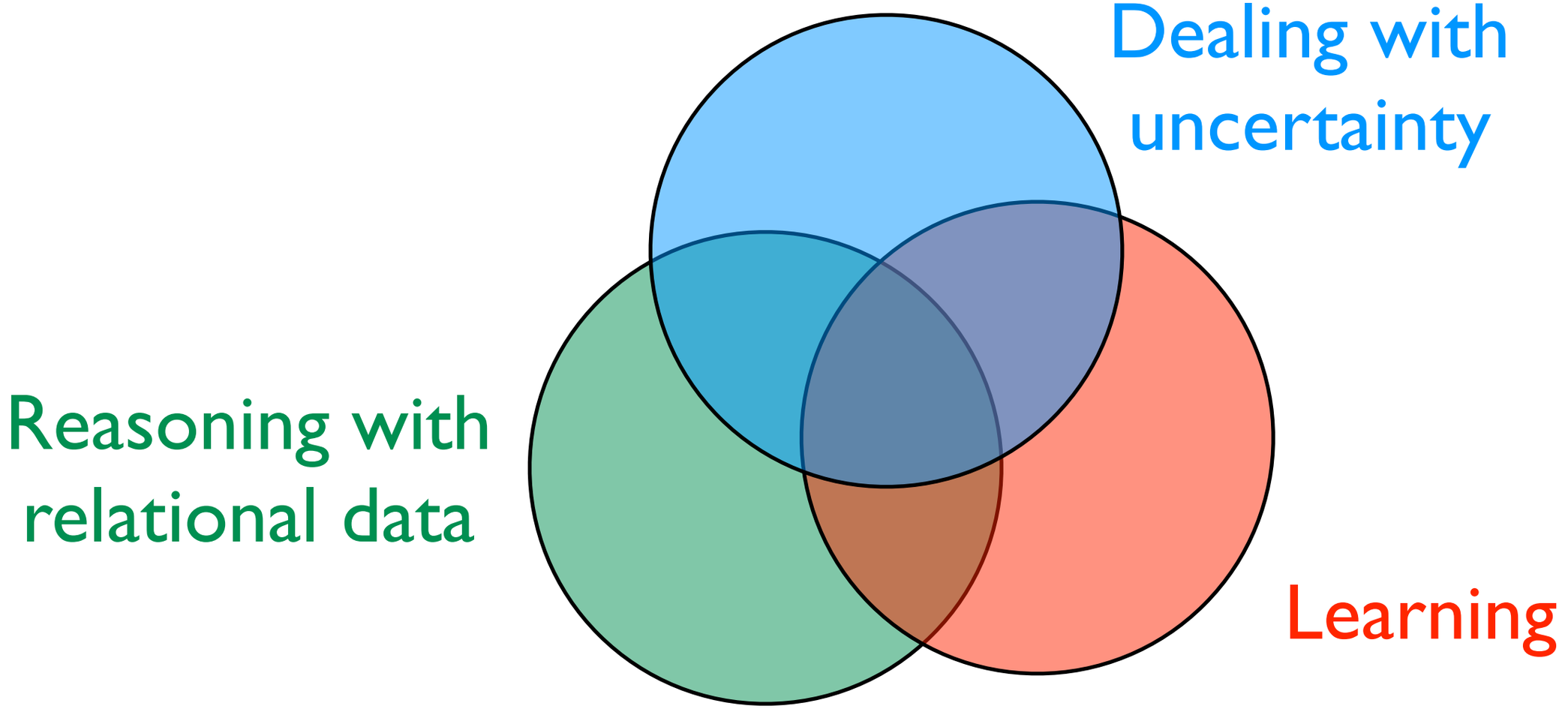
Analyzing Video Data



- Track people or objects over time? Even if temporarily hidden?
- Recognize activities?
- Infer object properties?

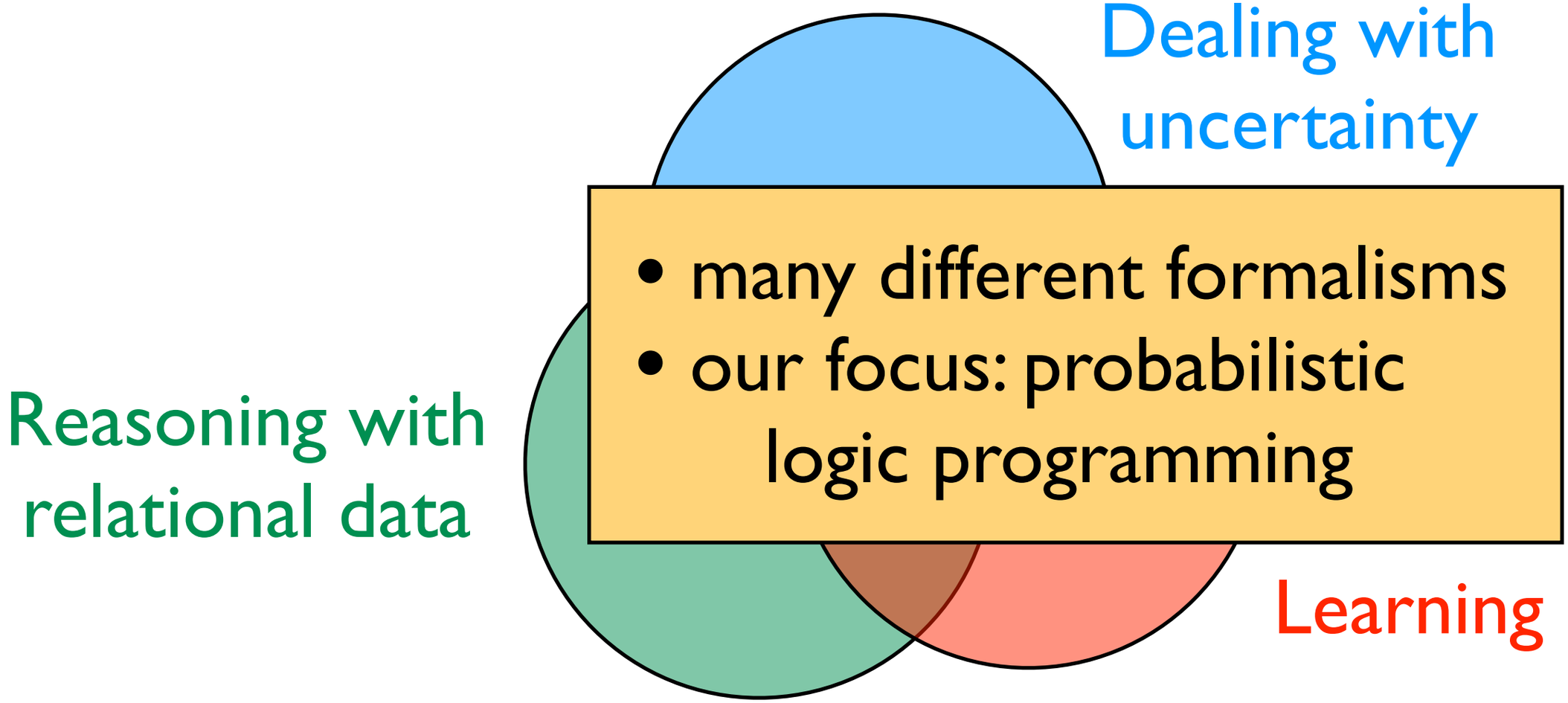
[Skarlatidis et al, TPLP 14;
Nitti et al, IROS 13, ICRA 14]

Common theme



Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

Common theme



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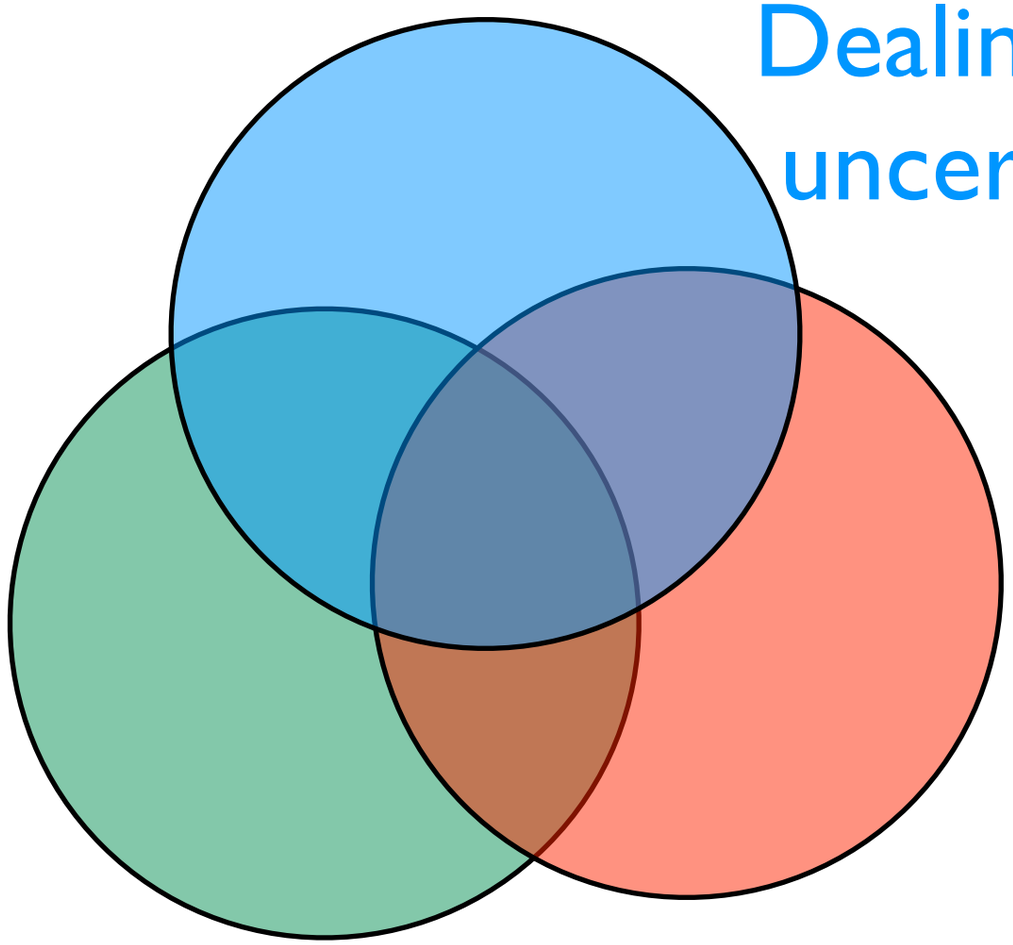
ProbLog

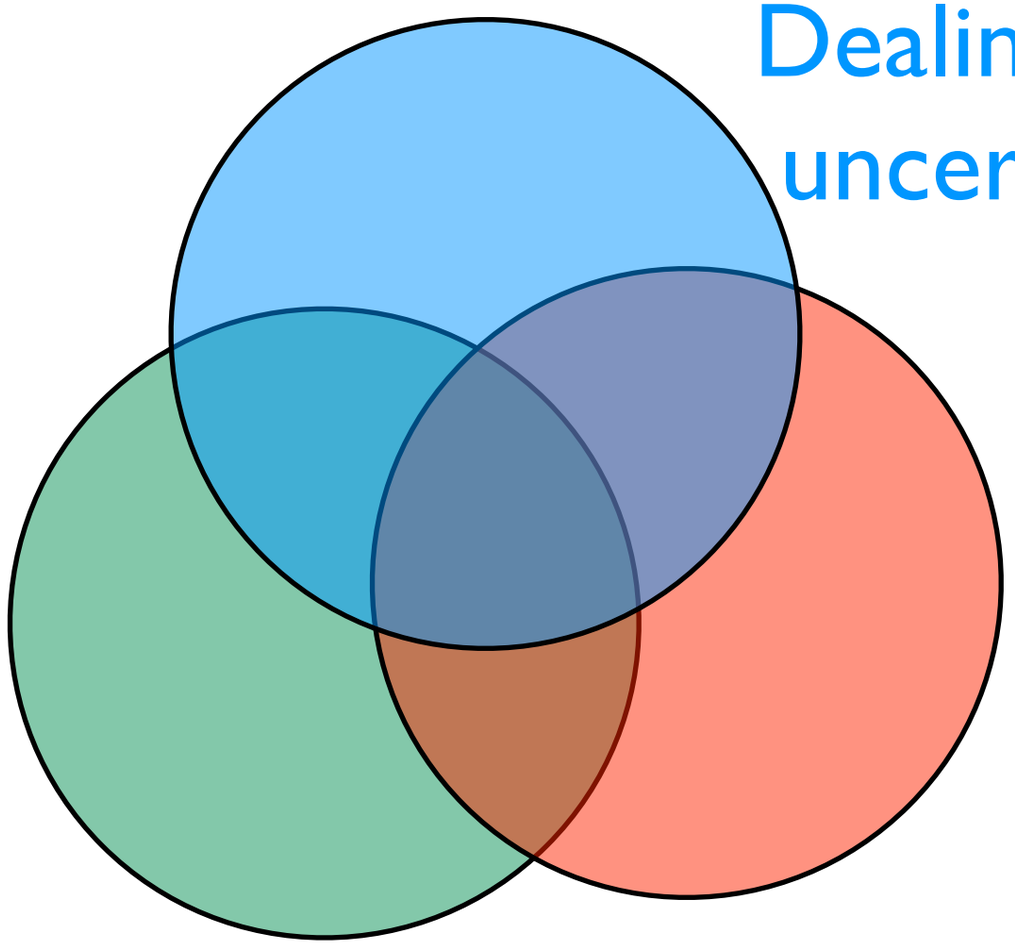
probabilistic Prolog



http://dtai.cs.kuleuven.be/problog/

ProbLog

probabilistic Prolog




```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X), smokes(Y).
```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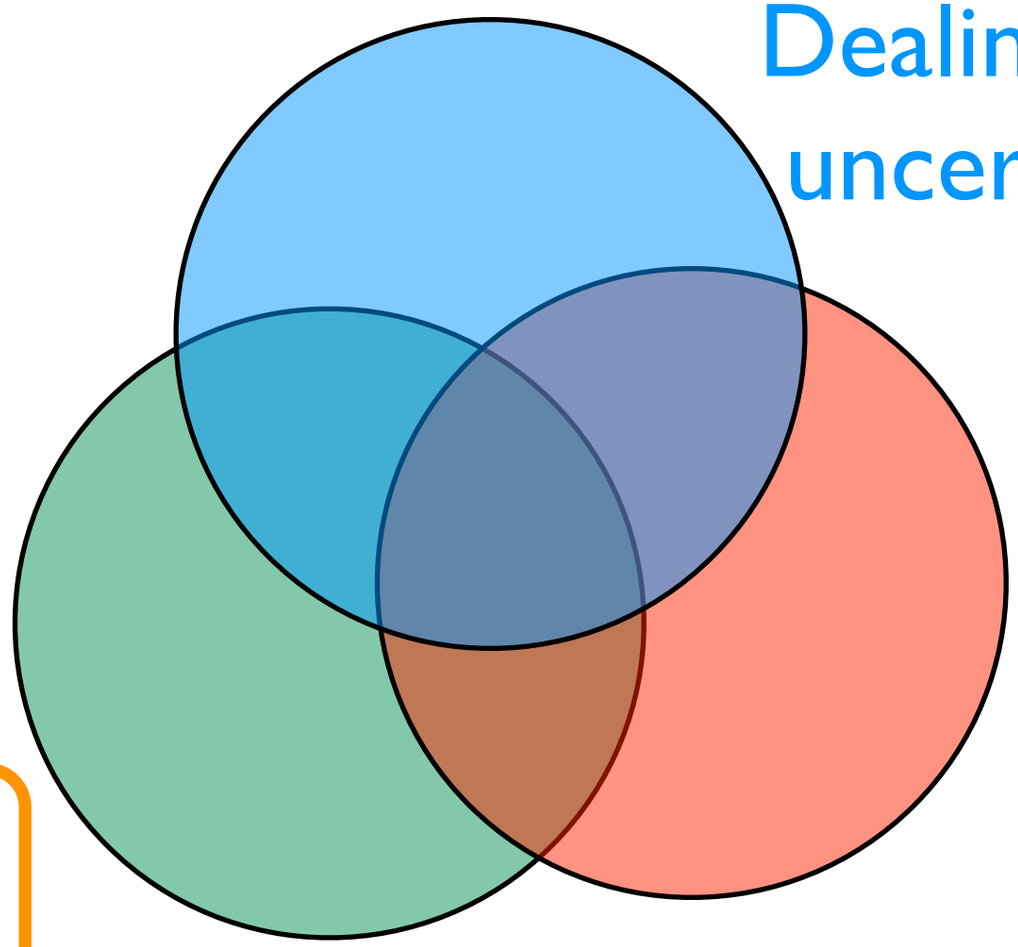
ProbLog

probabilistic Prolog




```
stress(ann).
influences(ann,bob).
influences(bob,carl).
```

one world

```
smokes(X) :- stress(X).
smokes(X) :-
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```


http://dtai.cs.kuleuven.be/problog/

ProbLog

probabilistic Prolog

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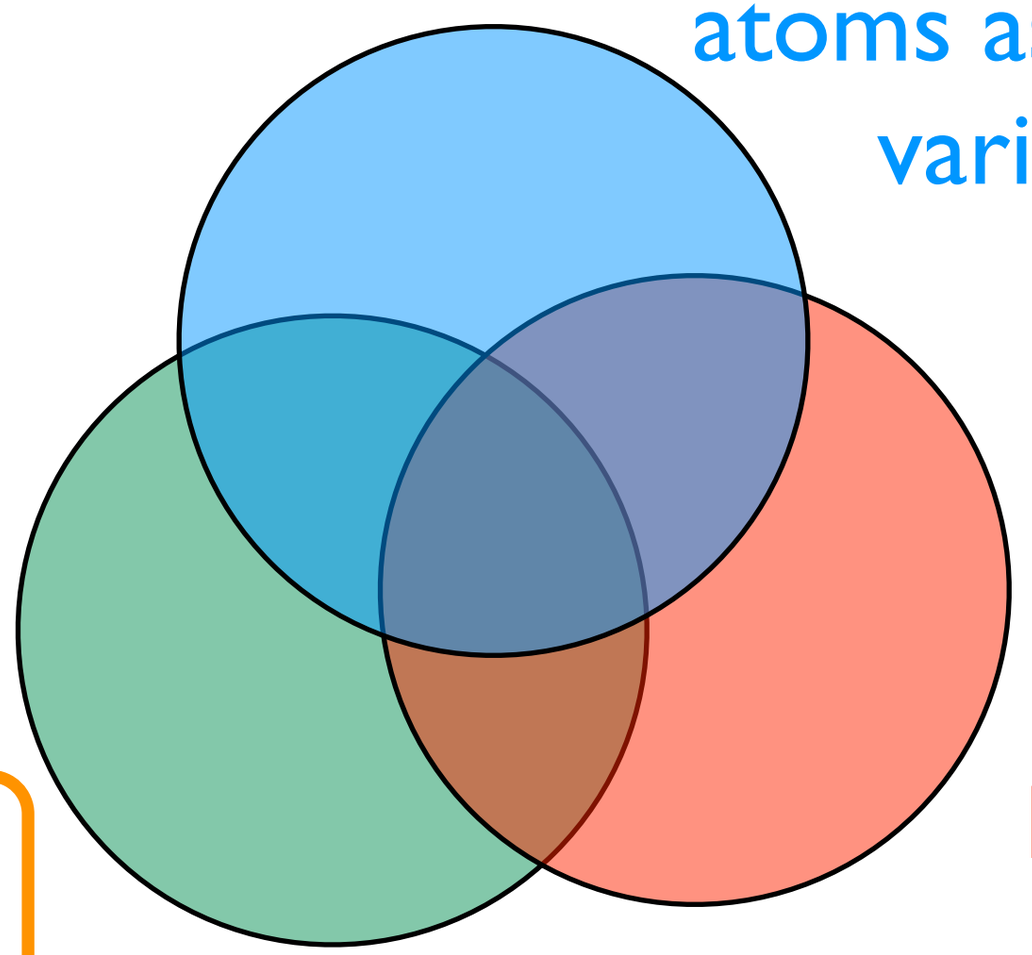
0.8::stress(ann).
0.6::influences(ann,bob).
0.2::influences(bob,carl).
```

atoms as random variables



Prolog / logic programming

Learning

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probabilistic Prolog

several possible worlds

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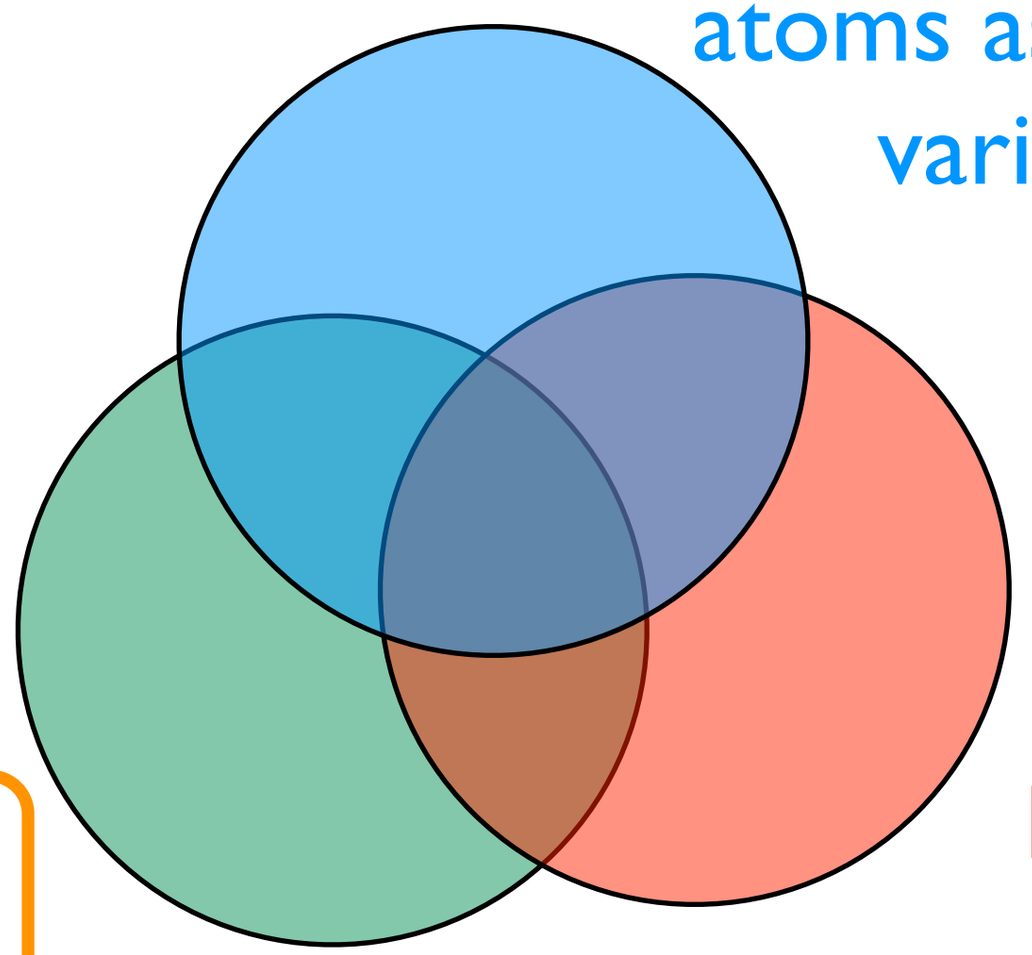

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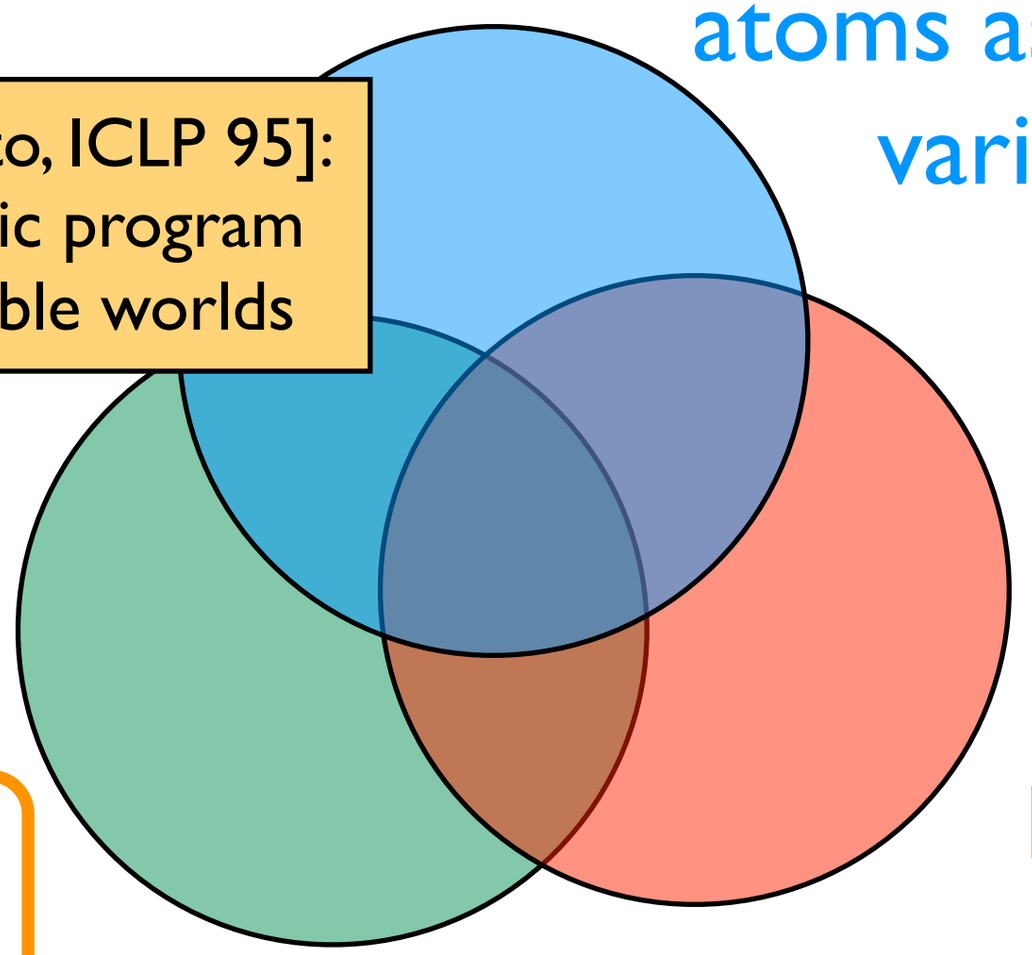
atoms as random variables

Distribution Semantics [Sato, ICLP 95]:
probabilistic choices + logic program
→ distribution over possible worlds



Prolog / logic programming

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probabilistic Prolog

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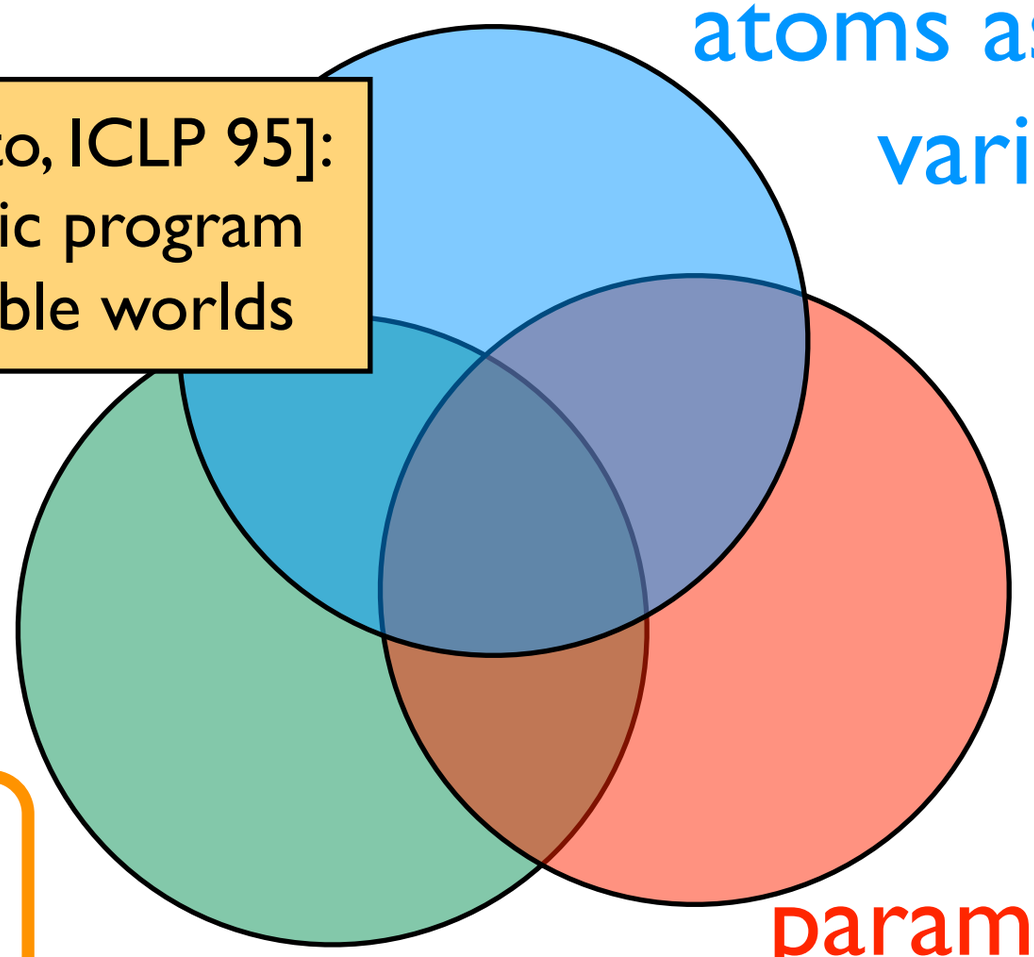

Prolog / logic programming

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one world

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smokes(X) :- stress(X).
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```

parameter learning, adapted relational learning techniques

http://dtai.cs.kuleuven.be/problog/

Probabilistic Prologs: Two Views

- Distribution semantics:
  - probability distribution over interpretations
  - degree of belief
- Stochastic Logic Programs (SLPs):
  - probability distribution over query answers
  - like in probabilistic grammars

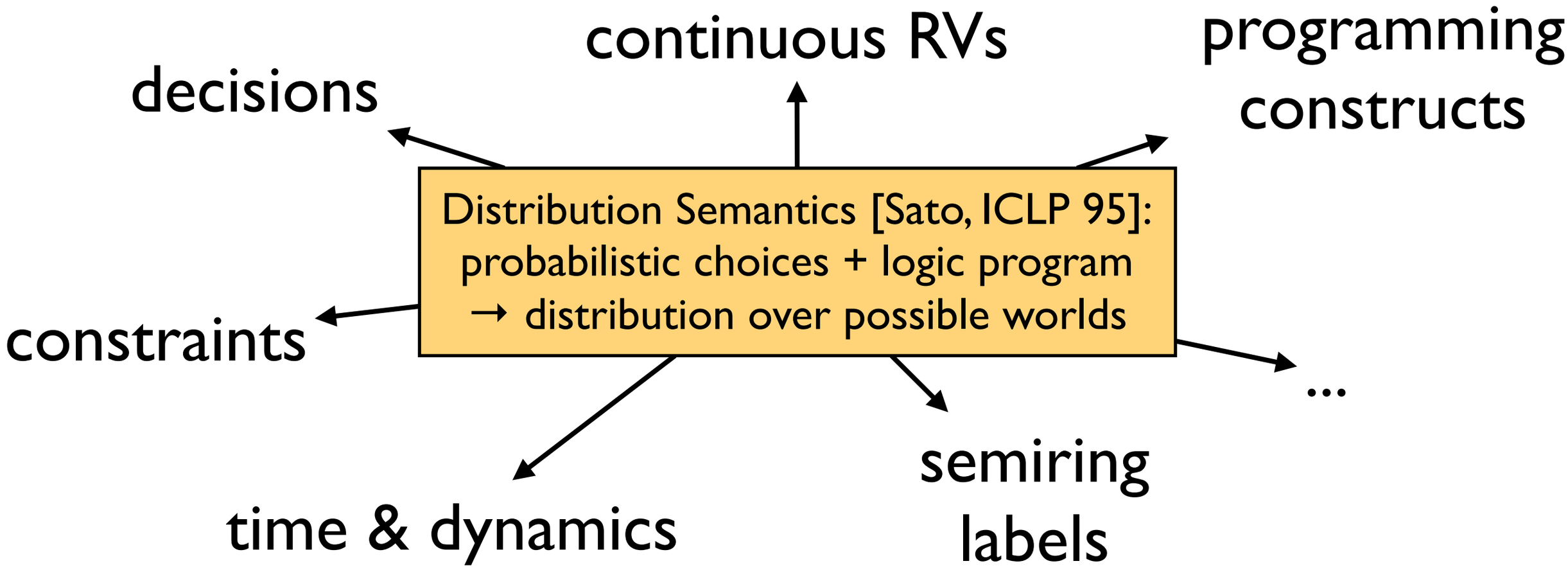
Probabilistic Logic Programming

Distribution Semantics [Sato, ICLP 95]:
probabilistic choices + logic program
→ distribution over possible worlds

e.g., PRISM, ICL, ProbLog, LPADs, CP-logic, ...

- multi-valued switches
- probabilistic alternatives
- probabilistic facts
- annotated disjunctions
- causal-probabilistic laws

Extensions of basic PLP



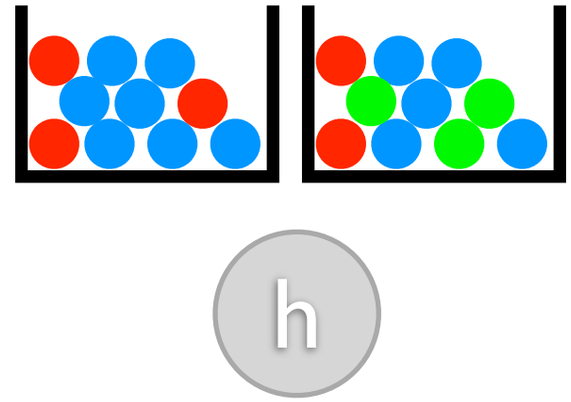
Roadmap

- Modeling
- Reasoning
- Language extensions
- Advanced topics

... with some detours on the way

ProbLog by example:

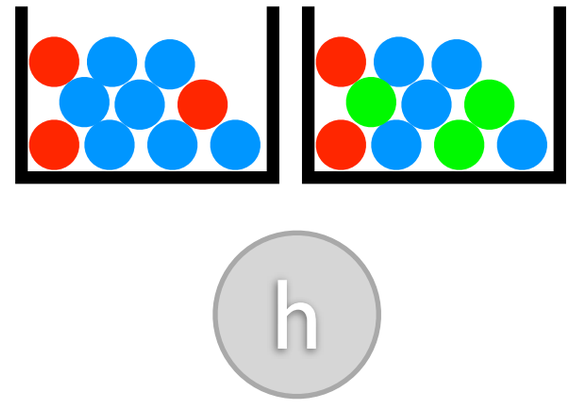
A bit of gambling



- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

ProbLog by example:

A bit of gambling



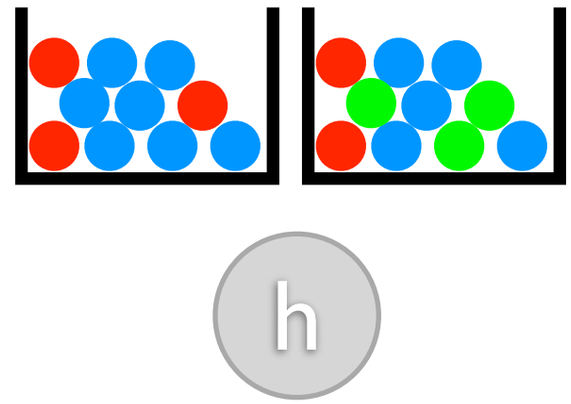
- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

```
0.4 :: heads.
```

probabilistic fact: heads is true with probability 0.4 (and false with 0.6)

ProbLog by example:

A bit of gambling



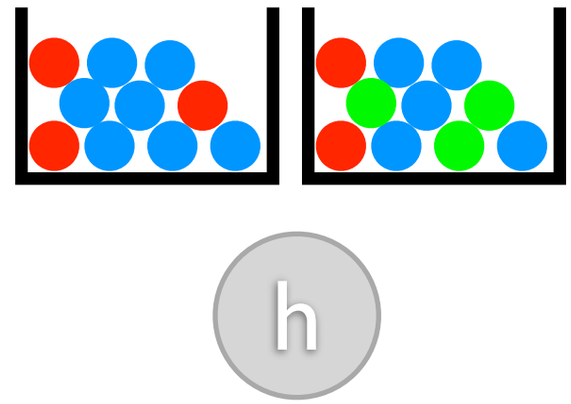
- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

```
0.4 :: heads.
0.3 :: col(1,red); 0.7 :: col(1,blue) <- true.
```

annotated disjunction: first ball is red with probability 0.3 and blue with 0.7

ProbLog by example:

A bit of gambling



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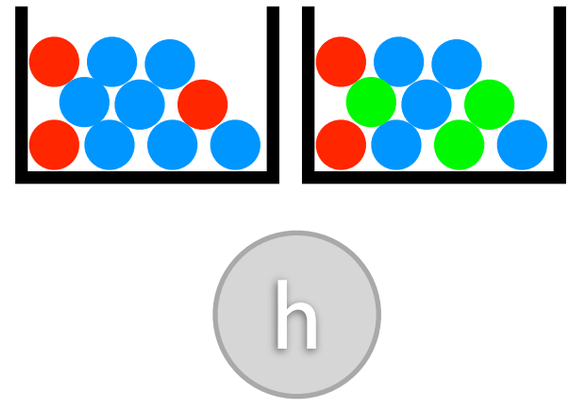
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue) <- true.
0.2 :: col(2,red); 0.3 :: col(2,green);
                   0.5 :: col(2,blue) <- true.
```

annotated disjunction: second ball is red with probability 0.2, green with 0.3, and blue with 0.5

ProbLog by example:

A bit of gambling



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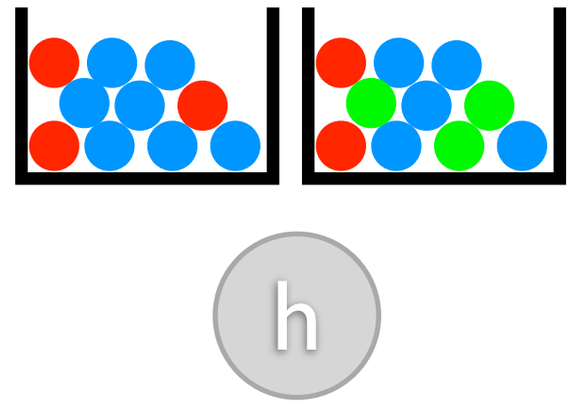
win :- heads, col(_,red).
```

logical rule encoding background knowledge

ProbLog by example:

A bit of gambling



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win :- heads, col(_,red).
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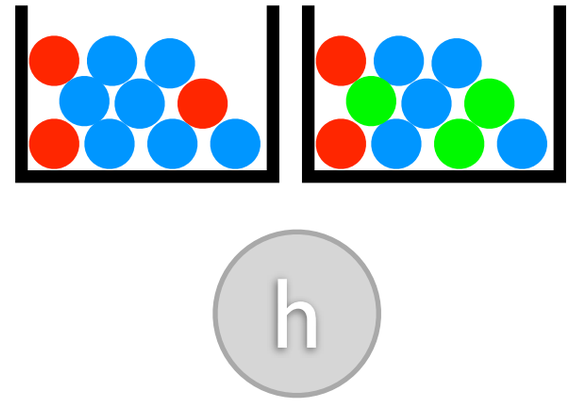

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```

probabilistic choices

```
win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

consequences

Questions

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue) <- true.
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win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

- Probability of **`win`**?
- Probability of **`win`** given **`col(2,green)`**?
- Most probable world where **`win`** is true?

Questions

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marginal probability

- Probability of `win`?

query

- Probability of `win` given `col(2,green)`?
- Most probable world where `win` is true?

Questions

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0.4 :: heads.

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```

marginal probability

- Probability of **`win`**?

conditional probability

- Probability of **`win`** given **`col(2,green)`**?

evidence

- Most probable world where **`win`** is true?

Questions

```
0.4 :: heads.

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MPE inference

Possible Worlds

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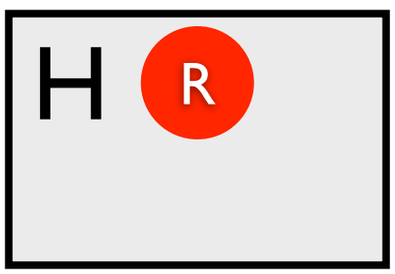
```

0.4 ×0.3



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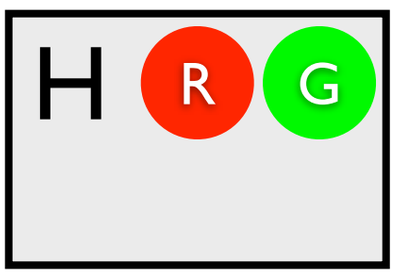

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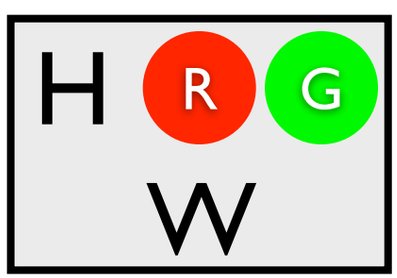

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$0.4 \times 0.3 \times 0.3$ $(1-0.4)$

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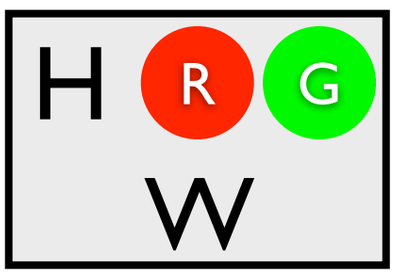

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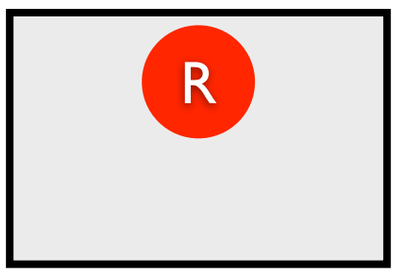

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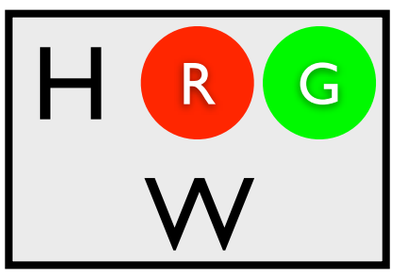
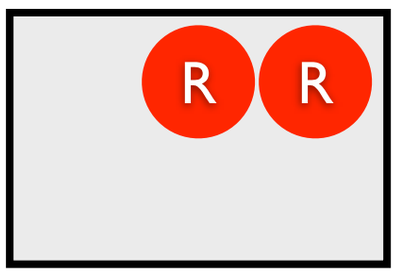


$(1-0.4) \times 0.3 \times 0.2$



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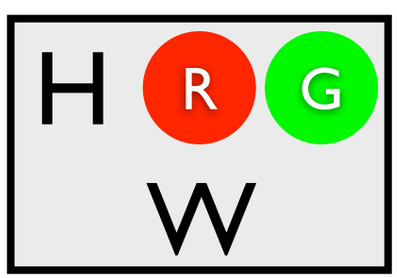

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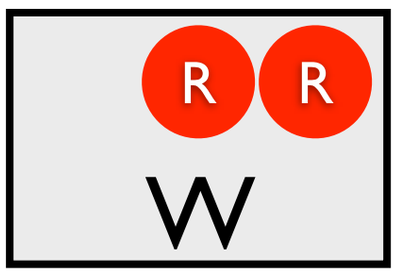

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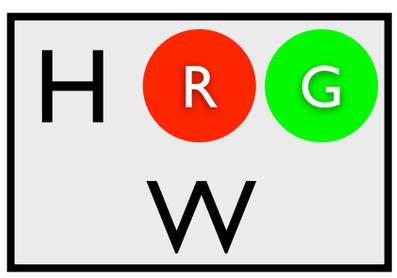
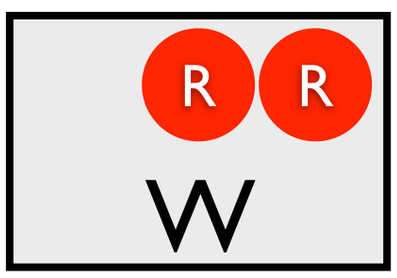


$(1-0.4) \times 0.3 \times 0.2$



$(1-0.4)$

Possible Worlds

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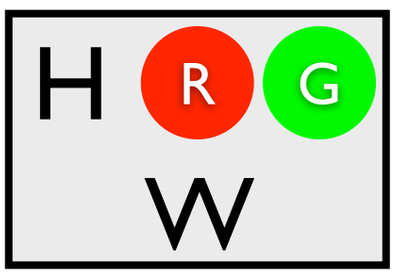
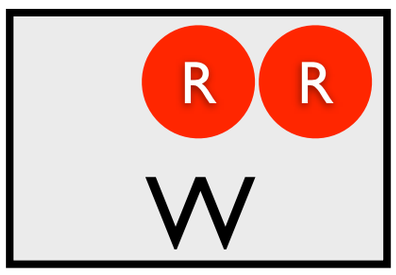
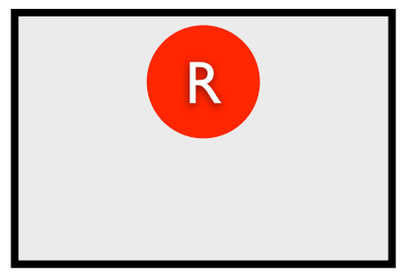


(1−0.4)×0.3 ×0.2



(1−0.4)×0.3



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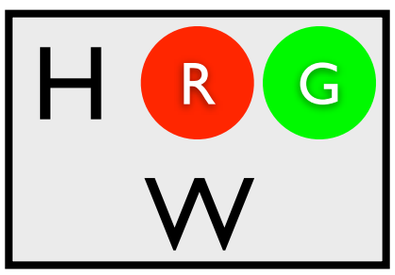
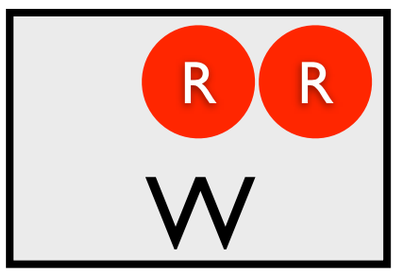
```

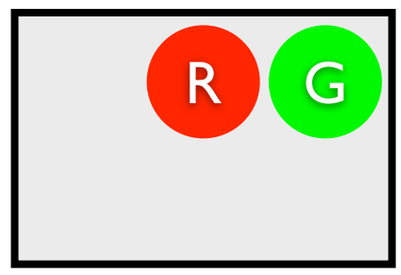
$0.4 \times 0.3 \times 0.3$



$(1-0.4) \times 0.3 \times 0.2$



$(1-0.4) \times 0.3 \times 0.3$



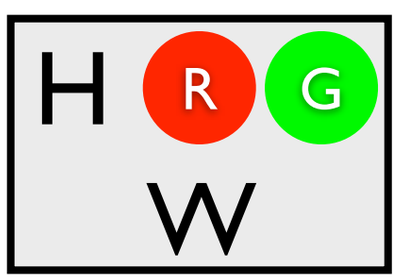
Possible Worlds

```
0.4 :: heads.

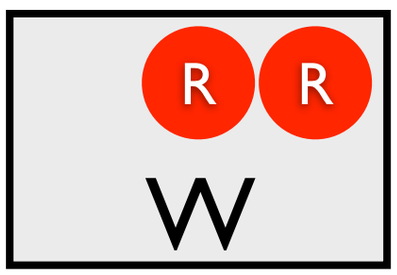
0.3 :: col(1,red); 0.7 :: col(1,blue) <- true.
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue) <- true.

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

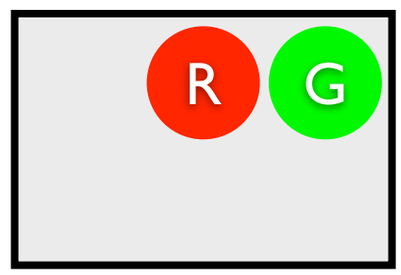
0.4 ×0.3 ×0.3



(1−0.4)×0.3 ×0.2



(1−0.4)×0.3 ×0.3



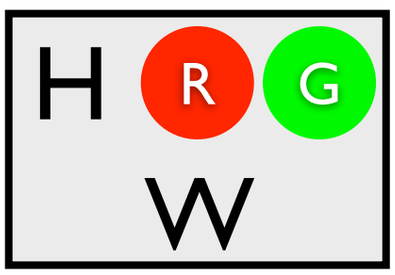
Possible Worlds

```
0.4 :: heads.

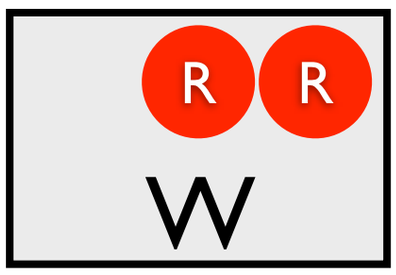
0.3 :: col(1,red); 0.7 :: col(1,blue) <- true.
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue) <- true.

win :- heads, col( ,red).
win :- col(1,C), col(2,C).
```

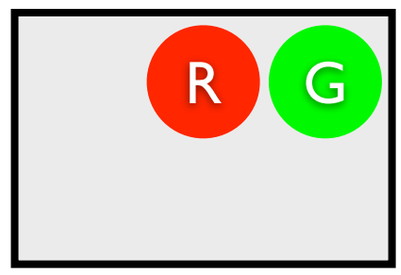
0.4 ×0.3 ×0.3



(1−0.4)×0.3 ×0.2

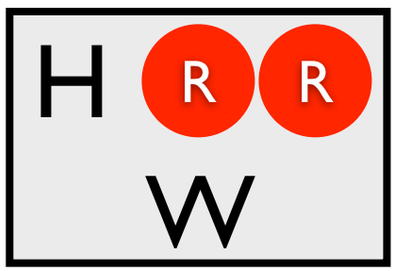


(1−0.4)×0.3 ×0.3

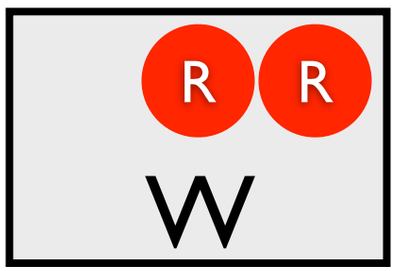


All Possible Worlds

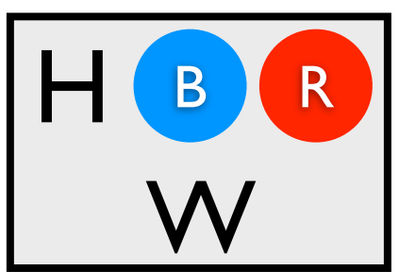
0.024



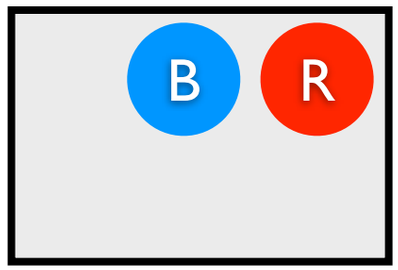
0.036



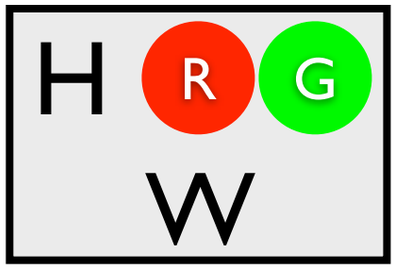
0.056



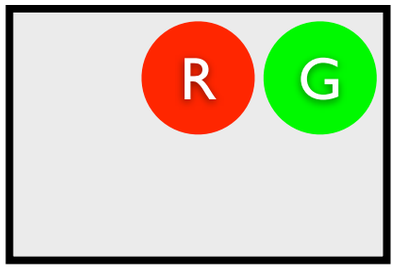
0.084



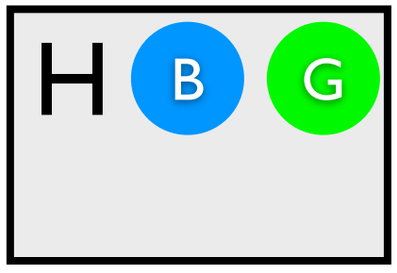
0.036



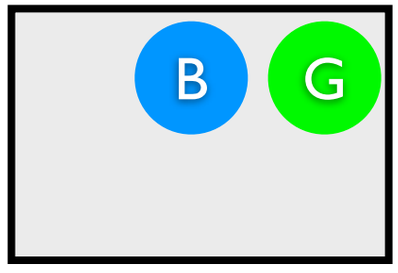
0.054



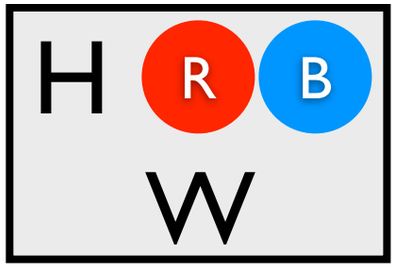
0.084



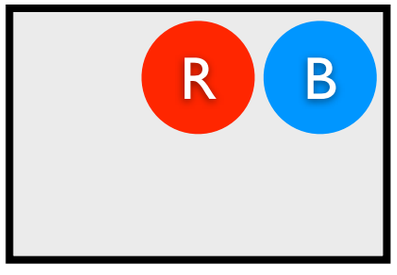
0.126



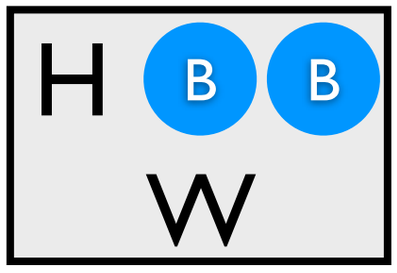
0.060



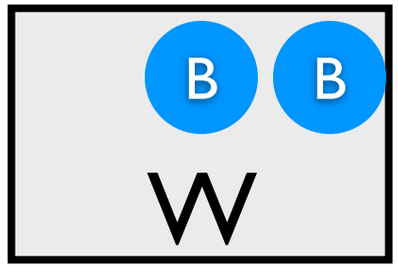
0.090



0.140

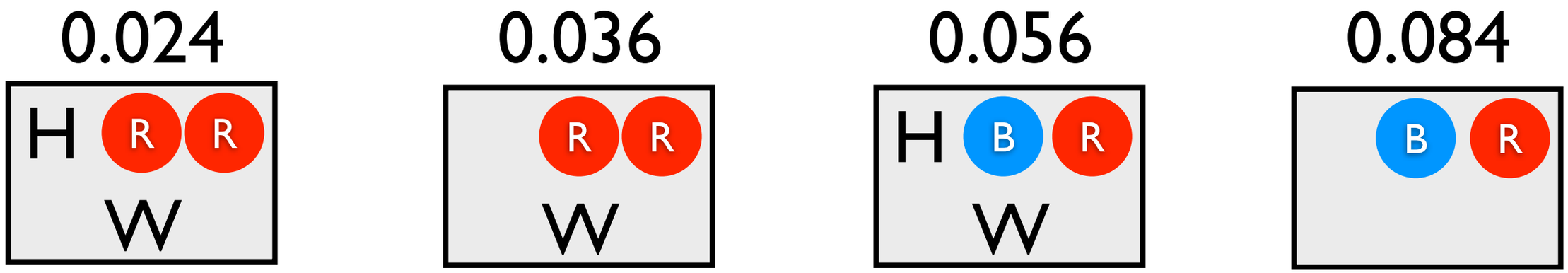


0.210



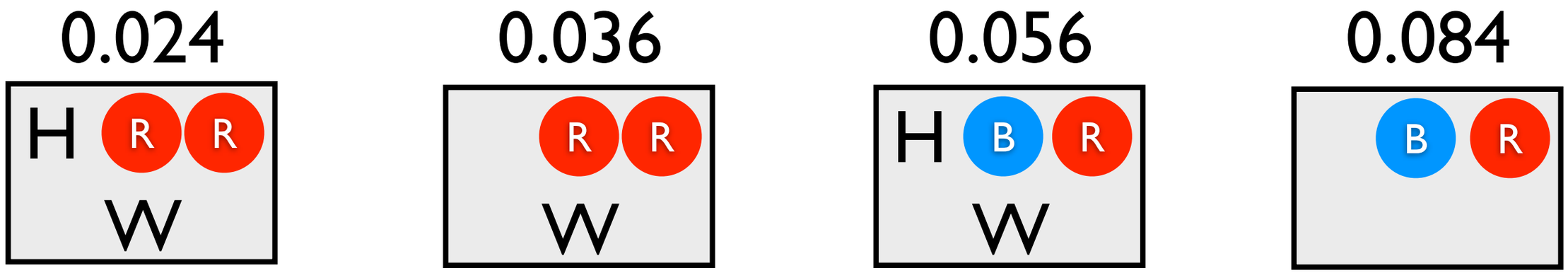
Most likely world where **win** is true?

MPE Inference



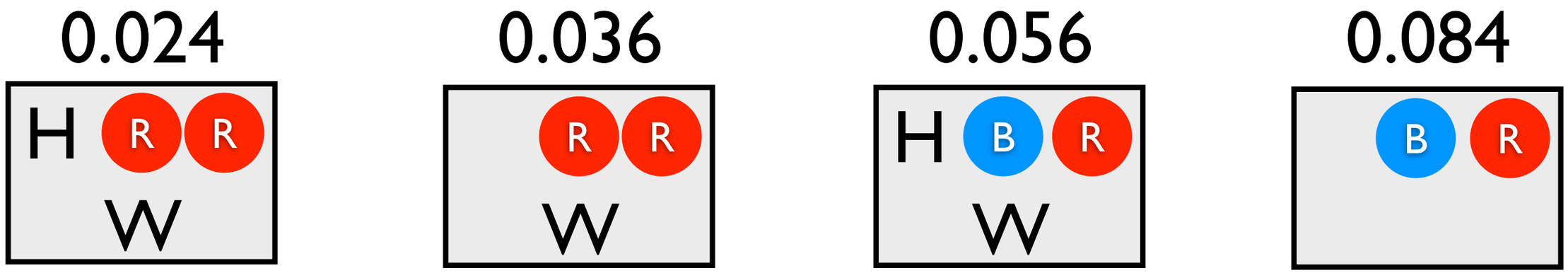
Most likely world where **win** is true?

MPE Inference



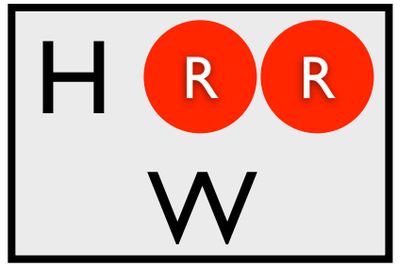
Most likely world where `col(2,blue)` is false?

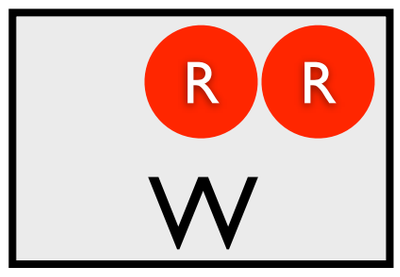
MPE Inference

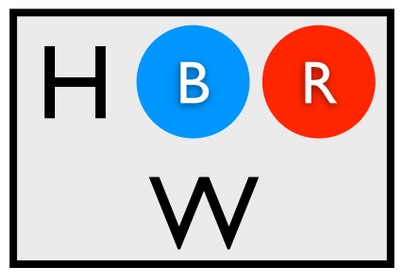


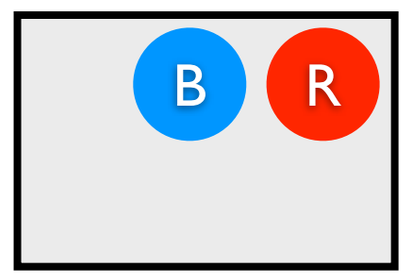
Most likely world where `col(2,blue)` is false?

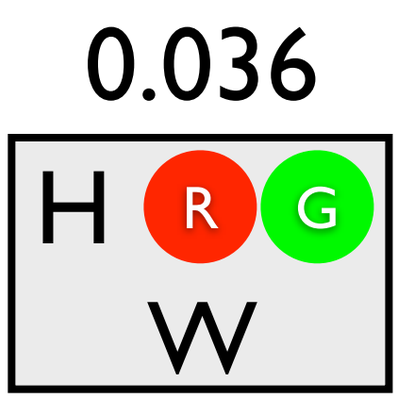
MPE Inference

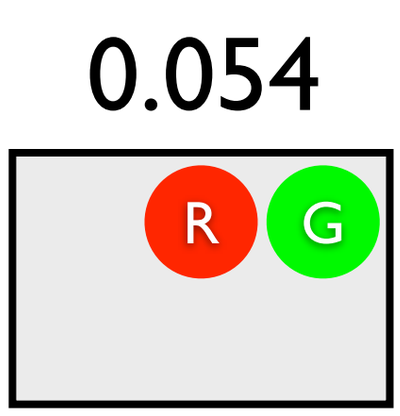


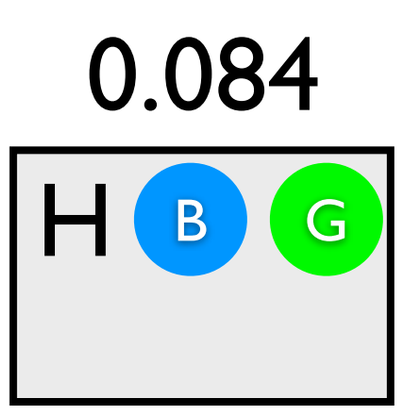


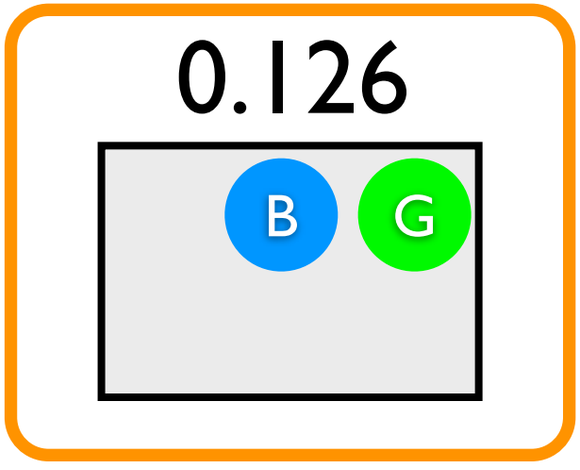


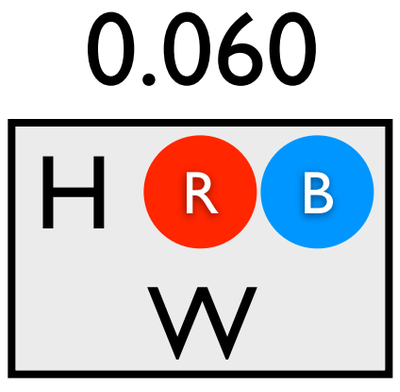


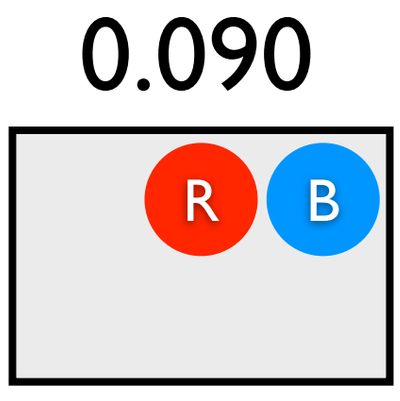


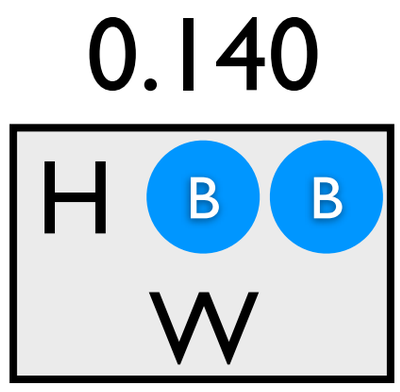


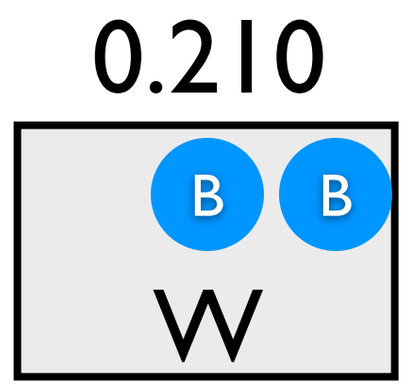






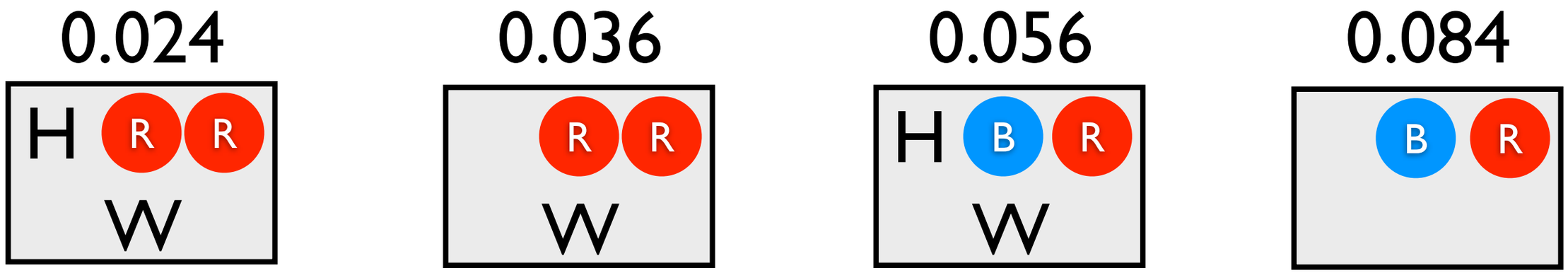






P(win)=?

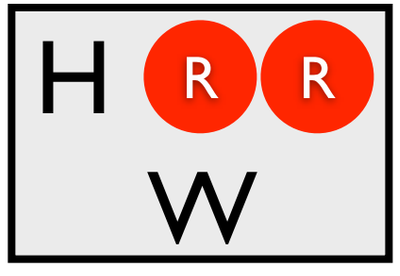
Marginal Probability



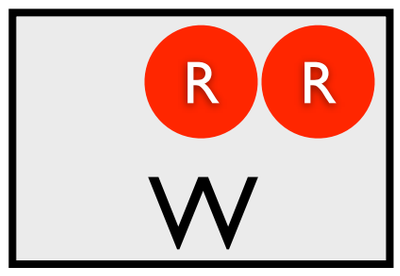
P(win)= Σ

Marginal Probability

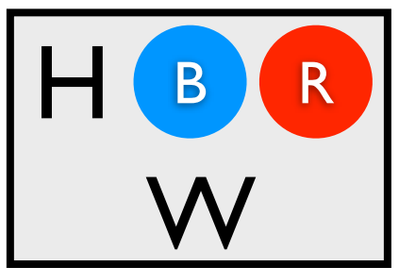
0.024



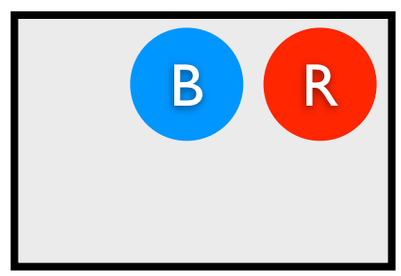
0.036



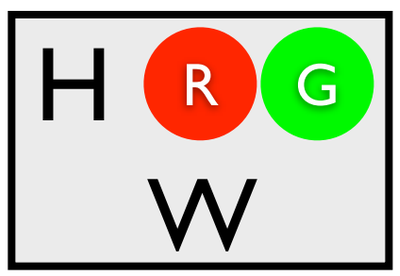
0.056



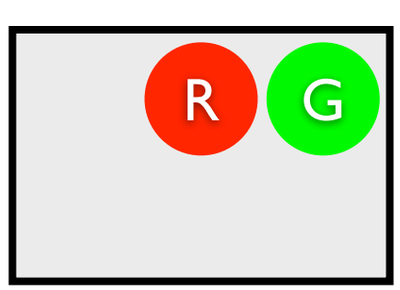
0.084



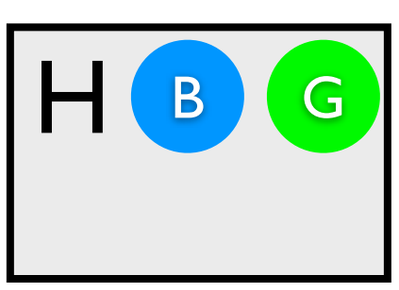
0.036



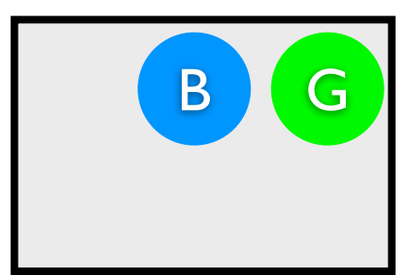
0.054



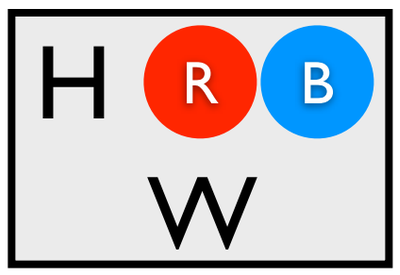
0.084



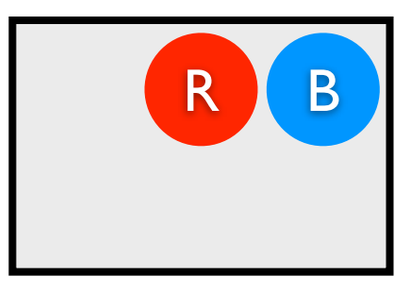
0.126



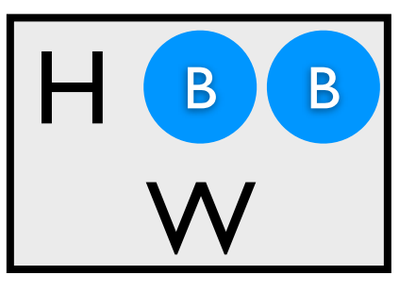
0.060



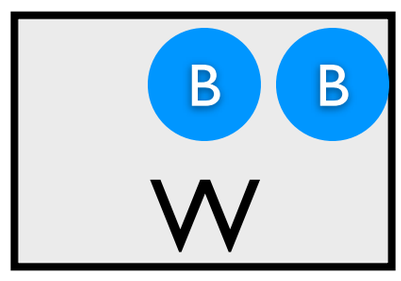
0.090



0.140



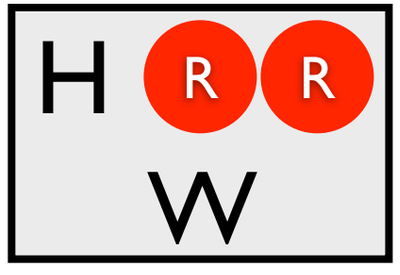
0.210



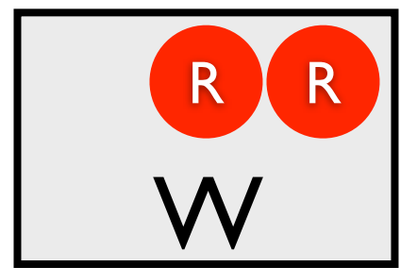
P(win)= Σ =0.562

Marginal Probability

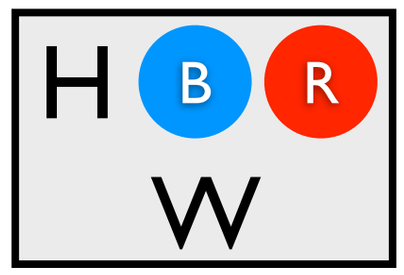
0.024



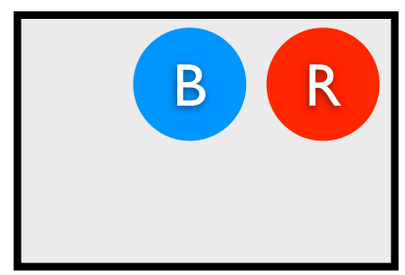
0.036



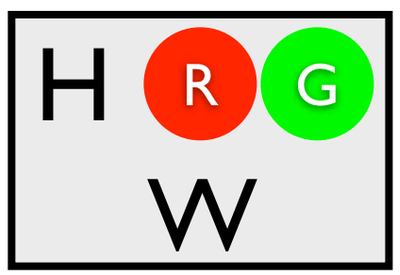
0.056



0.084



0.036



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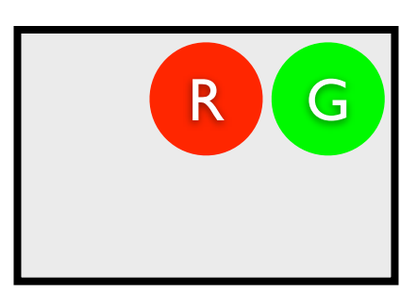

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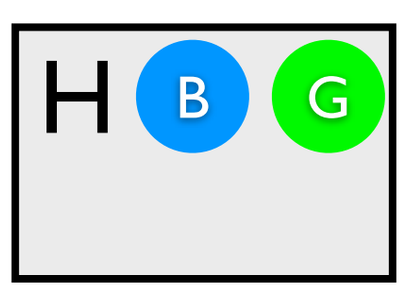
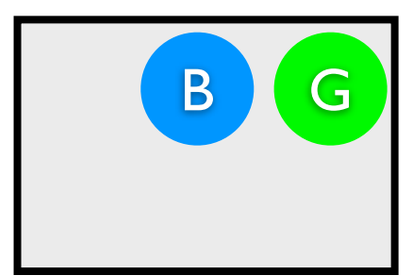
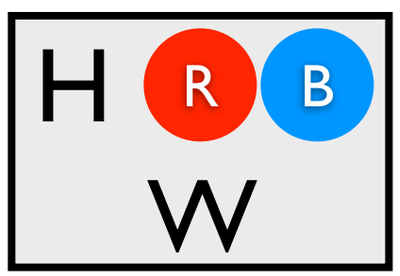


0.126



0.060



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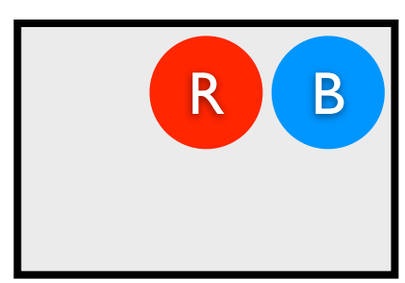
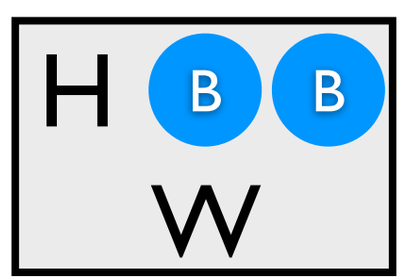
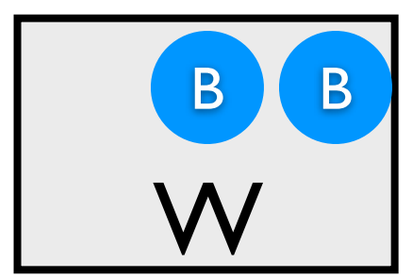


0.140



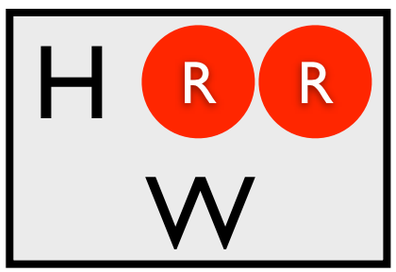
0.210



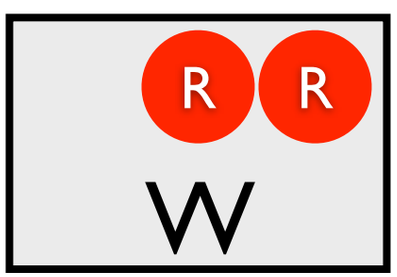
P(win|col(2,green))= ?

Conditional Probability

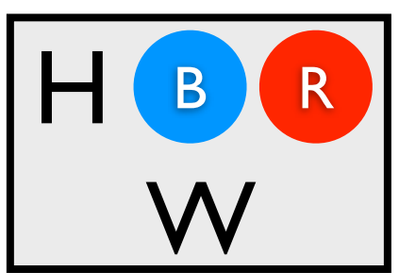
0.024



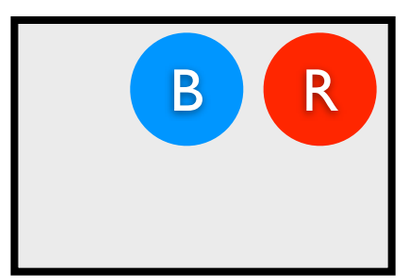
0.036



0.056



0.084



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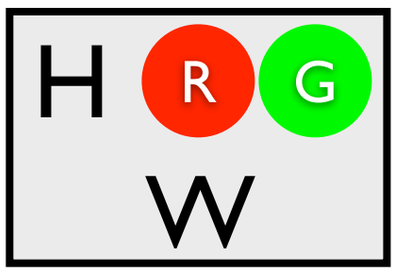

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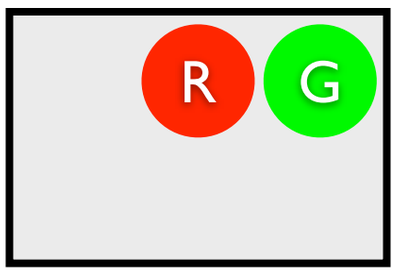
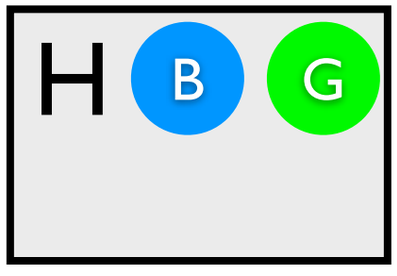
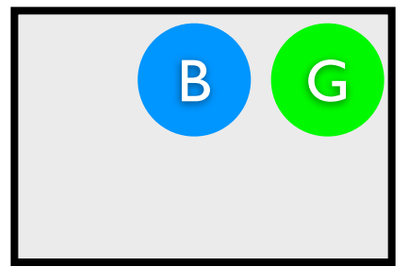


0.084



0.126



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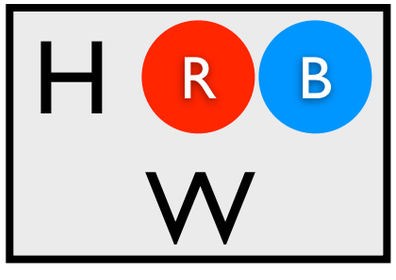
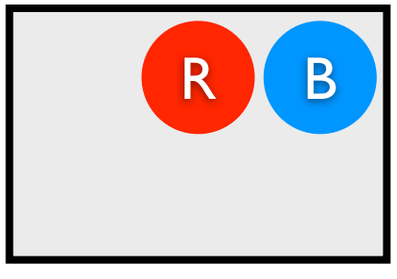
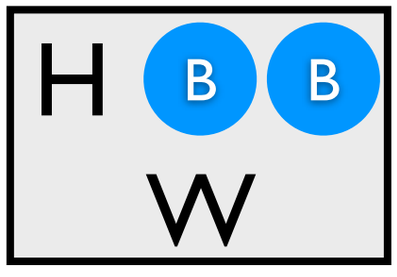


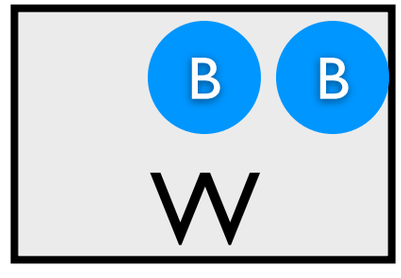
0.090



0.140



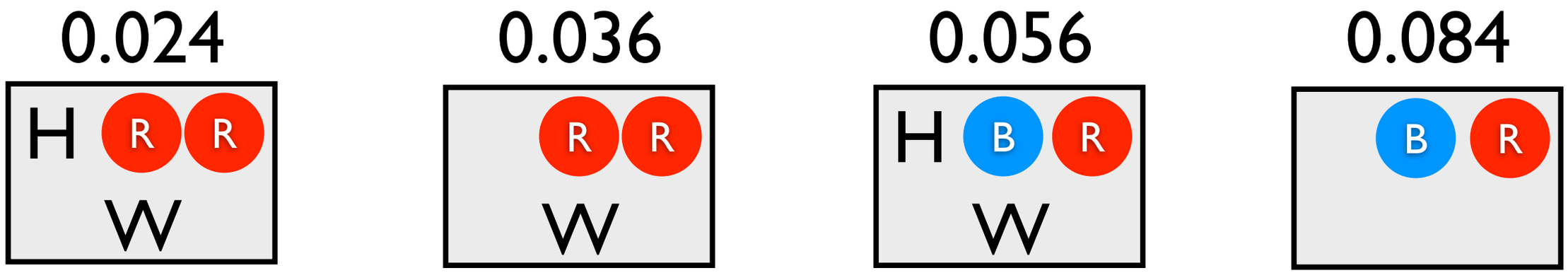
0.210



$$P(win|col(2,green)) = \Sigma/\Sigma$$

$$= P(win \wedge col(2,green))/P(col(2,green))$$

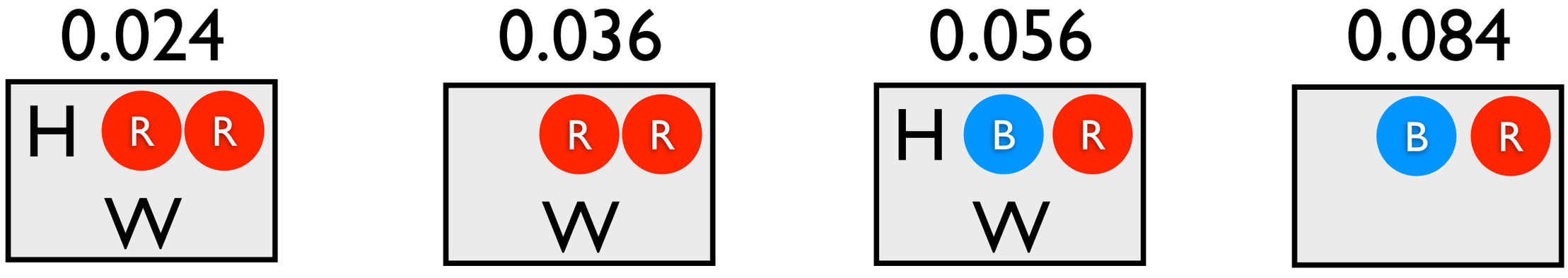
Conditional Probability



$$P(win|col(2,green)) = \Sigma/\Sigma$$

$$= P(win \wedge col(2,green))/P(col(2,green))$$

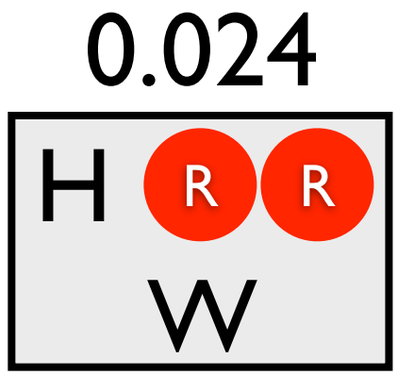
Conditional Probability

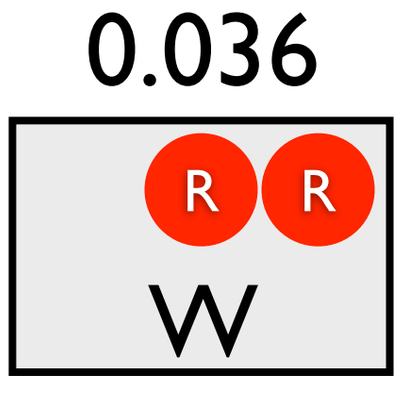


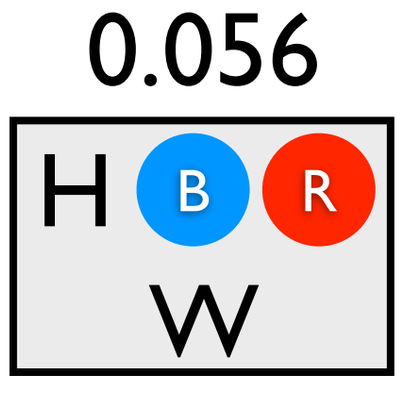
P(win|col(2,green))= ∑/∑

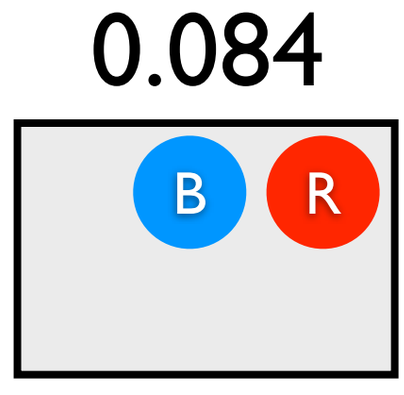
=0.036/0.3=0.12

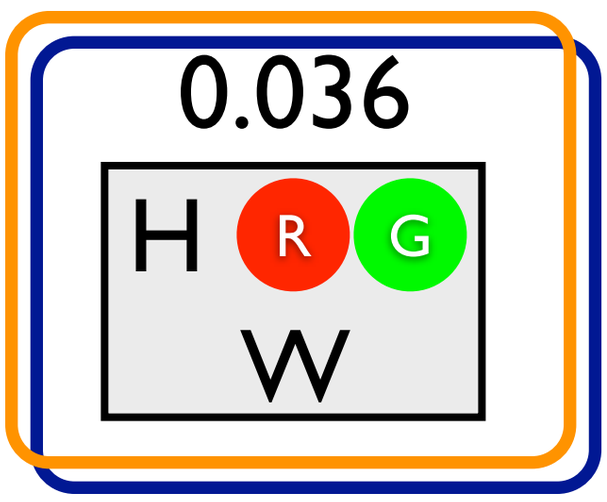
Conditional Probability

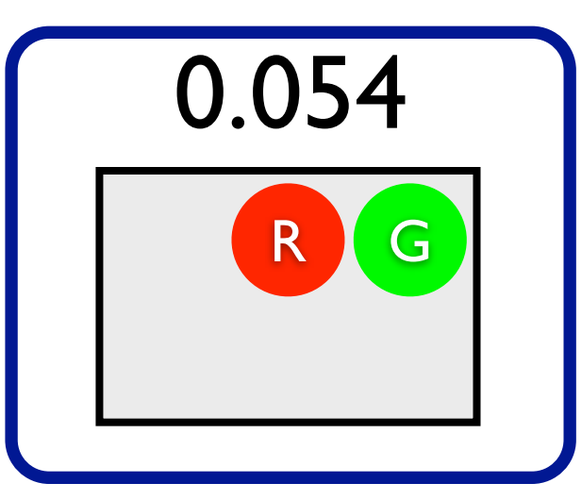


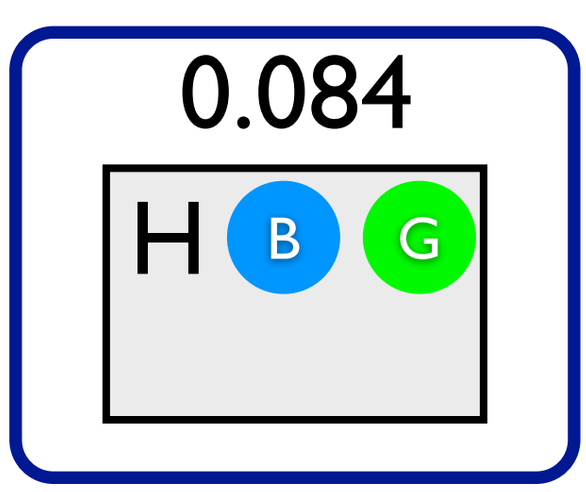


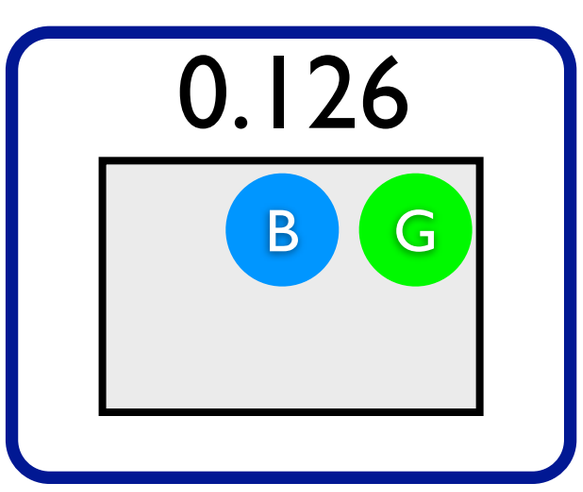


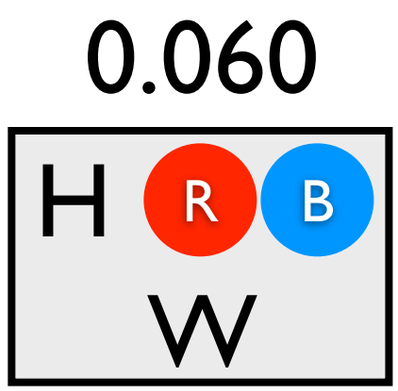


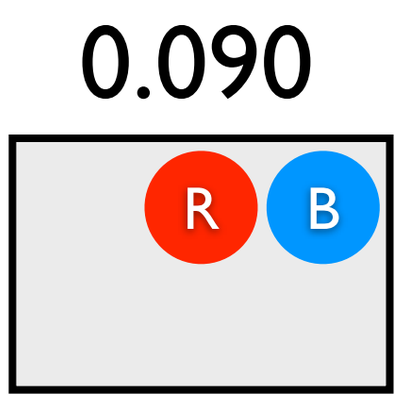


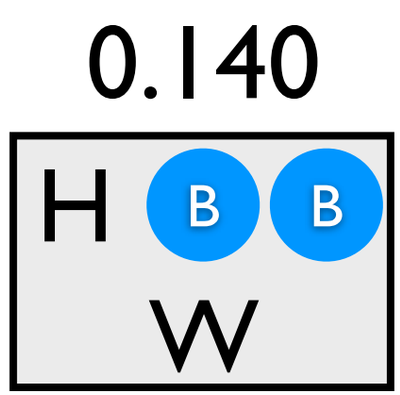


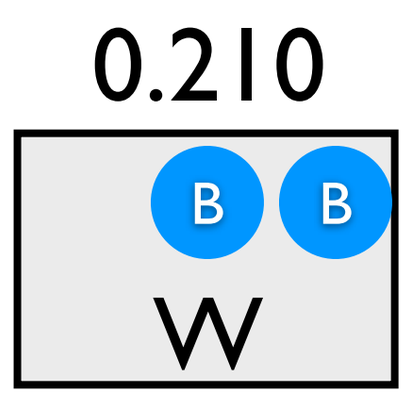












Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Prolog rules

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

sum over possible worlds where Q is true

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Prolog rules

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

sum over possible worlds where Q is true

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Prolog rules

probability of possible world

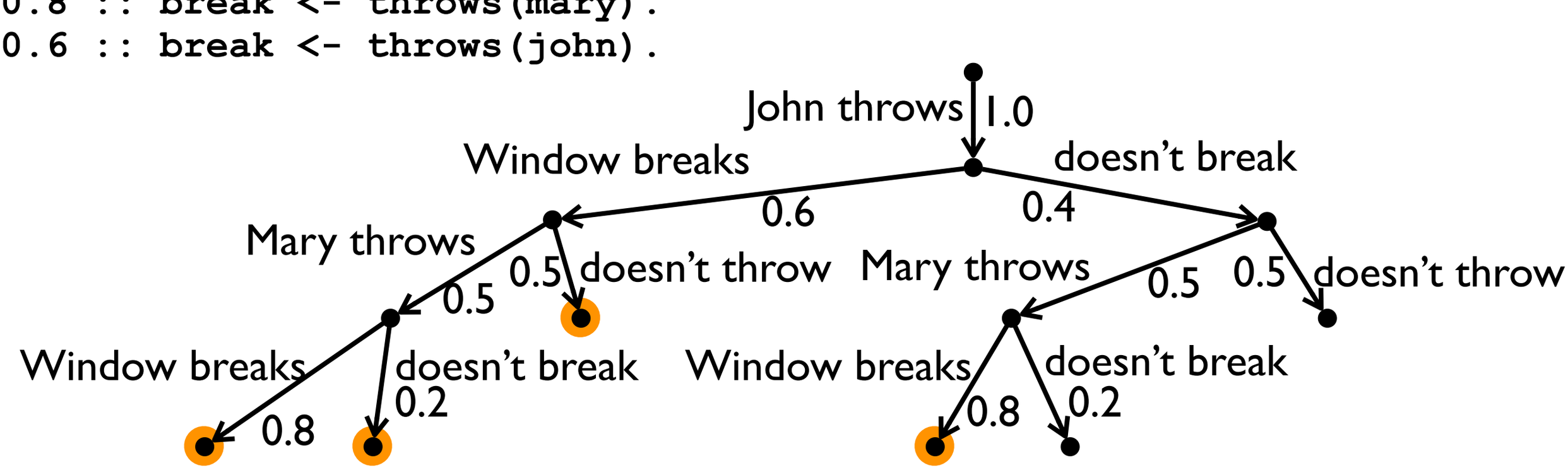
[Vennekens et al, ICLP 04]

Alternative view: CP-Logic

```
throws(john).
0.5::throws(mary).

0.8 :: break <- throws(mary).
0.6 :: break <- throws(john).
```

probabilistic causal laws



P(break)=0.6×0.5×0.8+0.6×0.5×0.2+0.6×0.5+0.4×0.5×0.8

Answering Questions

Given:

program

queries

evidence

?

Find:

marginal probabilities

conditional probabilities

MPE state

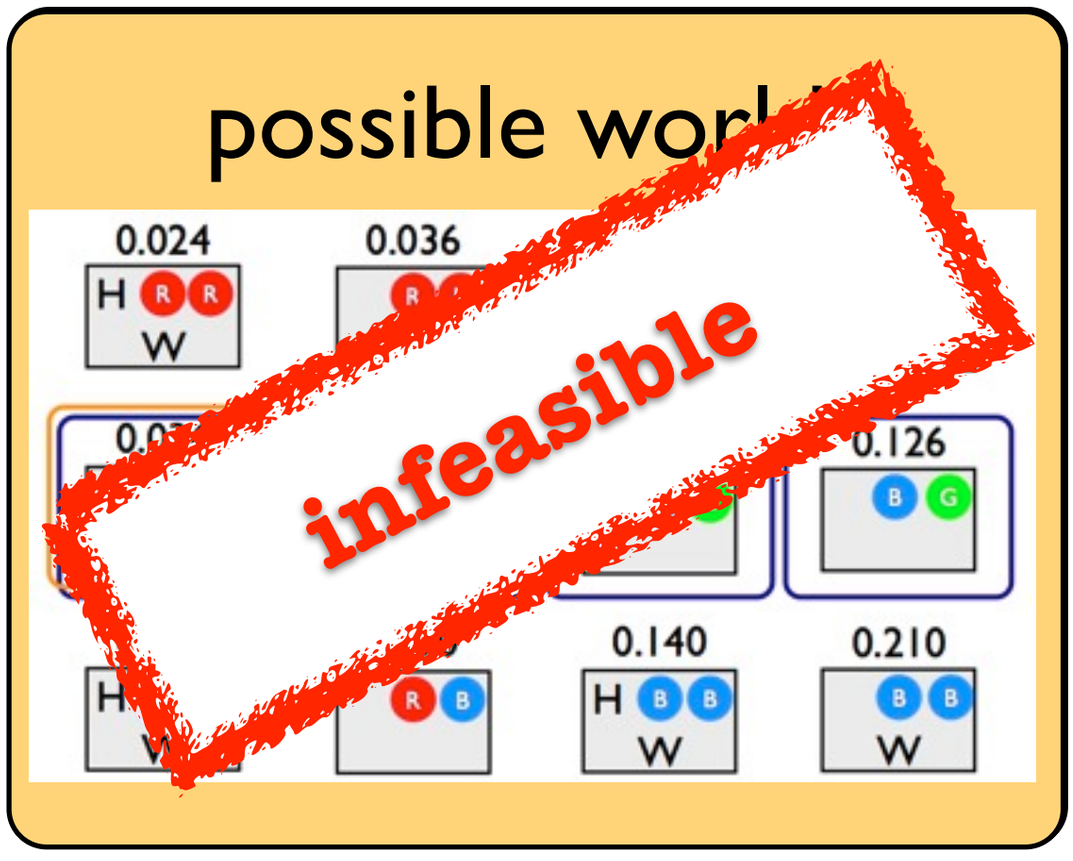
Answering Questions

Given:

program

queries

evidence



Find:

marginal probabilities

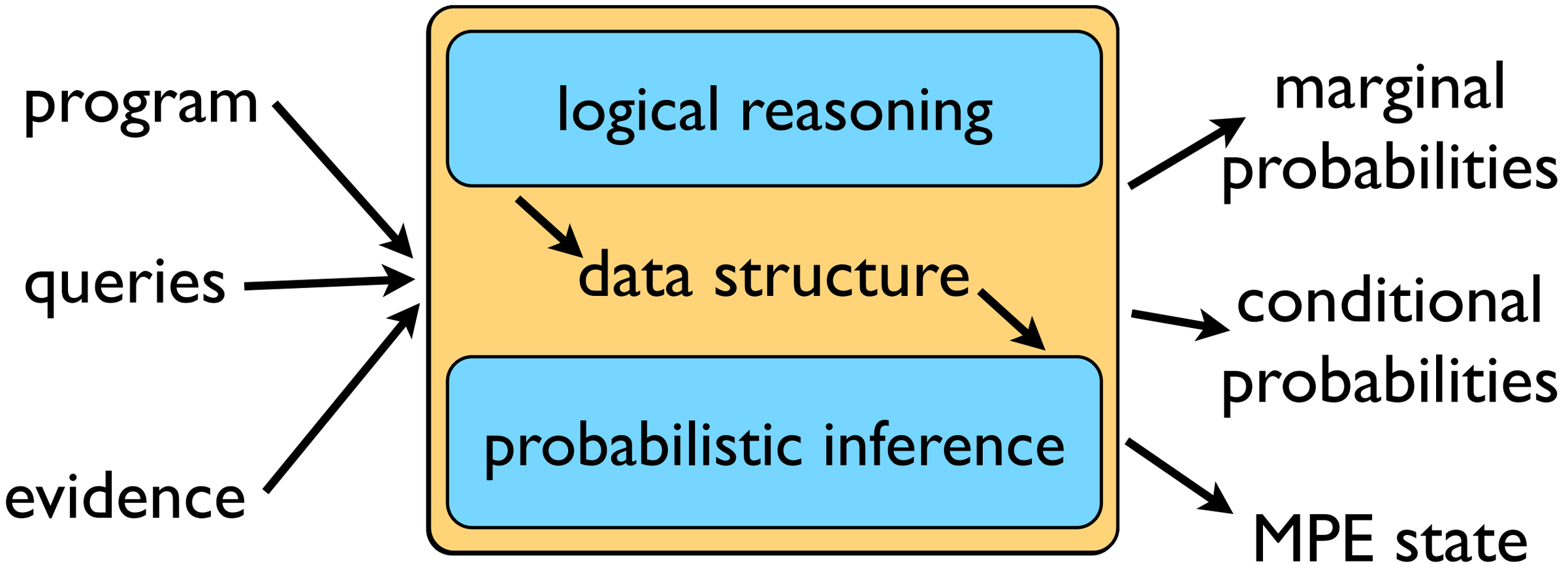
conditional probabilities

MPE state

Answering Questions

Given:

Find:

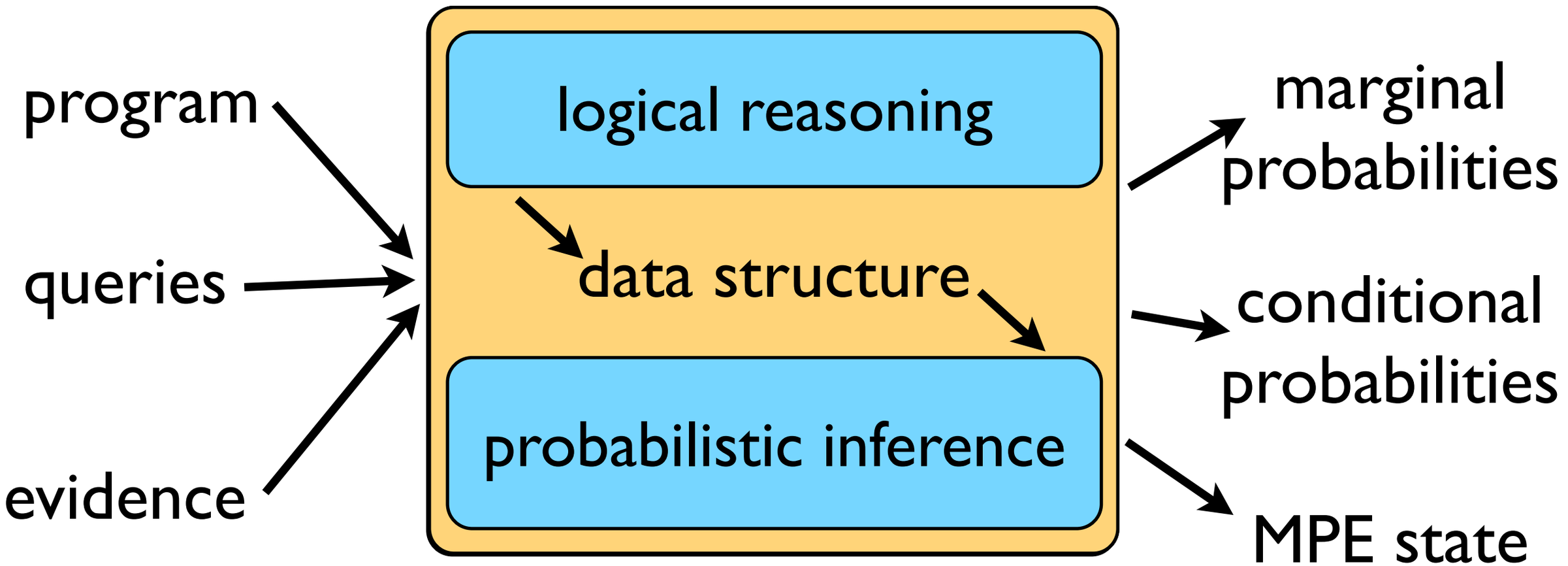


Answering Questions

more on this later

Given:

Find:



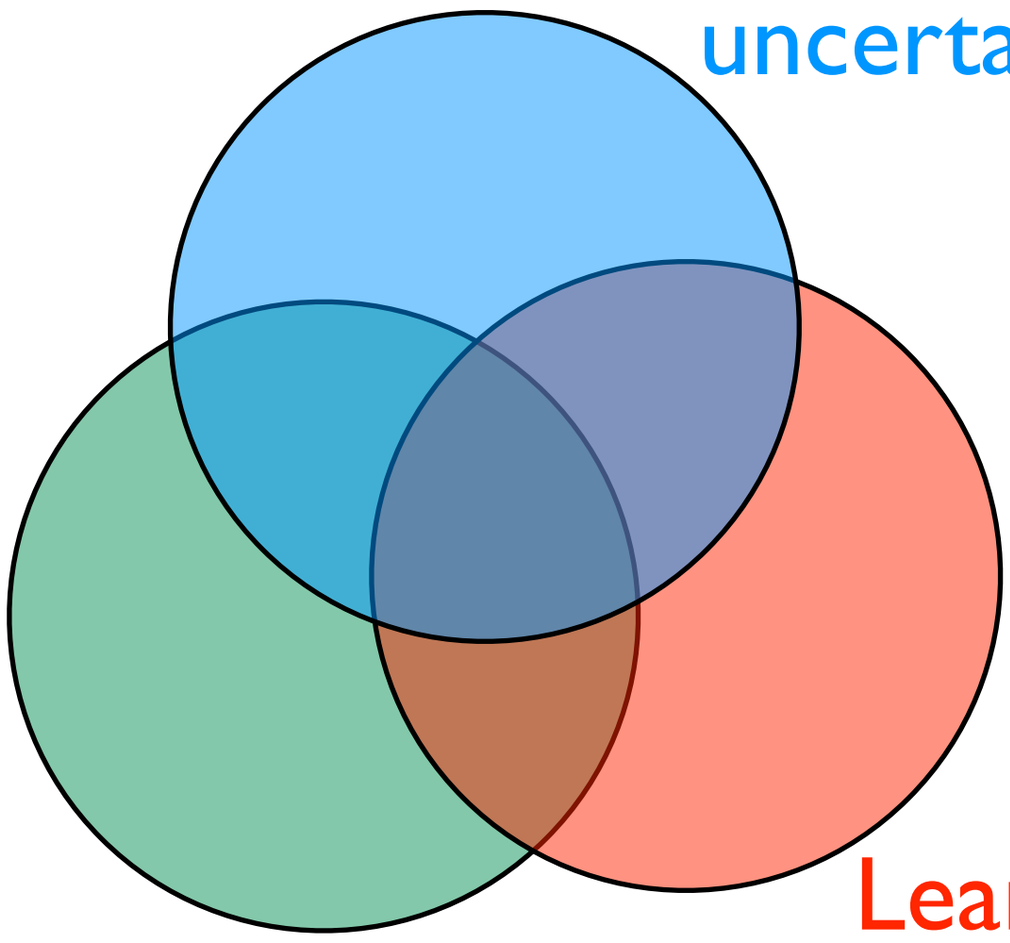
[Suciu et al 2011]

Probabilistic Databases

Dealing with uncertainty

Reasoning with relational data

Learning



[Suciu et al 2011]

Probabilistic Databases

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

bornIn

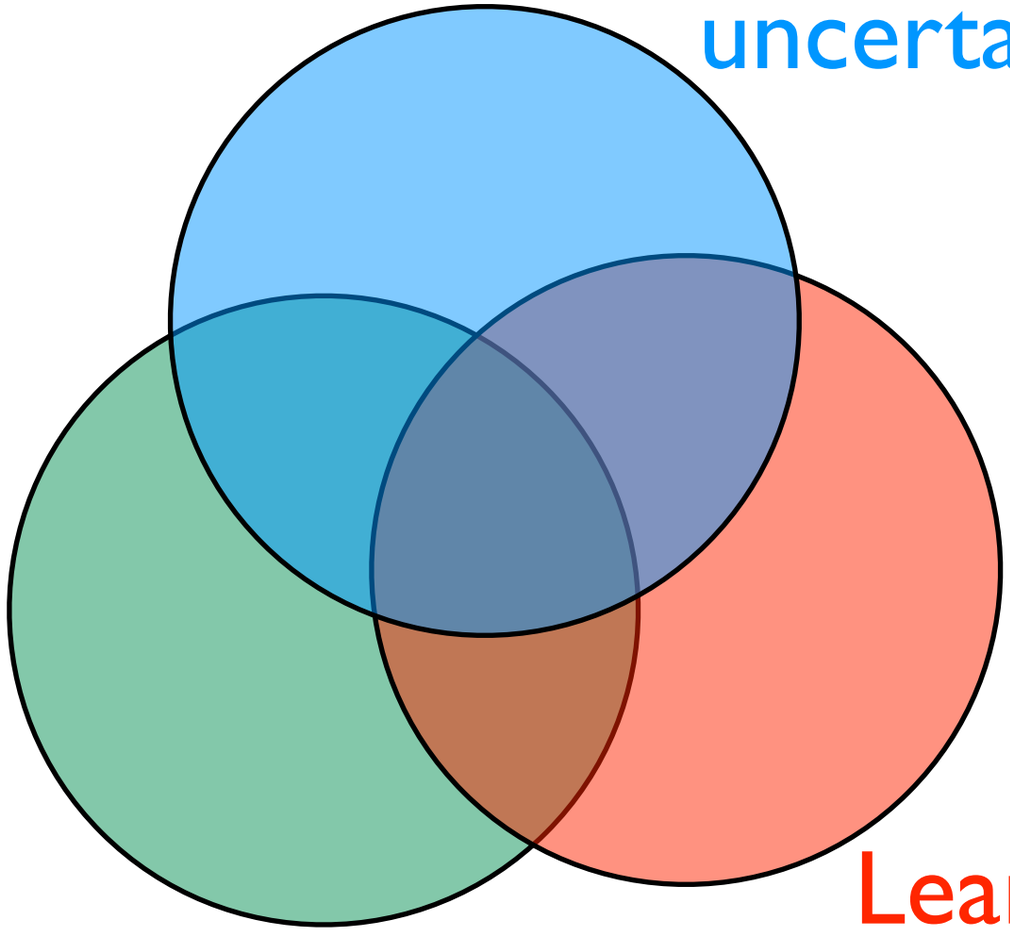
person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database

Dealing with uncertainty



Learning

[Suciu et al 2011]

Probabilistic Databases

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

one world

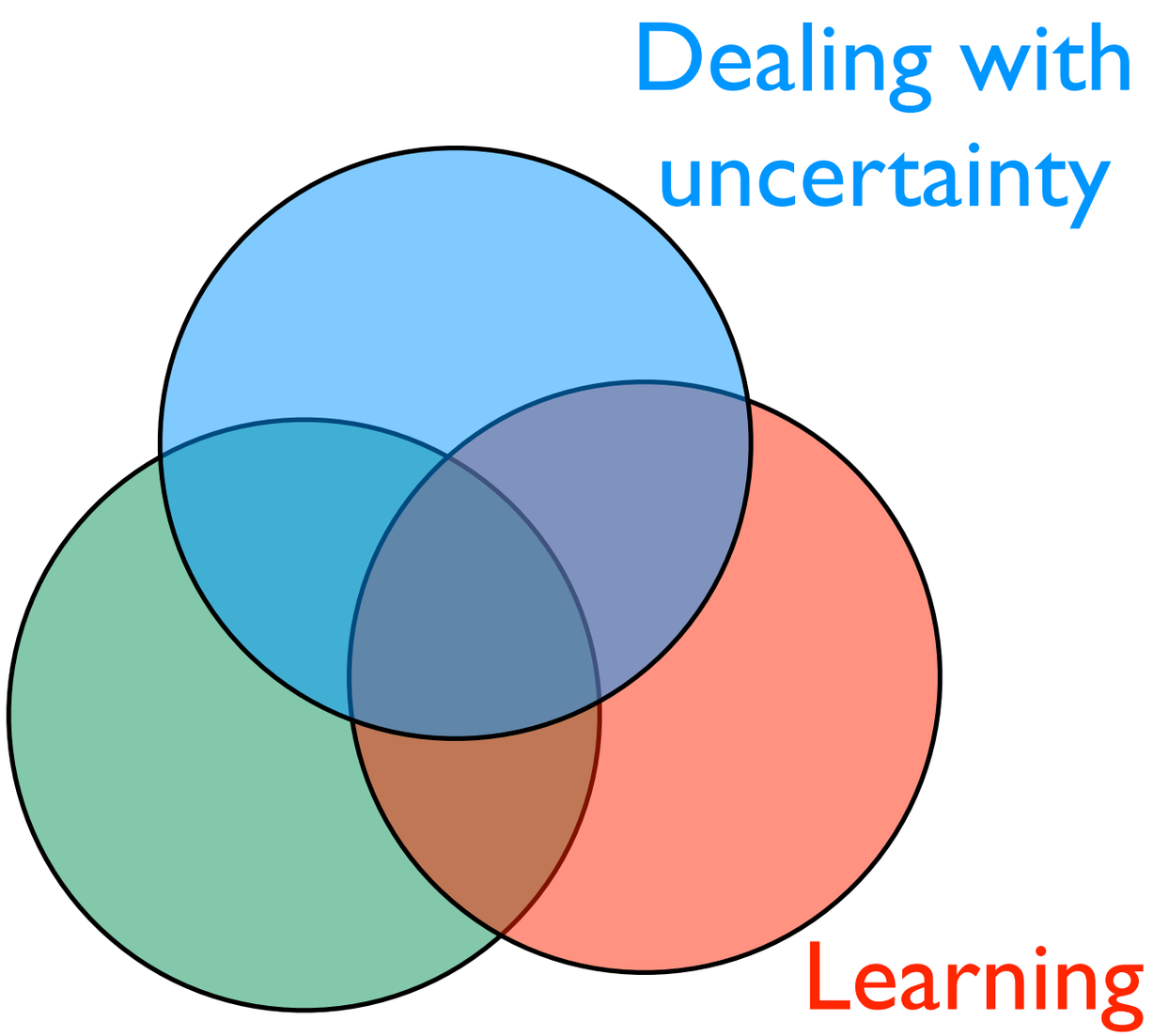
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



[Suciu et al 2011]

Probabilistic Databases

bornIn

person	city	P
ann	london	0.87
bob	york	0.95
eve	new york	0.90
tom	paris	0.56

cityIn

city	country	P
london	uk	0.99
york	uk	0.75
paris	usa	0.40

tuples as random variables

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

one world

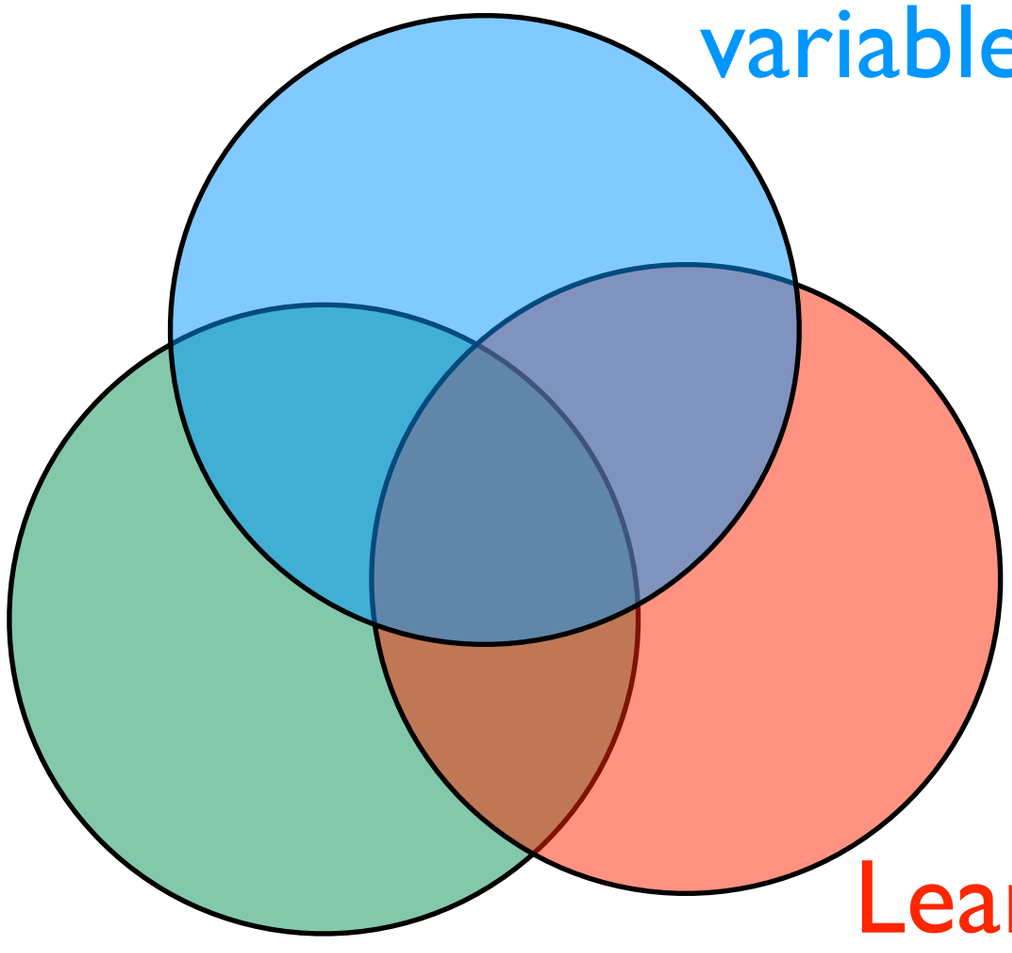
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



Learning

[Suciu et al 2011]

Probabilistic Databases

several possible worlds

bornIn

person	city	P
ann	london	0.87
bob	york	0.95
eve	new york	0.90
tom	paris	0.56

cityIn

city	country	P
london	uk	0.99
york	uk	0.75
paris	usa	0.40

tuples as random

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

one world

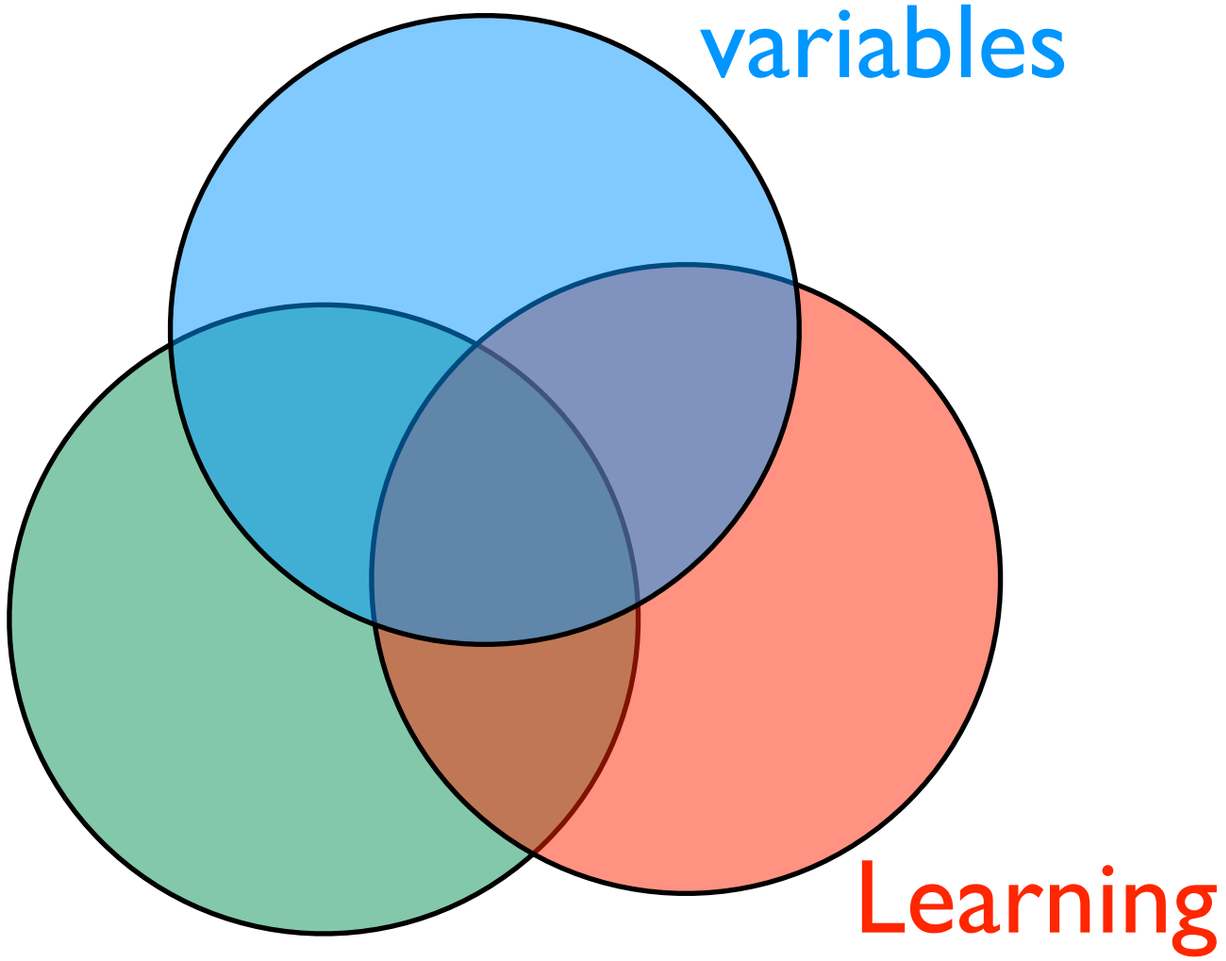
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



[Suciu et al 2011]

Probabilistic Databases

several possible worlds

bornIn

person	city	P
ann	london	0.87
bob	york	0.95
		0.90
		0.56

cityIn

city	country	P
london	uk	0.99
york	uk	0.75
paris	usa	0.40

tuples as random

probabilistic tables + database queries
→ distribution over possible worlds

```
select
from bornIn x, cityIn y
where x.city=y.city
```

one world

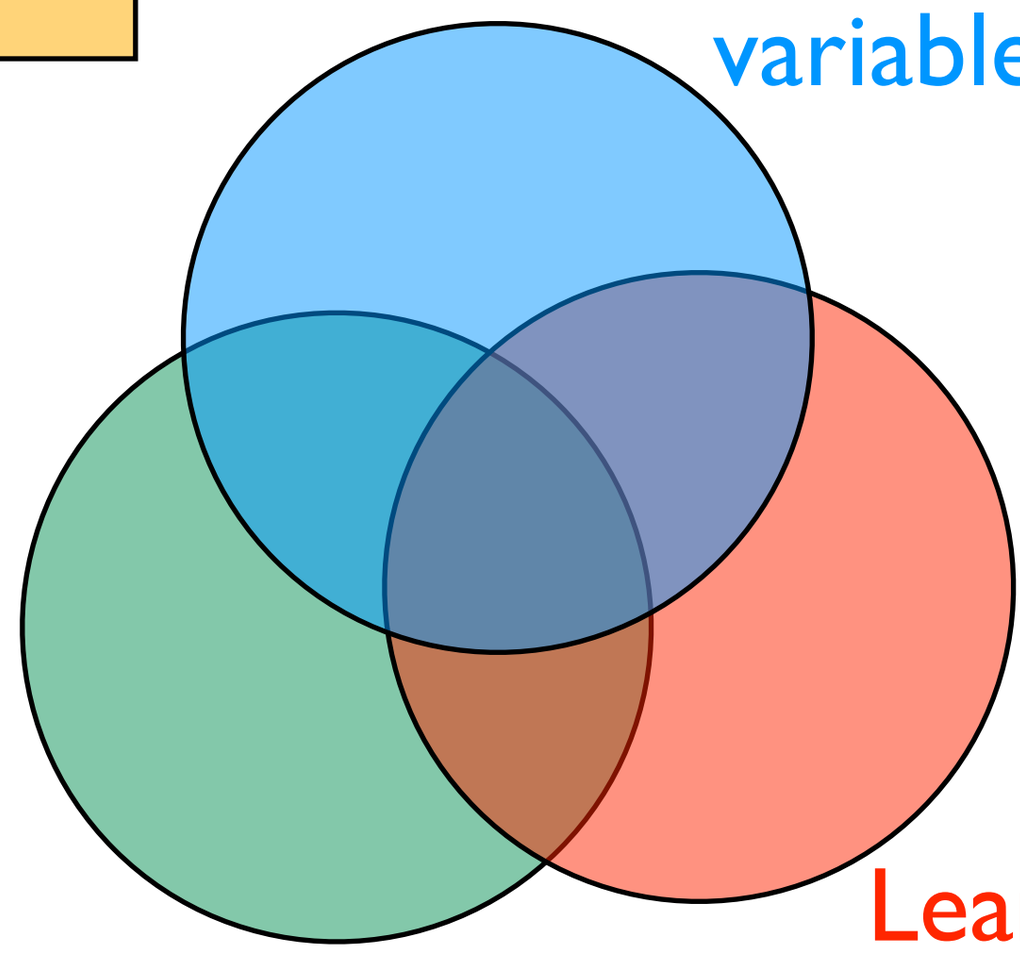
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



Example: Information Extraction

Recently-Learned Facts twitter Refresh

instance	iteration	date learned	confidence
kelly_andrews is a female	826	29-mar-2014	98.7
investment_next_year is an economic sector	829	10-apr-2014	95.3
shibenik is a geopolitical entity that is an organization	829	10-apr-2014	97.2
quality_web_design_work is a character trait	826	29-mar-2014	91.0
mercedes_benz_cls_by_carlsson is an automobile manufacturer	829	10-apr-2014	95.2
social_work is an academic program at the university rutgers_university	827	02-apr-2014	93.8
dante wrote the book the_divine_comedy	826	29-mar-2014	93.8
willie_aames was born in the city los_angeles	831	16-apr-2014	100.0
kitt_peak is a mountain in the state or province arizona	831	16-apr-2014	96.9
greenwich is a park in the city london	831	16-apr-2014	100.0

instances for many different relations

degree of certainty

NELL: http://rtw.ml.cmu.edu/rtw/

Querying: relational database

ProducesProduct

Company	Product
sony	walkman
microsoft	mac_os_x
ibm	personal_computer
microsoft	mac_os
adobe	adobe_indesign
adobe	adobe_dreamweaver
...	...

HeadquarteredIn

Company	City
microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

[Example from Suciu et al 2011]

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microsoft	redmond
ibm	san_jose
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honda	torrance
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egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```


[Example from Suciu et al 2011]

Querying: relational database

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microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company
personal_computer	ibm
adobe_indesign	adobe
adobe_dreamweaver	adobe

[Example from Suciu et al 2011]

Querying: relational database

ProducesProduct

Company	Product
sony	walkman
microsoft	mac_os_x
ibm	personal_computer
microsoft	mac_os
adobe	adobe_indesign
adobe	adobe_dreamweaver
...	...

HeadquarteredIn

Company	City
microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
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```

Product	Company
personal_computer	ibm
adobe_indesign	adobe
adobe_dreamweaver	adobe

[Example from Suciu et al 2011]

Querying: relational database

ProducesProduct

Company	Product
sony	walkman
microsoft	mac_os_x
ibm	personal_computer
microsoft	mac_os
adobe	adobe_indesign
adobe	adobe_dreamweaver
...	...

HeadquarteredIn

Company	City
microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company
personal_computer	ibm
adobe_indesign	adobe
adobe_dreamweaver	adobe

[Example from Suciu et al 2011]

Querying: relational database

ProducesProduct

Company	Product
sony	walkman
microsoft	mac_os_x
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adobe	adobe_indesign
adobe	adobe_dreamweaver
...	...

HeadquarteredIn

Company	City
microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company
personal_computer	ibm
adobe_indesign	adobe
adobe_dreamweaver	adobe

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

same query -

probabilities handled implicitly

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.96x0.99=0.95

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.9x0.93=0.83

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
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adobe	adobe_indesign	0.9
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...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

0.87x0.93=0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
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microsoft	redmond	1.00
ibm	san_jose	0.99
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honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

answer tuples ranked by probability

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```


[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
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...	...	...

[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

```
0.96::producesProduct(sony,walkman).
0.96::producesProduct(microsoft,mac_os_x).
0.96::producesProduct(ibm,personal_computer).
0.9::producesProduct(microsoft,mac_os).
0.9::producesProduct(adobe,adobe_indesign).
0.87::producesProduct(adobe,adobe_dreamweaver).
...
```


[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and y.City='san_jose'
```

PDB with tuple-level uncertainty in ProbLog?

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and y.City='san_jose'
```

```
result(Product,Company) :-
    producesProduct(Company,Product),
    headquarteredIn(Company,san_jose).
query(result(_,_)).
```

PDB with tuple-level uncertainty in ProbLog?

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and y.City='san_jose'
```

```
result(Product,Company) :-
    producesProduct(Company,Product),
    headquarteredIn(Company,san_jose).
query(result(_,_)).
```

PDB with tuple-level uncertainty in ProbLog?

```
0.96::producesProduct(sony,walkman).
0.96::producesProduct(microsoft,mac_os_x).
0.96::producesProduct(ibm,personal_computer).
0.9::producesProduct(microsoft,mac_os).
0.9::producesProduct(adobe,adobe_indesign).
0.87::producesProduct(adobe,adobe_dreamweaver).
...
1.00::headquarteredIn(microsoft,redmond).
0.99::headquarteredIn(ibm,san_jose).
0.93::headquarteredIn(emirates_airlines,dubai).
0.93::headquarteredIn(honda,torrance).
0.93::headquarteredIn(horizon,seattle).
0.93::headquarteredIn(egyptair,cairo).
0.93::headquarteredIn(adobe,san_jose).
...
      result(Product,Company) :- producesProduct(Company,Product),
                                 headquarteredIn(Company,san_jose).

      query(result(_,_)).
```

PDB with attribute-level uncertainty in ProbLog?

color

item	color	P
mug	green blue	0.65 0.35
plate	pink red purple	0.23 0.14 0.63

PDB with attribute-level uncertainty in ProbLog?

color

item	color	P
mug	green blue	0.65 0.35
plate	pink red purple	0.23 0.14 0.63

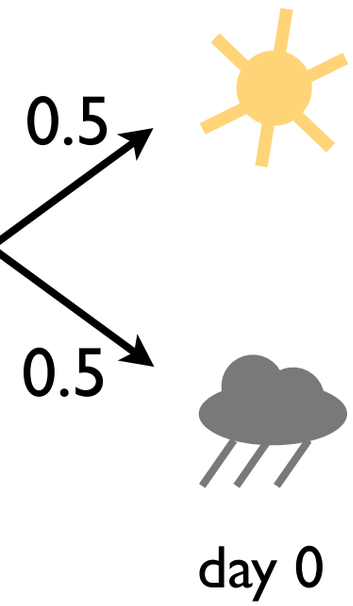
```
0.65::color(mug,green); 0.35::color(mug,blue) <- true.
0.23::color(plate,pink); 0.14::color(plate,red);
                                0.63::color(plate,purple) <- true.
```

ProbLog by example:

Rain or sun?

ProbLog by example:

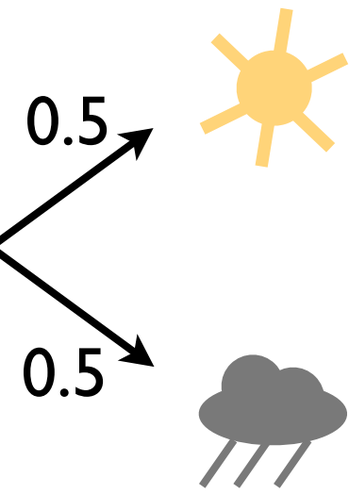
Rain or sun?



day 0

ProbLog by example:

Rain or sun?

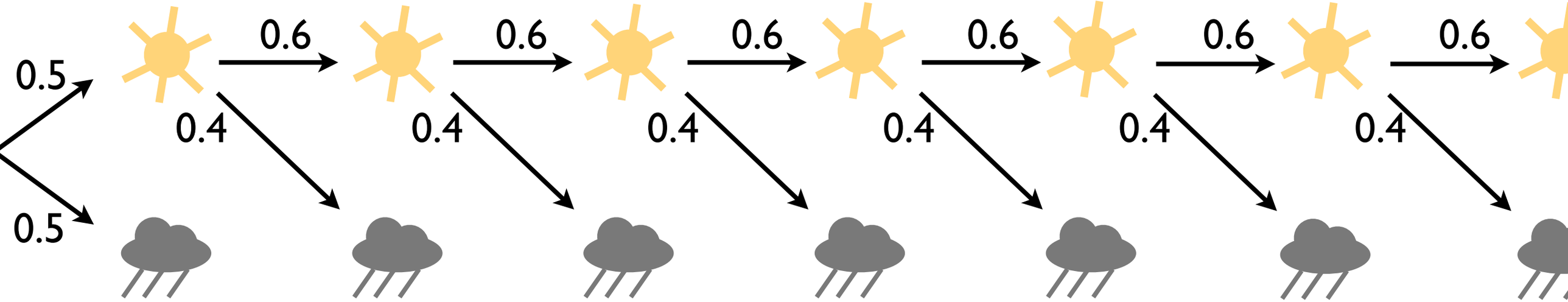


day 0

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.
```

ProbLog by example:

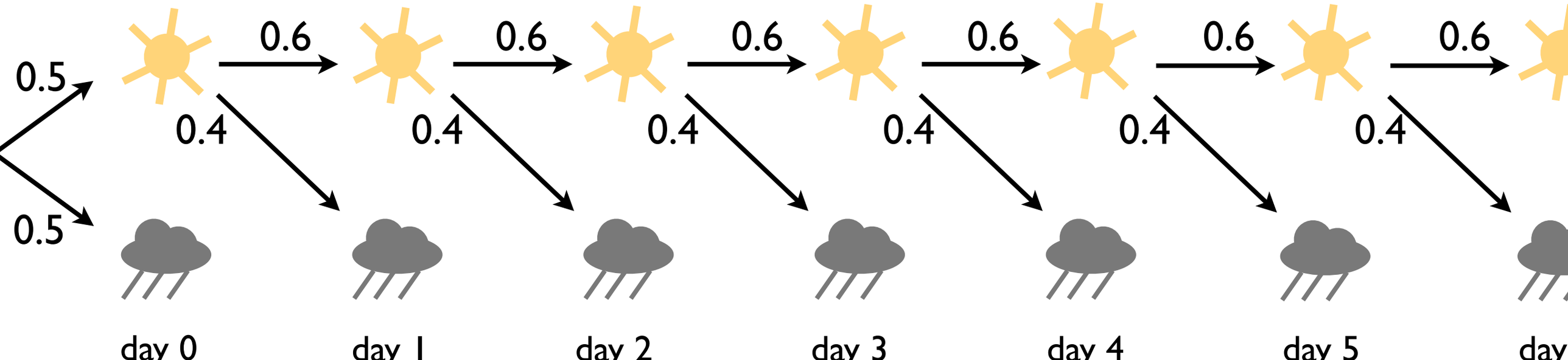
Rain or sun?




```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.
```

ProbLog by example:

Rain or sun?

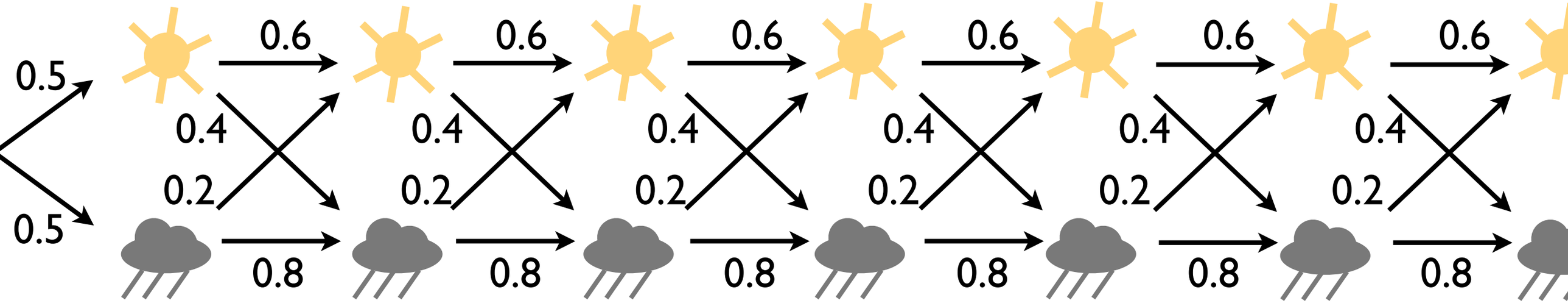



```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                    <- T>0, Tprev is T-1, weather(sun,Tprev).
```

ProbLog by example:

Rain or sun?

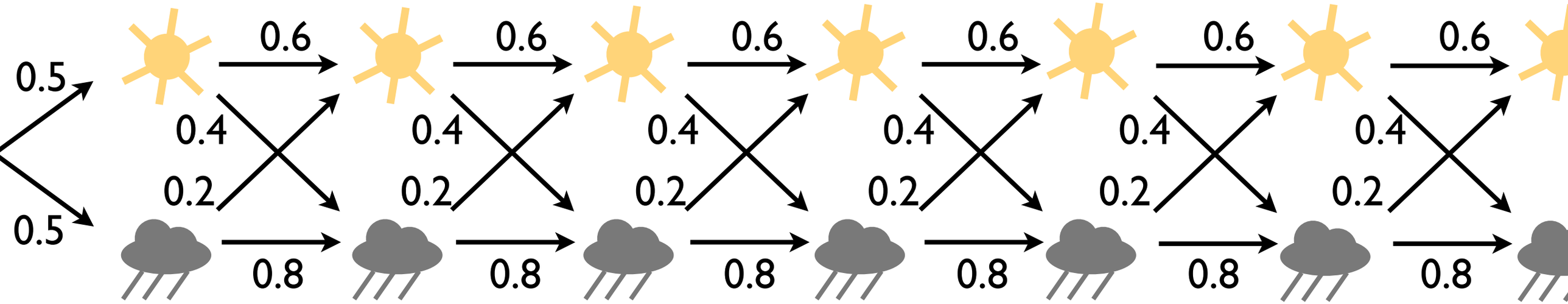



```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(sun,Tprev).
```

ProbLog by example:

Rain or sun?

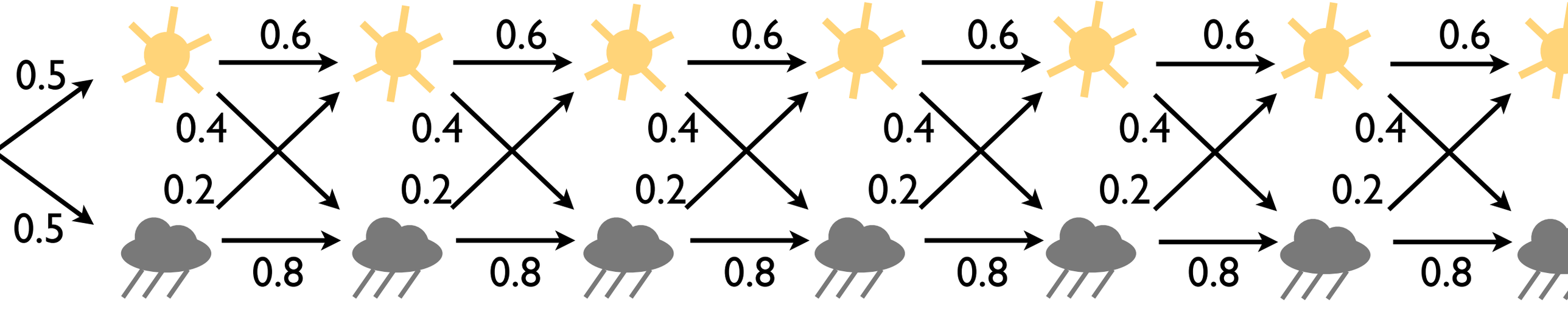



```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(sun,Tprev).
0.2::weather(sun,T) ; 0.8::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(rain,Tprev).
```

ProbLog by example:

Rain or sun?



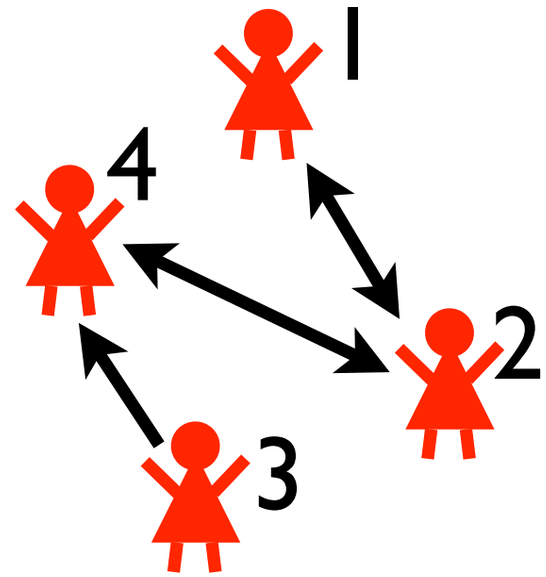

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(sun,Tprev).
0.2::weather(sun,T) ; 0.8::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(rain,Tprev).
```

infinite possible worlds! BUT: finitely many partial worlds suffice to answer any given ground query

ProbLog by example:

Friends & smokers

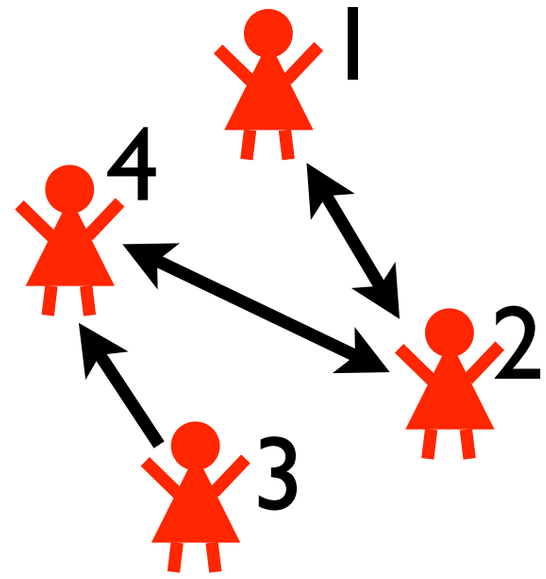



```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Friends & smokers



typed probabilistic facts

= a probabilistic fact for each grounding

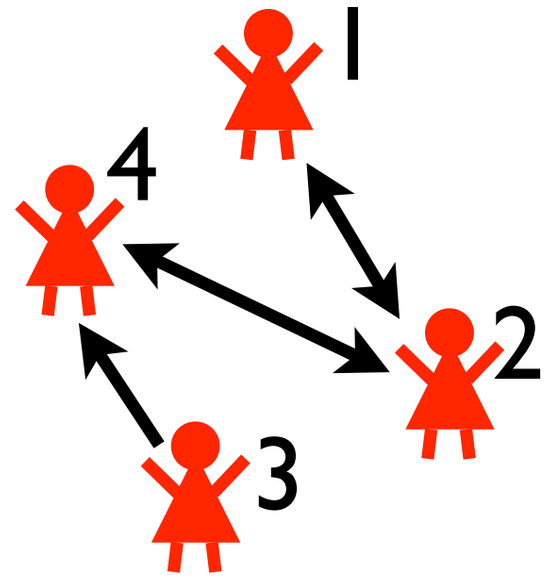
```
0.3::stress(X):- person(X).
0.2::influences(X,Y):-
                person(X), person(Y).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Friends & smokers



typed probabilistic facts

= a probabilistic fact for each grounding

```
0.3::stress(X):- person(X).
0.2::influences(X,Y):-
              person(X), person(Y).
```

```
0.3::stress(1).
0.3::stress(2).
0.3::stress(3).
0.3::stress(4).
```

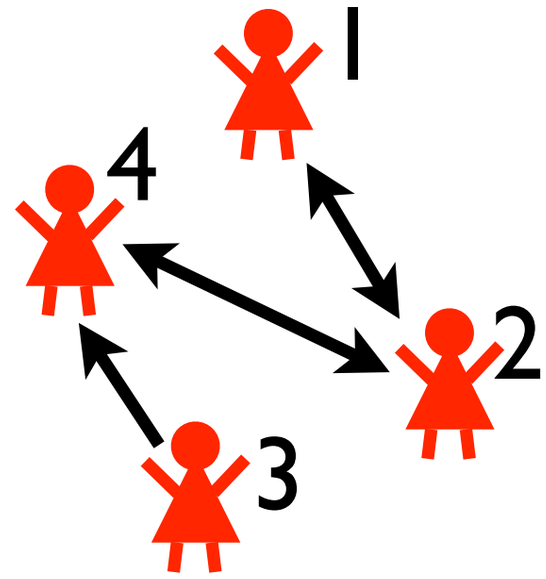
```
0.2::influences(1,1).
0.2::influences(1,2).
0.2::influences(1,3).
0.2::influences(1,4).
0.2::influences(2,1).
...
0.2::influences(4,2).
0.2::influences(4,3).
0.2::influences(4,4).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Friends & smokers




```
0.3::stress(X):- person(X).
0.2::influences(X,Y):-
              person(X), person(Y).

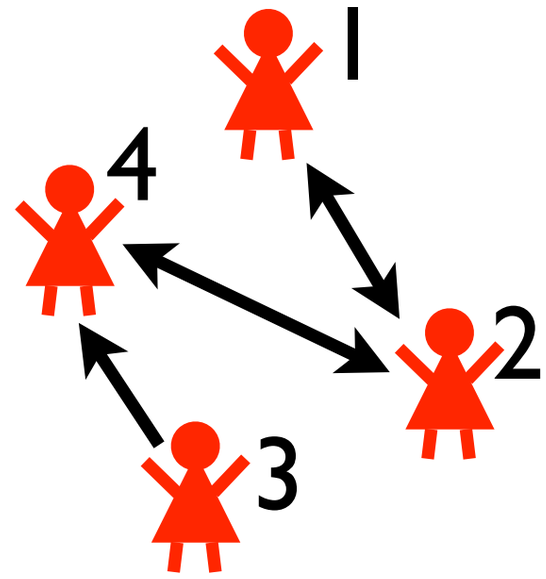
smokes(X) :- stress(X).
smokes(X) :-
     friend(X,Y), influences(Y,X), smokes(Y).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Friends & smokers




```
0.3::stress(X):- person(X).
0.2::influences(X,Y):-
              person(X), person(Y).

smokes(X)  :- stress(X).
smokes(X)  :-
     friend(X,Y), influences(Y,X), smokes(Y).

0.4::asthma(X) <- smokes(X).
```

```
person(1).
person(2).
person(3).
person(4).

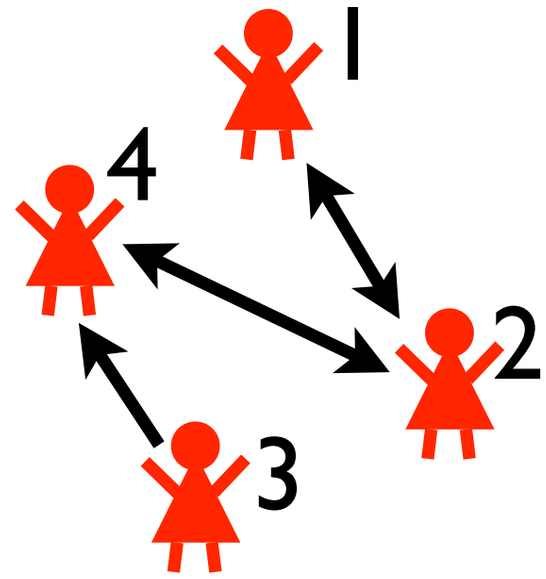
friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

annotated disjunction with implicit head atom:

with probability 0.6, nothing happens

ProbLog by example:

Friends & smokers




```
0.3::stress(X):- person(X).
0.2::influences(X,Y):-
              person(X), person(Y).

smokes(X) :- stress(X).
smokes(X) :-
     friend(X,Y), influences(Y,X), smokes(Y).

0.4::asthma(X) <- smokes(X).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).
```

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight), P is 1.0/Weight.
```

flexible probability:
computed from the weight of the item

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).
```

```
1/6::pack(skis).
1/4::pack(boots).
1/3::pack(helmet).
1/2::pack(gloves).
```

```
P::pack(Item) :- weight(Item,Weight), P is 1.0/Weight.
```

flexible probability:
computed from the weight of the item

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).
```

list of all items

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).

excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
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P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

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excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

pack first item, decrease limit by its weight, and continue with rest of items

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).

excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

do **not** pack first item,
continue with rest of items

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).

excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

no items left: did we add too much?

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).

excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

Summary: ProbLog Syntax

- input database: ground facts

```
person(bob).
```

- probabilistic facts

```
0.5::stress(bob).
```

- typed probabilistic facts (body deterministic)

```
0.5::stress(X) :- person(X).
```

- flexible probabilities

```
P::pack(Item) :- weight(Item,W),
                 P is 1.0/W.
```

- annotated disjunctions

```
0.4::asthma(X) <- smokes(X).
```

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.
```

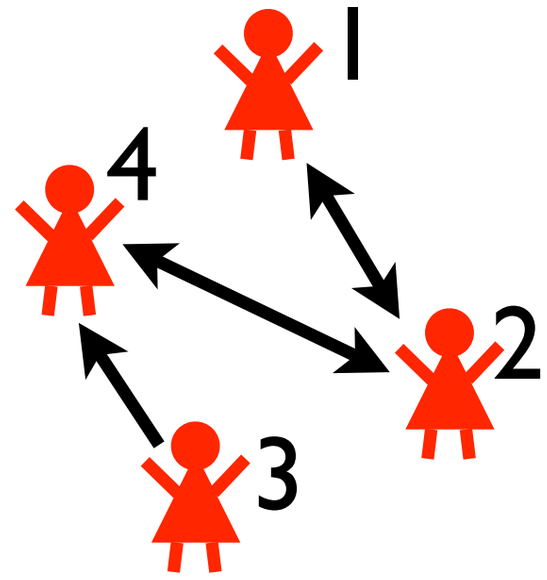
- Prolog clauses

```
smokes(X) :- influences(Y,X), smokes(Y).
```

```
excess([I|R],Limit) :- \+pack(I), excess(R,Limit).
```

ProbLog example:

Friends & smokers



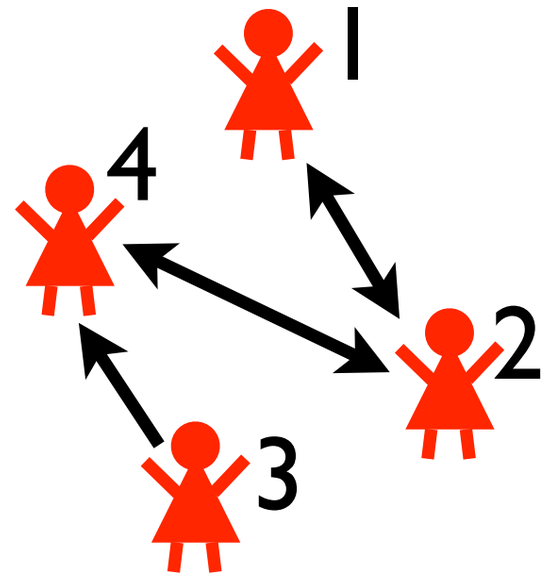

```
0.5::stress(1).
0.1::stress(2).
0.8::stress(3).
0.3::stress(4).

0.9::friend(1,2).
0.8::friend(2,1).
0.3::friend(2,4).
0.7::friend(3,4).
0.1::friend(4,2).

smokes(X) :- stress(X).
smokes(X) :-
     friend(Y,X), smokes(Y).
```

ProbLog example:

Friends & smokers

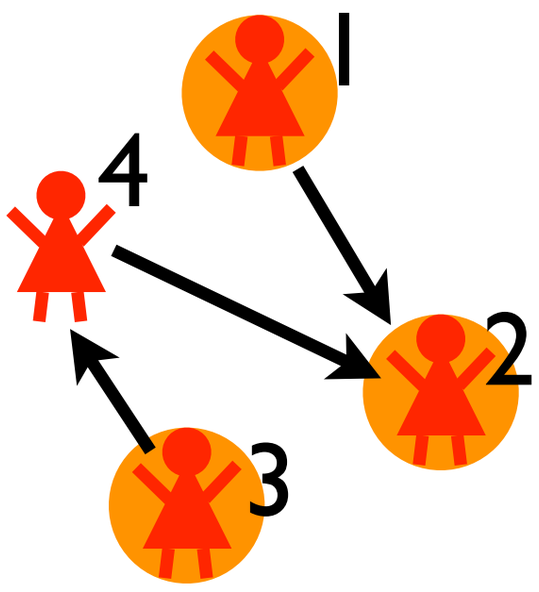



```
0.5::stress(1).
0.1::stress(2).
0.8::stress(3).
0.3::stress(4).

0.9::friend(1,2).
0.8::friend(2,1).
0.3::friend(2,4).
0.7::friend(3,4).
0.1::friend(4,2).

smokes(X) :- stress(X).
smokes(X) :-
     friend(Y,X), smokes(Y).
```

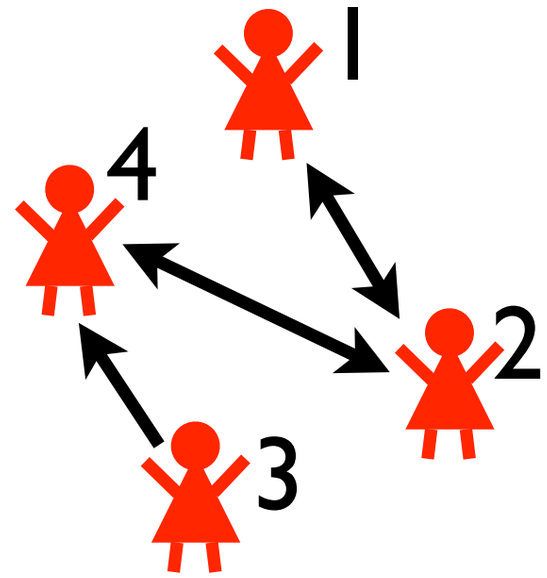
example possible world




```
friend(1,2).
friend(3,4).
friend(4,2).
stress(1).
stress(2).
stress(3).
smokes(1).
smokes(2).
smokes(3).
smokes(4).
```

ProbLog example:

Friends & smokers

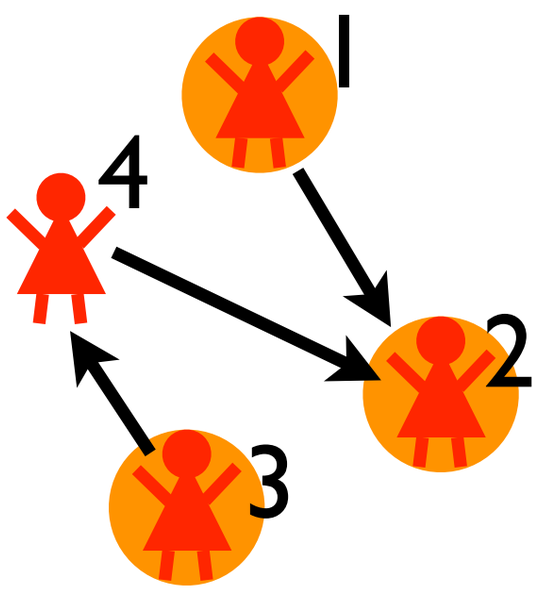



```
0.5::stress(1).
0.1::stress(2).
0.8::stress(3).
0.3::stress(4).

0.9::friend(1,2).
0.8::friend(2,1).
0.3::friend(2,4).
0.7::friend(3,4).
0.1::friend(4,2).

smokes(X) :- stress(X).
smokes(X) :-
     friend(Y,X), smokes(Y).
```

example possible world

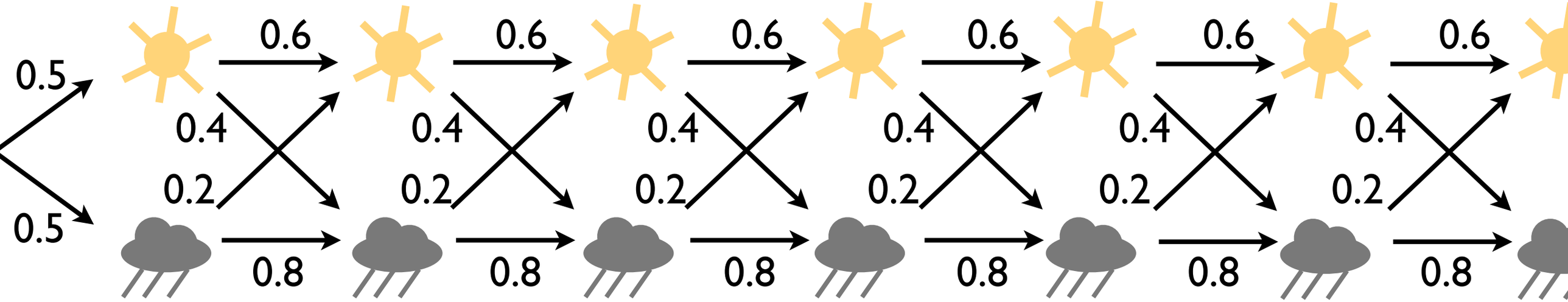



```
friend(1,2).
friend(3,4).
friend(4,2).
stress(1).
stress(2).
stress(3).
smokes(1).
smokes(2).
smokes(3).
smokes(4).
```

- several instances of **`smokes(X)`** in same world
- **`smokes(2)`** : multiple derivations in same world
- distribution over worlds not (always) a distribution over computations / answers

ProbLog by example:

Rain or sun?




```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

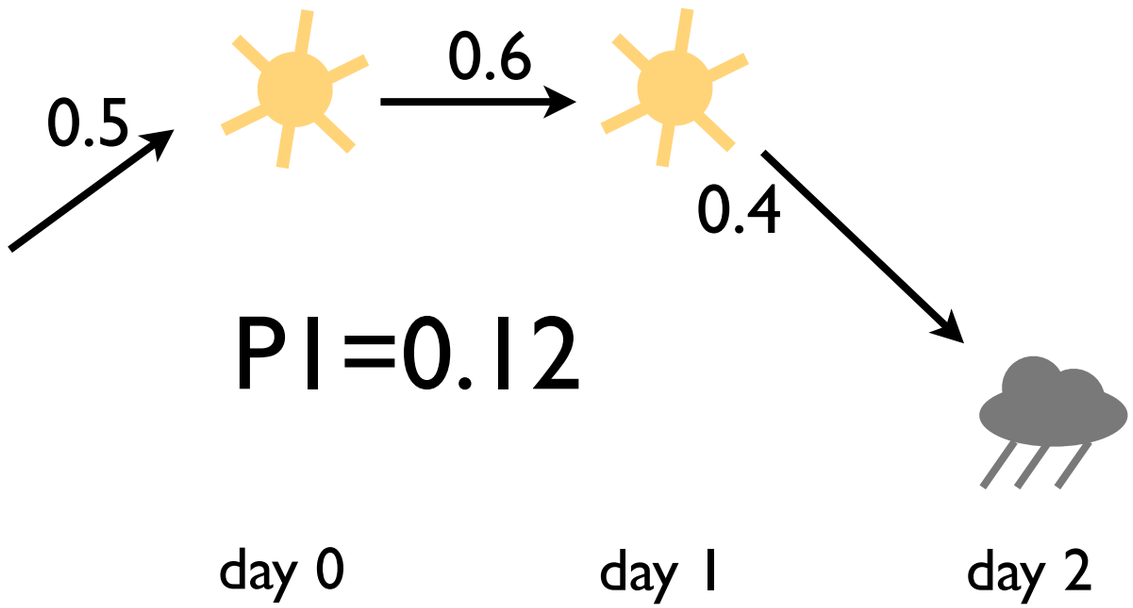
0.6::weather(sun,T) ; 0.4::weather(rain,T)
                    <- T>0, Tprev is T-1, weather(sun,Tprev).
0.2::weather(sun,T) ; 0.8::weather(rain,T)
                    <- T>0, Tprev is T-1, weather(rain,Tprev).
```

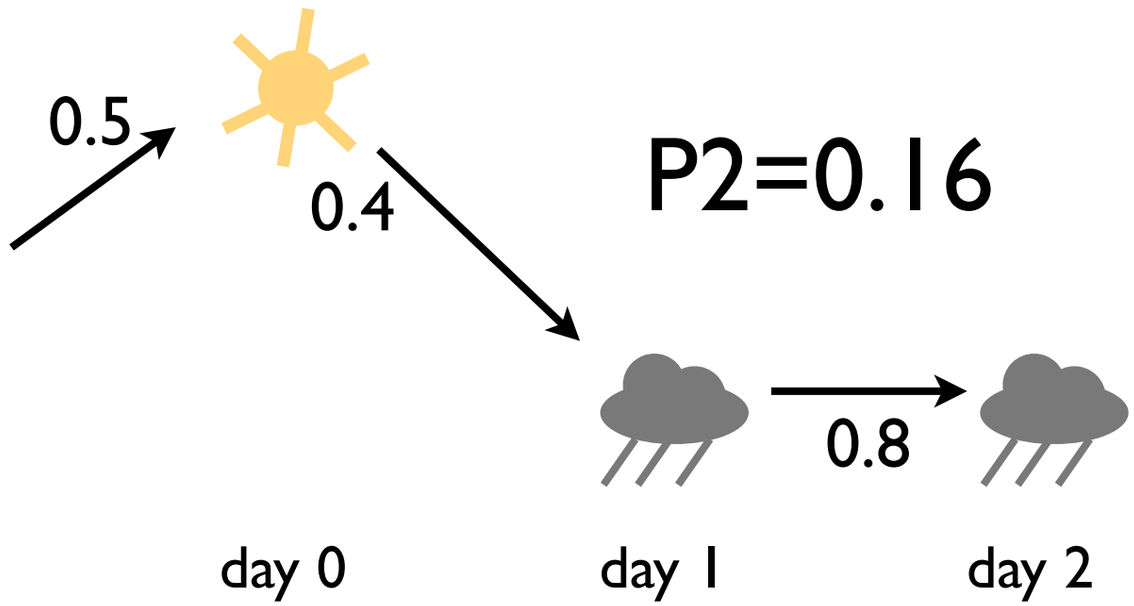
≤1 proof for a ground query per possible world → distribution over worlds is distribution over derivations!

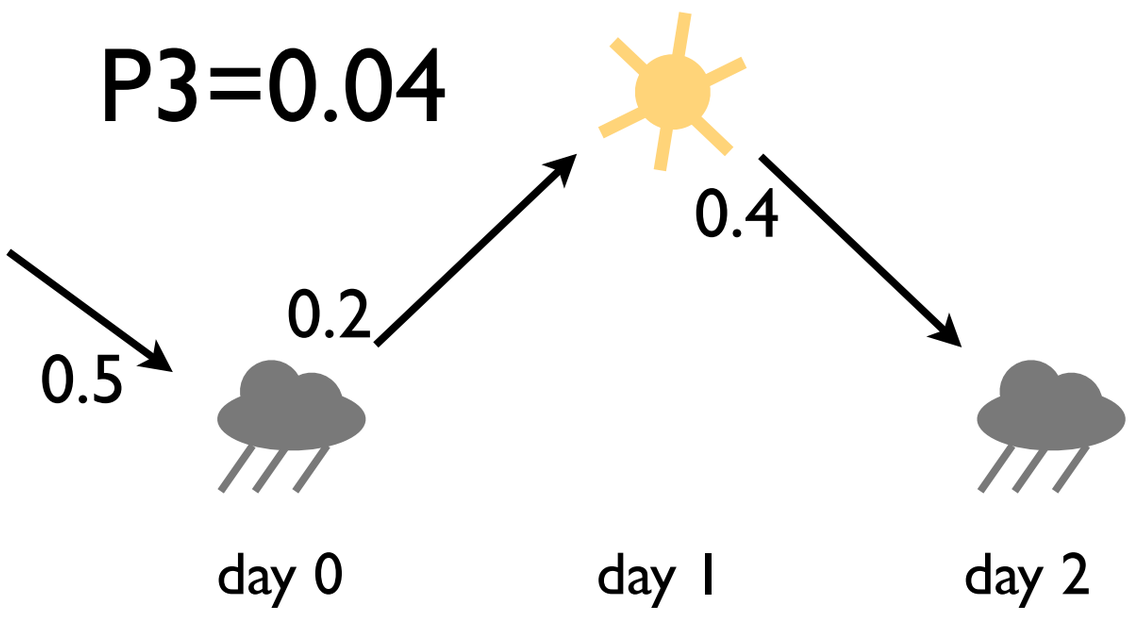
Possible worlds

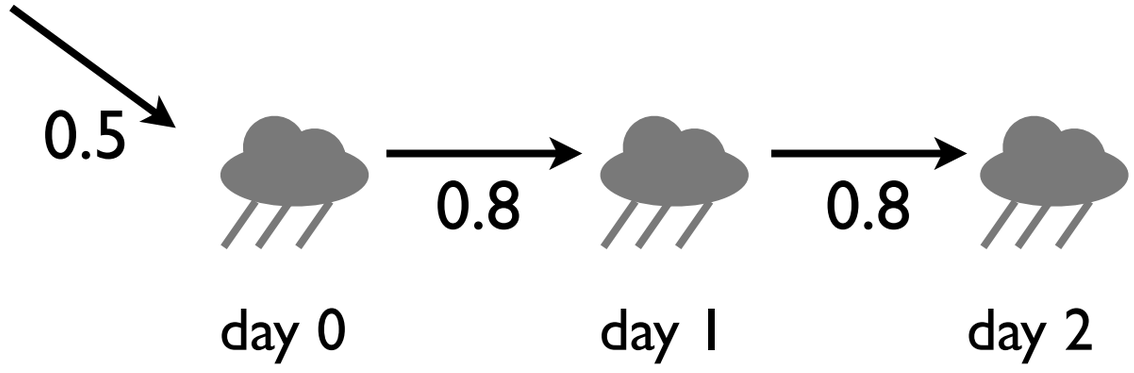
```
?- weather(rain,2).
```

P=P1+P2+P3+P4









Mutually Exclusive Rules:

no two rules apply simultaneously

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                <- T>0, Tprev is T-1, weather(sun,Tprev).

0.2::weather(sun,T) ; 0.8::weather(rain,T)
                <- T>0, Tprev is T-1, weather(rain,Tprev).
```

Mutually Exclusive Rules:

no two rules apply simultaneously

first rule for day 0, others for later days

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                <- T>0, Tprev is T-1, weather(sun,Tprev).

0.2::weather(sun,T) ; 0.8::weather(rain,T)
                <- T>0, Tprev is T-1, weather(rain,Tprev).
```

Mutually Exclusive Rules:

no two rules apply simultaneously

first rule for day 0, others for later days

day 0: either sun or rain

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                <- T>0, Tprev is T-1, weather(sun,Tprev).

0.2::weather(sun,T) ; 0.8::weather(rain,T)
                <- T>0, Tprev is T-1, weather(rain,Tprev).
```

Mutually Exclusive Rules:

no two rules apply simultaneously

first rule for day 0, others for later days

day 0: either sun or rain

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                    <- T>0, Tprev is T-1, weather(sun,Tprev).

0.2::weather(sun,T) ; 0.8::weather(rain,T)
                    <- T>0, Tprev is T-1, weather(rain,Tprev).
```

rules for T>0 cover mutually exclusive cases on previous day

[Sato and Kameya, JAIR 01]

PRISM

- Another probabilistic Prolog based on the distribution semantics
- Mutual exclusiveness assumption
  - allows for efficient inference by dynamic programming, cf. probabilistic grammars
  - but excludes certain models, e.g., smokers
- http://sato-www.cs.titech.ac.jp/prism/

PRISM

- "multi-valued random switches" = annotated disjunctions with body *true*
- switch gives fresh result on each call
- Prolog rules
- limited support for negation (compiling away)

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain]).

:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

```
weather(W,Time) :-
  Time >= 0,
  msw(init,W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
   Now < T,
   msw(tr(WNow),WNext),
   Next is Now+1,
   w(Next,T,WNext,WT).
```

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

```
:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

```
weather(W,Time) :-
  Time >= 0,
  msw(init,W0),
  w(0,Time,W0,W).

w(T,T,W,W).
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   msw(tr(WNow),WNext),
   Next is Now+1,
   w(Next,T,WNext,WT).
```

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

```
:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

probability distributions

```
weather(W,Time) :-
  Time >= 0,
  msw(init,W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
   Now < T,
   msw(tr(WNow),WNext),
   Next is Now+1,
   w(Next,T,WNext,WT).
```

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

```
:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

probability distributions

```
weather(W,Time) :-
   Time >= 0,
   msw(init,W0),
   w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

set **W0** to random value of **init**

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

```
:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

probability distributions

```
weather(W,Time) :-
   Time >= 0,
   msw(init,W0),
   w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

set **`W0`** to random value of **`init`**

set **`WNext`** to random value of **`tr(WNow)`**, using **fresh** value on every call

Weather in PRISM / ProbLog

```
values(init,[sun,rain]).
values(tr(_),[sun,rain]).

:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

```
weather(W,Time) :-
  Time >= 0,
  msw(init,W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

```
0.5::init(sun) ;
  0.5::init(rain) <- true.
0.6::tr(T,sun,sun) ;
  0.4::tr(T,sun,rain) <- true.
0.2::tr(T,rain,sun) ;
  0.8::tr(T,rain,rain) <- true.
```

```
weather(W,Time) :-
  Time >= 0,
  init(W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    tr(Now,WNow,WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

ProbLog needs to explicitly use different facts at each call

Probabilistic Prologs: Two Views

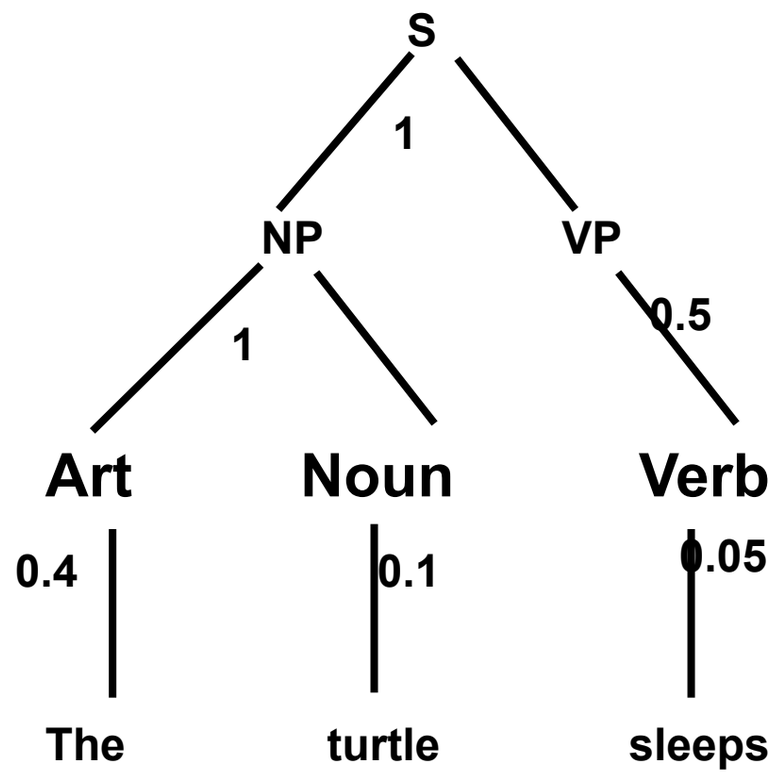
- Distribution semantics:
  - probability distribution over interpretations
  - degree of belief
- Stochastic Logic Programs (SLPs):
  - probability distribution over query answers
  - like in probabilistic grammars

Probabilistic Prologs: Two Views

- Distribution semantics:
  - probability distribution over interpretations
  - degree of belief
- Stochastic Logic Programs (SLPs):
  - probability distribution over query answers
  - like in probabilistic grammars

Probabilistic Context Free Grammars

1.0 : S -> NP, VP
1.0 : NP -> Art, Noun
0.6 : Art -> a
0.4 : Art -> the
0.1 : Noun -> turtle
0.1 : Noun -> turtles
…
0.5 : VP -> Verb
0.5 : VP -> Verb, NP
0.05 : Verb -> sleep
0.05 : Verb -> sleeps
….



P(parse tree) = 1x1x.5x.1x.4x.05

PCFGs

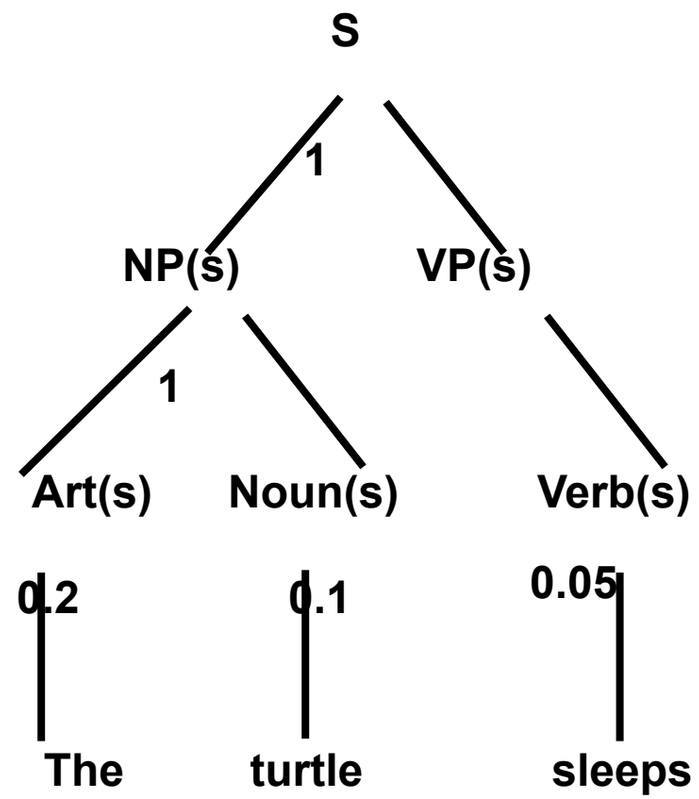
$P(parse\ tree) = \prod_i p_i^{c_i}$
where p_i is the probability of rule i
and c_i the number of times
it is used in the parse tree

$P(sentence) = \sum_{p:parsetree} P(p)$

Observe that derivations always succeed, that is
$S \to T, Q$ and $T \to R, U$
always yields
$S \to R, U, Q$

Probabilistic Definite Clause Grammar

1.0 S -> NP(Num), VP(Num)
1.0 NP(Num) -> Art(Num), Noun(Num)
0.6 Art(sing) -> a
0.2 Art(sing) -> the
0.2 Art(plur) -> the
0.1 Noun(sing) -> turtle
0.1 Noun(plur) -> turtles
…
0.5 VP(Num) -> Verb(Num)
0.5 VP(Num) -> Verb(Num), NP(Num)
0.05 Verb(sing) -> sleeps
0.05 Verb(plur) -> sleep
….



P(derivation tree) = 1x1x.5x.1x .2 x.05

Stochastic Logic Programs

[58][Muggleton, Cussens]

In SLP notation

1

```
sentence(A, B) :- noun_phrase(C, A, D), verb_phrase(C, D, B).
noun_phrase(A, B, C) :- article(A, B, D), noun(A, D, C).
verb_phrase(A, B, C) :- intransitive_verb(A, B, C).
```

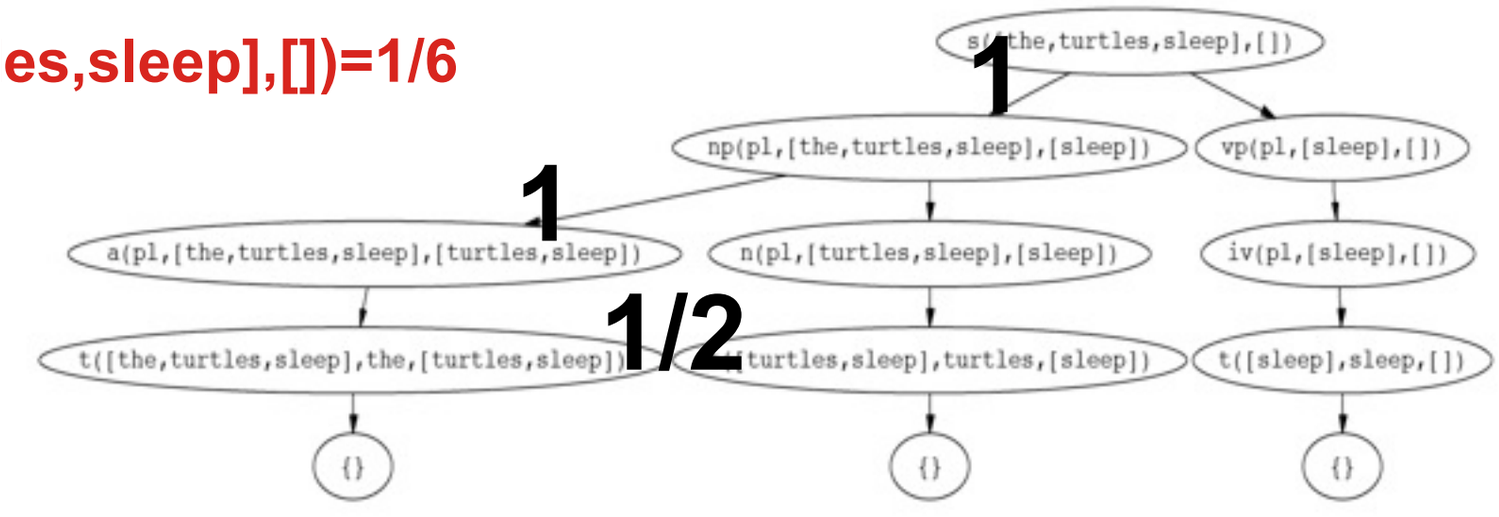
1/3

```
article(singular, A, B) :- terminal(A, a, B).
article(singular, A, B) :- terminal(A, the, B).
article(plural, A, B)   :- terminal(A, the, B).
```

1/2

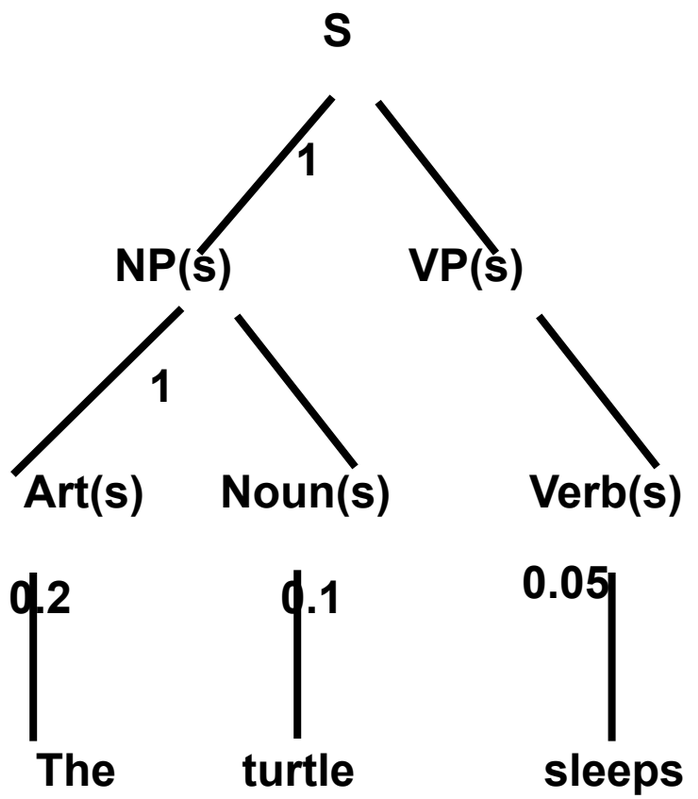
```
noun(singular, A, B) :- terminal(A, turtle, B).
noun(plural, A, B)   :- terminal(A, turtles, B).
intransitive_verb(singular, A, B) :- terminal(A, sleeps, B).
intransitive_verb(plural, A, B) :- terminal(A, sleep, B).
terminal([A|B],A,B).
```

P(s([the,turtles,sleep],[])=1/6



Probabilistic DCG

1.0 S -> NP(Num), VP(Num)
1.0 NP(Num) -> Art(Num), Noun(Num)
0.6 Art(sing) -> a
0.2 Art(sing) -> the
0.2 Art(plur) -> the
0.1 Noun(sing) -> turtle
0.1 Noun(plur) -> turtles
…
0.5 VP(Num) -> Verb(Num)
0.5 VP(Num) -> Verb(Num), NP(Num)
0.05 Verb(sing) -> sleeps
0.05 Verb(plur) -> sleep
….



P(derivation tree) = 1x1x.5x.1x .2 x.05

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1.0 S -> NP(Num), VP(Num)
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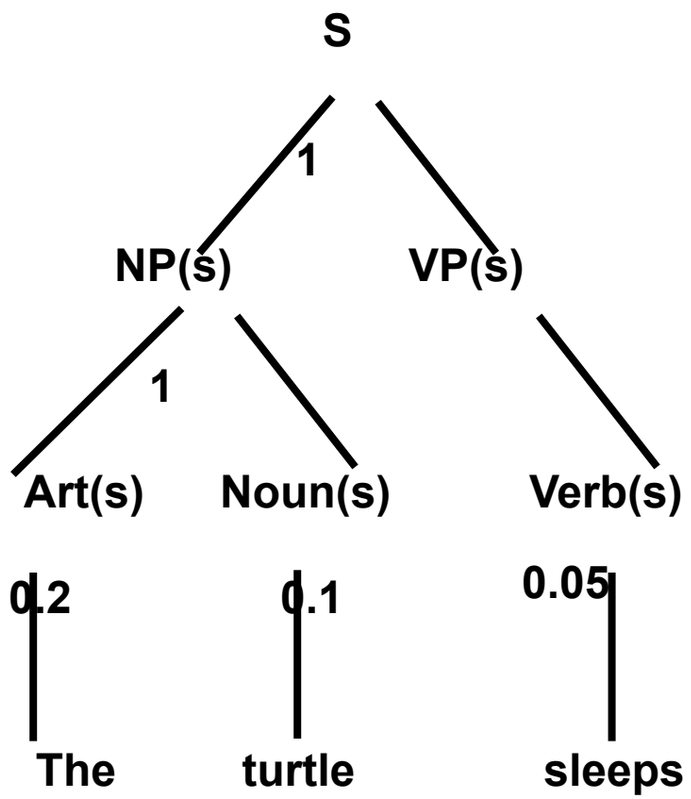


P(derivation tree) = 1x1x.5x.1x .2 x.05

What about "A turtles sleeps" ?

SLPs

$P_d(derivation) = \prod_i p_i^{c_i}$
where p_i is the probability of rule i
and c_i the number of times
it is used in the parse tree

Observe that some derivations now fail due to unification,
$np(Num) \rightarrow art(Num), noun(Num)$ and $art(sing) \rightarrow a$
$noun(plural) \rightarrow turtles$

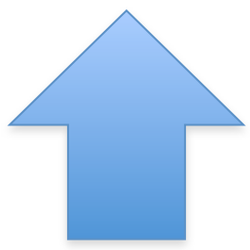
Normalization necessary
$P_s(proof) = \frac{P_d(proof)}{\sum_i P_d(proof_i)}$

ProPPR

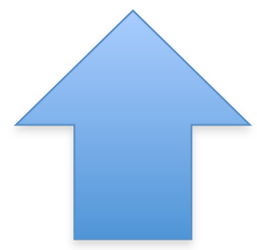
- A variation on SLPs
- Integrating concepts from Personalized Page Rank
- Fast inference and rule learning abilities
- Used by CMU group for NELL (Never Ending Learning)
- See [Wang et al., CIKM 13, arXiv:1404.3301]

Sample ProPPR program….

```
about(X,Z) :- handLabeled(X,Z)          # base.
about(X,Z) :- sim(X,Y),about(Y,Z)       # prop.
sim(X,Y) :- links(X,Y)                  # sim,link.
sim(X,Y) :-
    hasWord(X,W),hasWord(Y,W),
    linkedBy(X,Y,W)                     # sim,word.
linkedBy(X,Y,W) :- true                 # by(W).
```

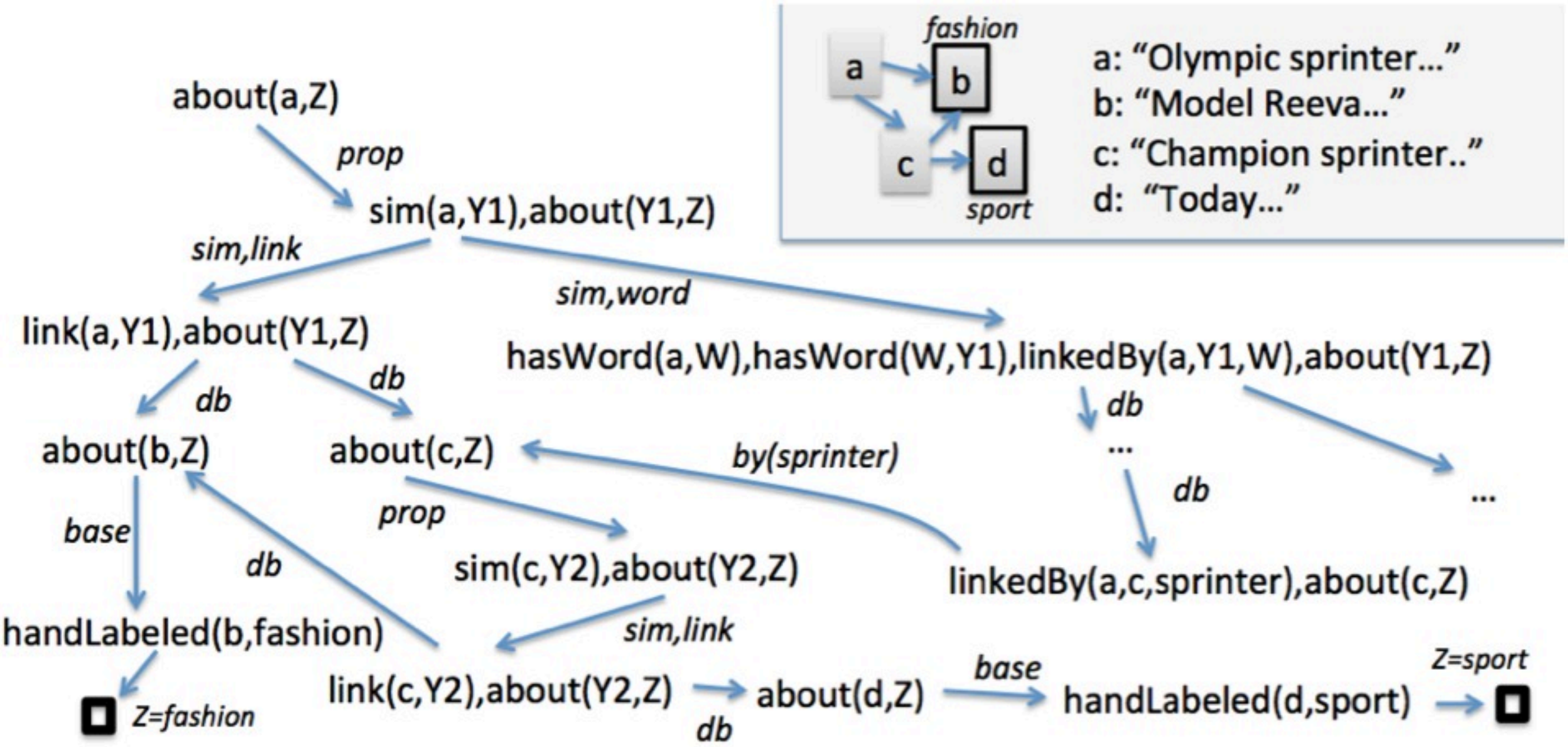


Horn rules



features of rules
(vars from head ok)

[Slide by William Cohen]



.. and search space…

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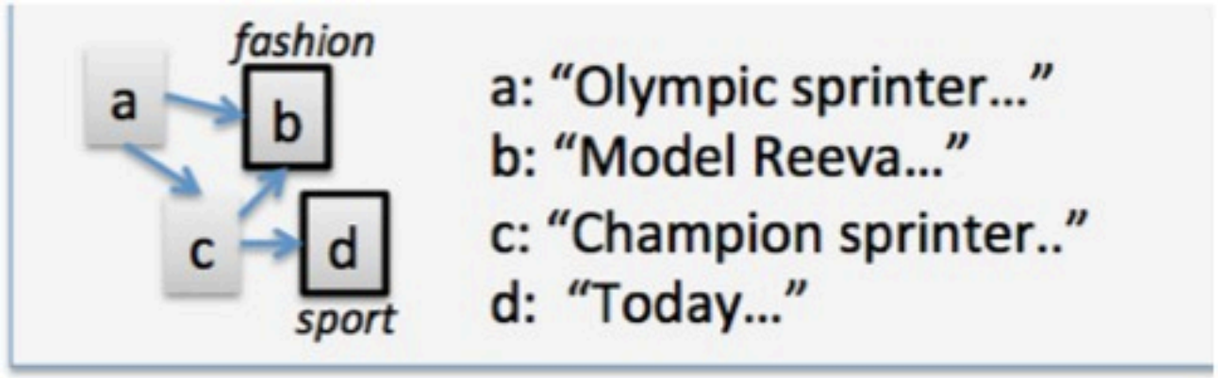
linkedBy(X,Y,W) :- true                 # by(W).
```

[Slide by William Cohen]

D'oh! This is a graph!



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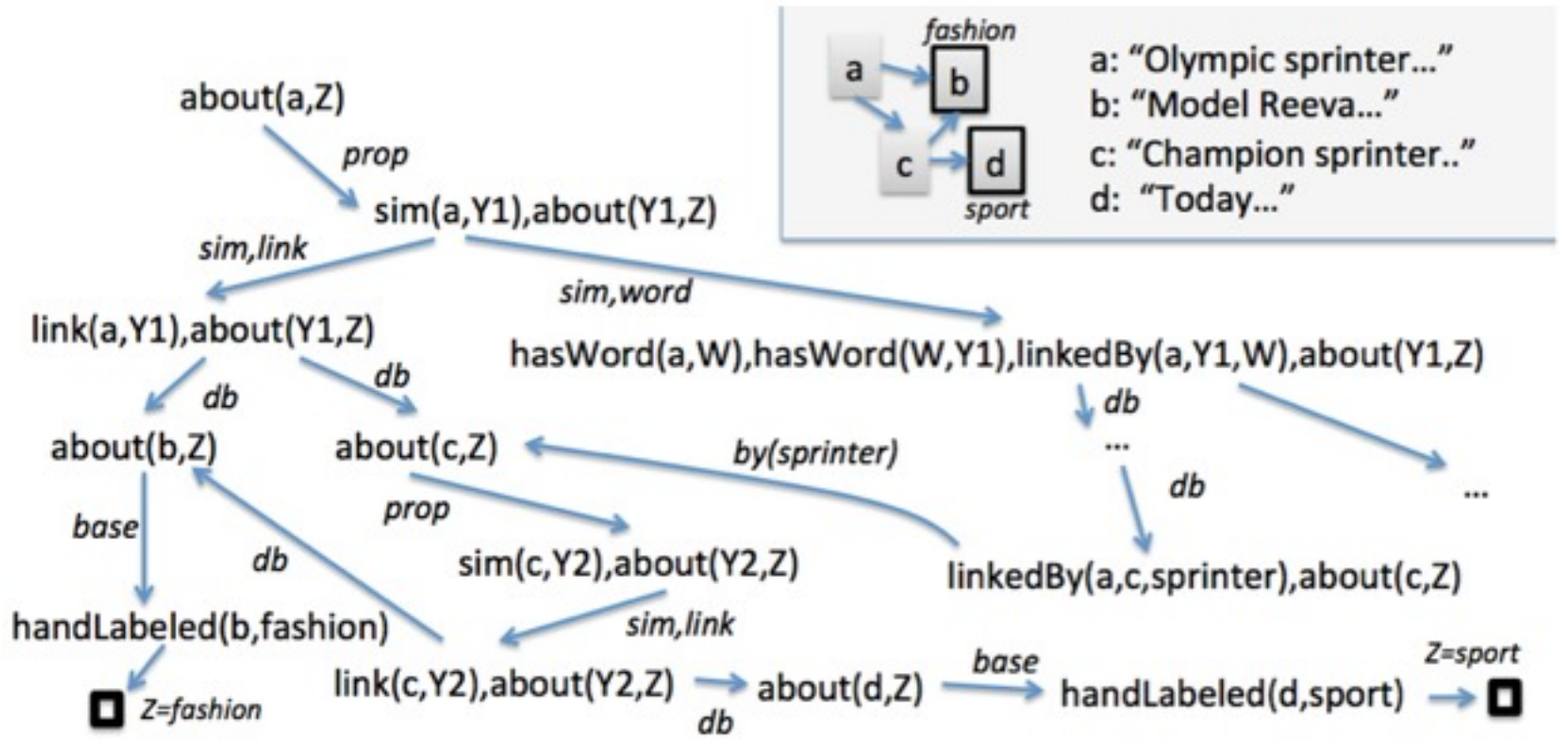
- Score for a query soln (e.g., “Z=sport” for “about(a,Z)”) depends on probability of reaching a □ node*
  - learn transition probabilities based on features of the rules
  - implicit “reset” transitions with (p≥α) back to query node
- Looking for answers supported by many short proofs



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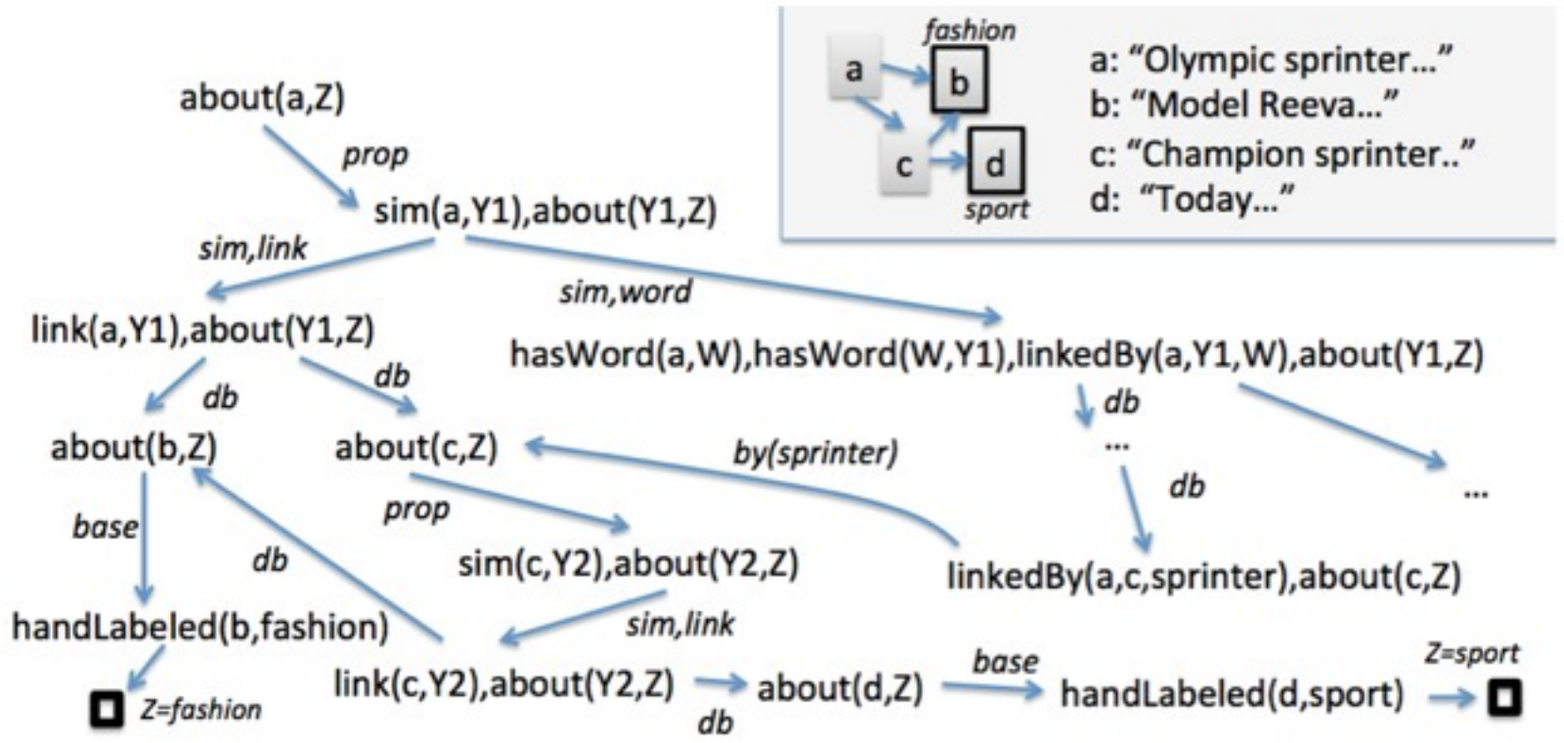
*Exactly as in Stochastic Logic Programs [Cussens, 2001]



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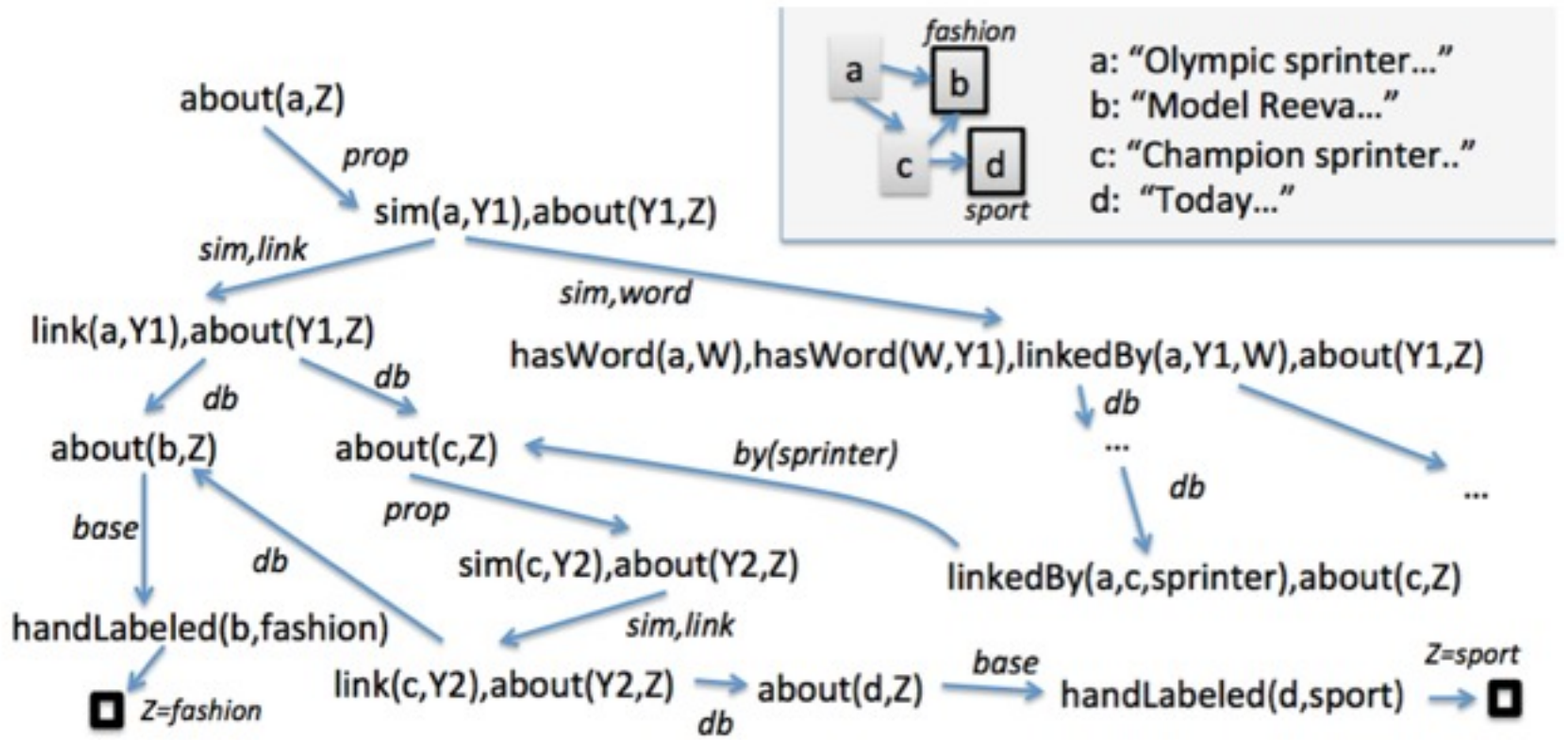

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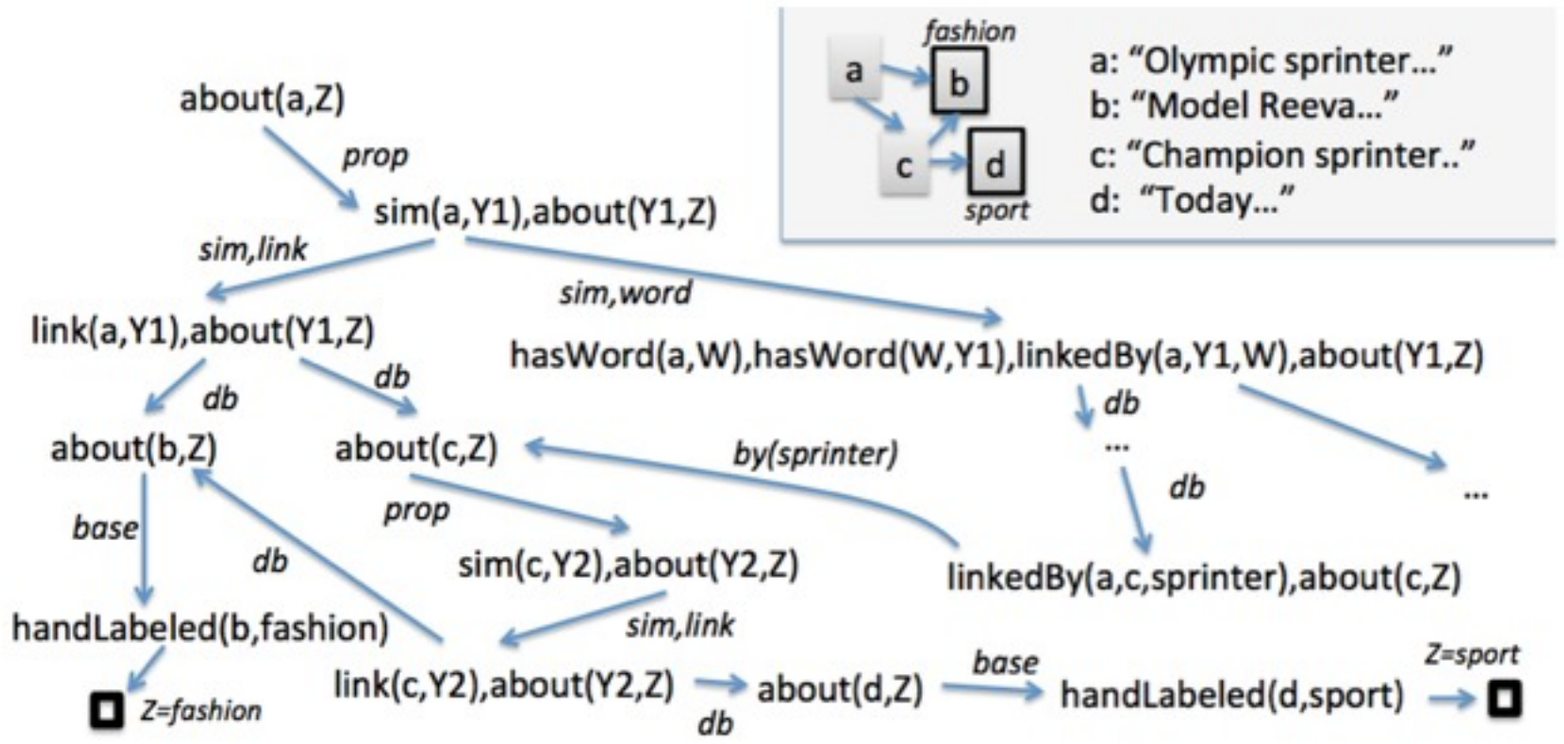
Learning: supervised variant of personalized PageRank (Backstrom & Leskovic, 2011)



[Slide by William Cohen]

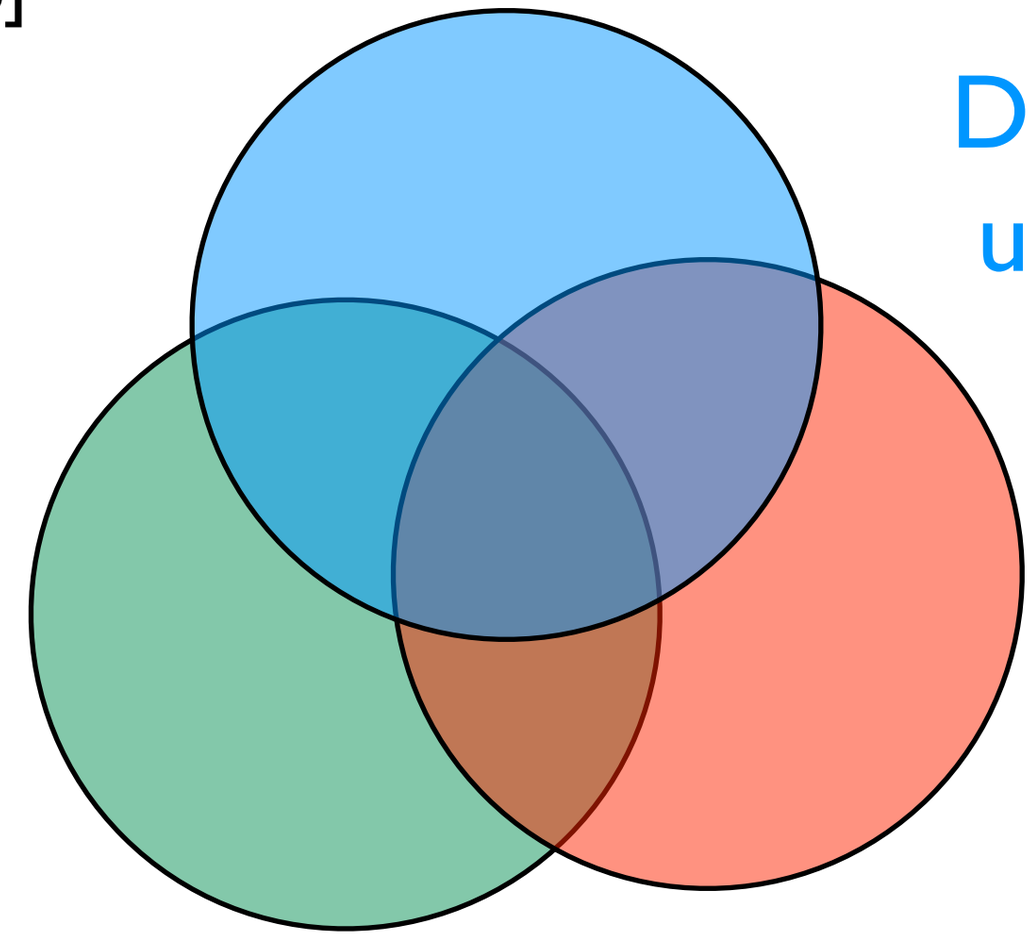
Probabilistic Programming Languages outside LP

- IBAL [Pfeffer 01]
- Figaro [Pfeffer 09]
- Church [Goodman et al 08]
- BLOG [Milch et al 05]
- and many more appearing recently

Church

probabilistic functional programming

[Goodman et al, UAI 08]



Dealing with uncertainty

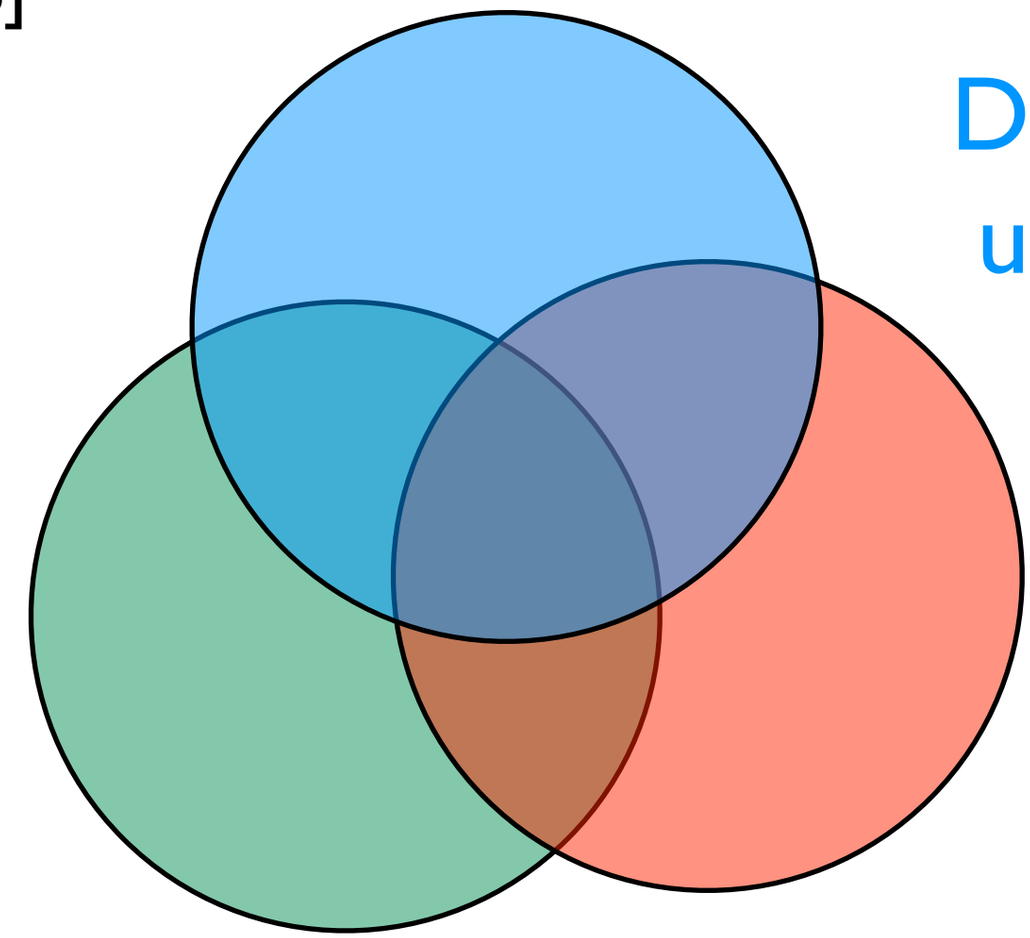
Reasoning with relational data

Learning

Church

probabilistic functional programming

[Goodman et al, UAI 08]



Dealing with uncertainty

functional programming

Learning

```
(define plus5 (lambda (x) (+ x 5)))

(map plus5 '(1 2 3))
```

Church

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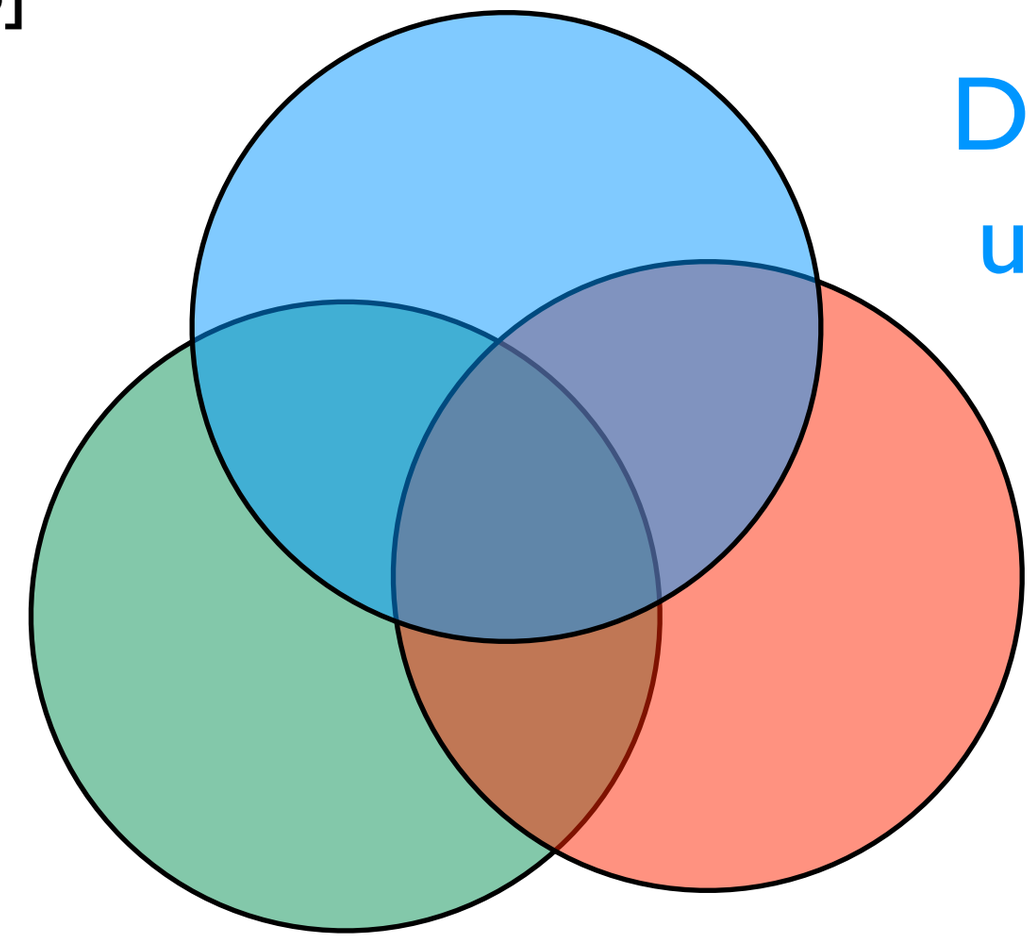

Dealing with uncertainty

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Learning

one execution

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Church

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(define randplus5
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                 (+ x 5)
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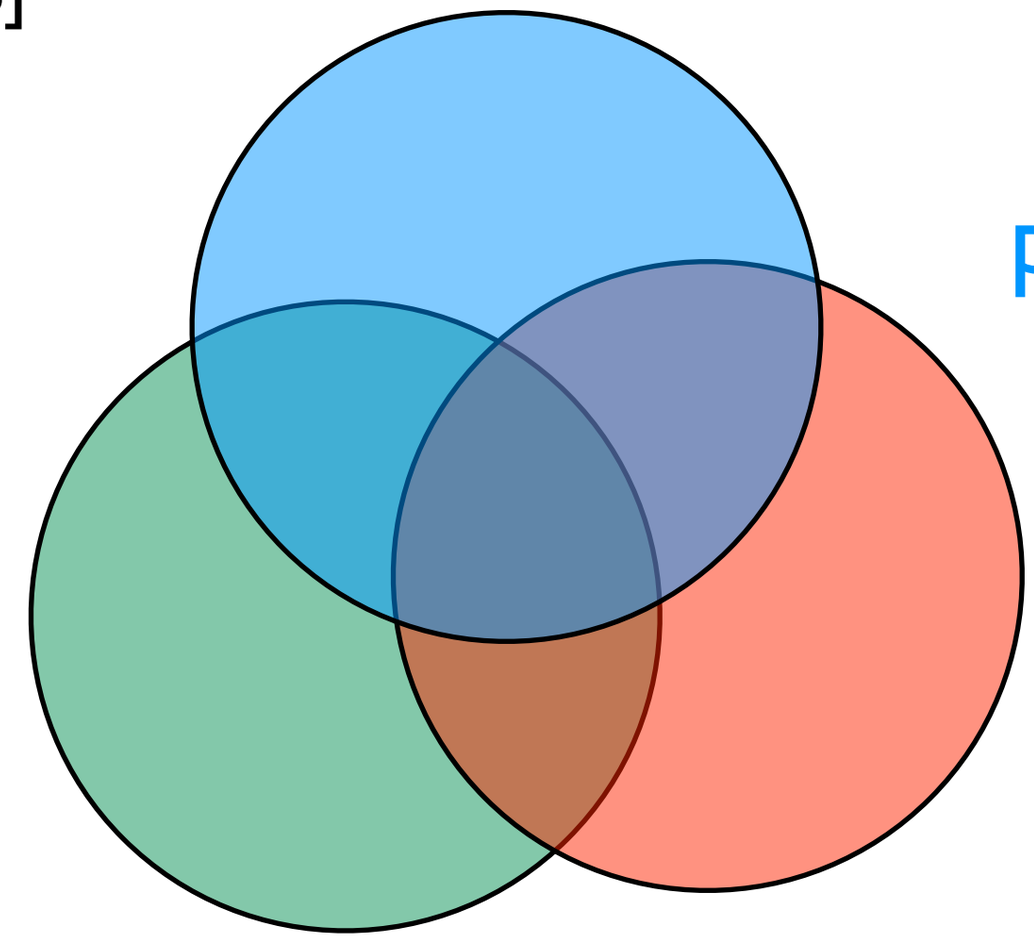
```



random primitives

functional programming

Learning

one execution

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(define plus5 (lambda (x) (+ x 5)))

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Church

probabilistic functional programming

[Goodman et al, UAI 08]

several possible executions

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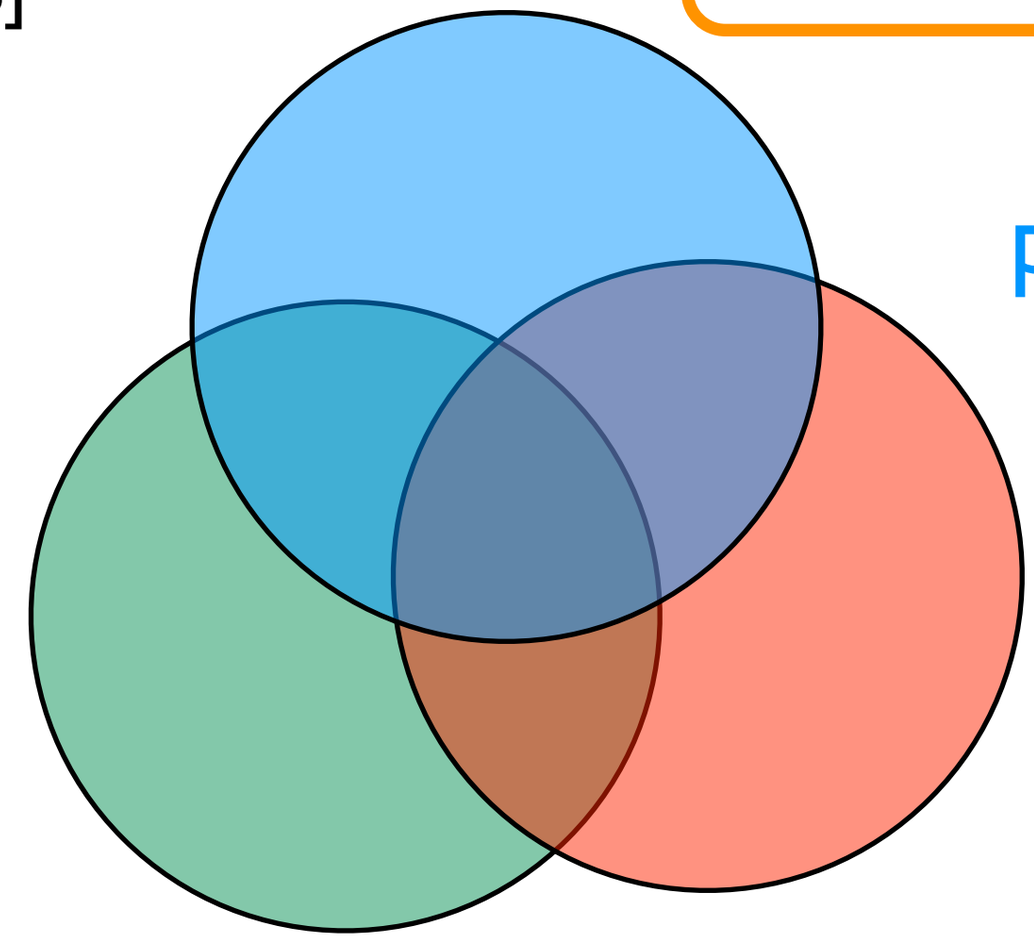

random primitives

functional programming

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Church

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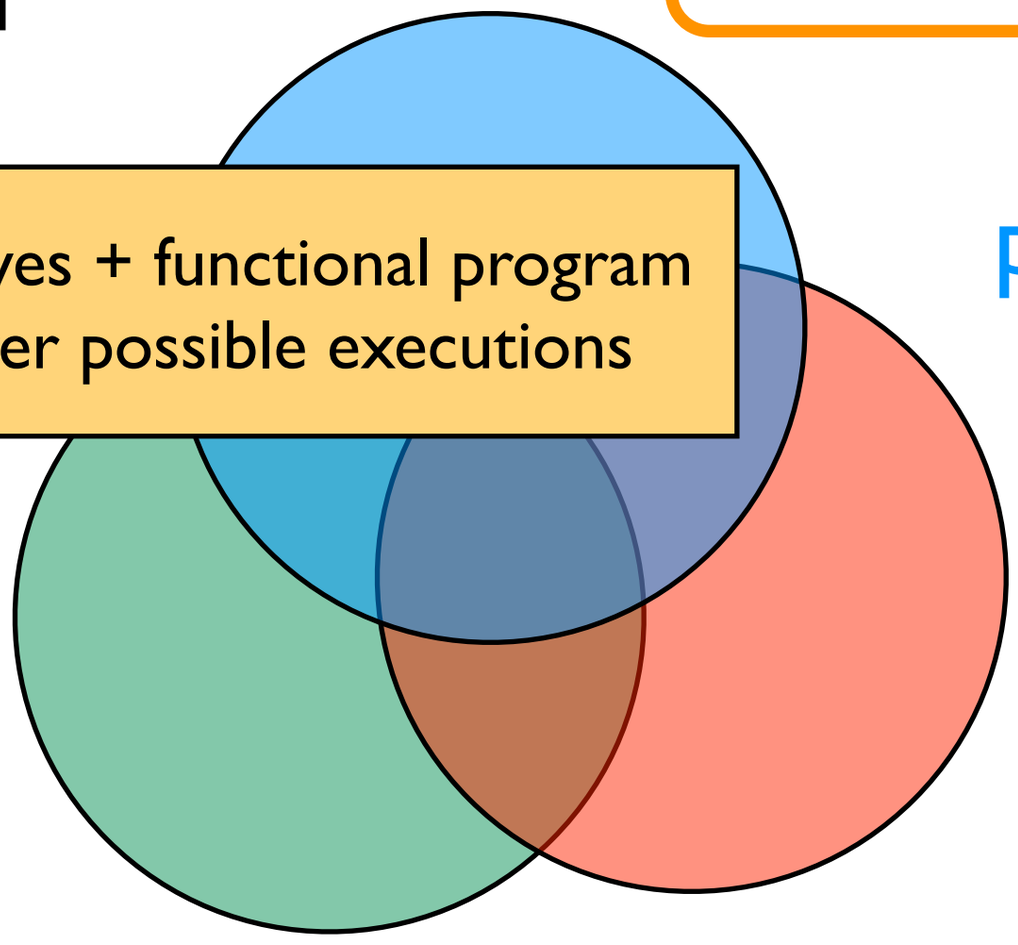

probabilistic primitives + functional program
→ distribution over possible executions

random primitives

functional programming

Learning

one execution

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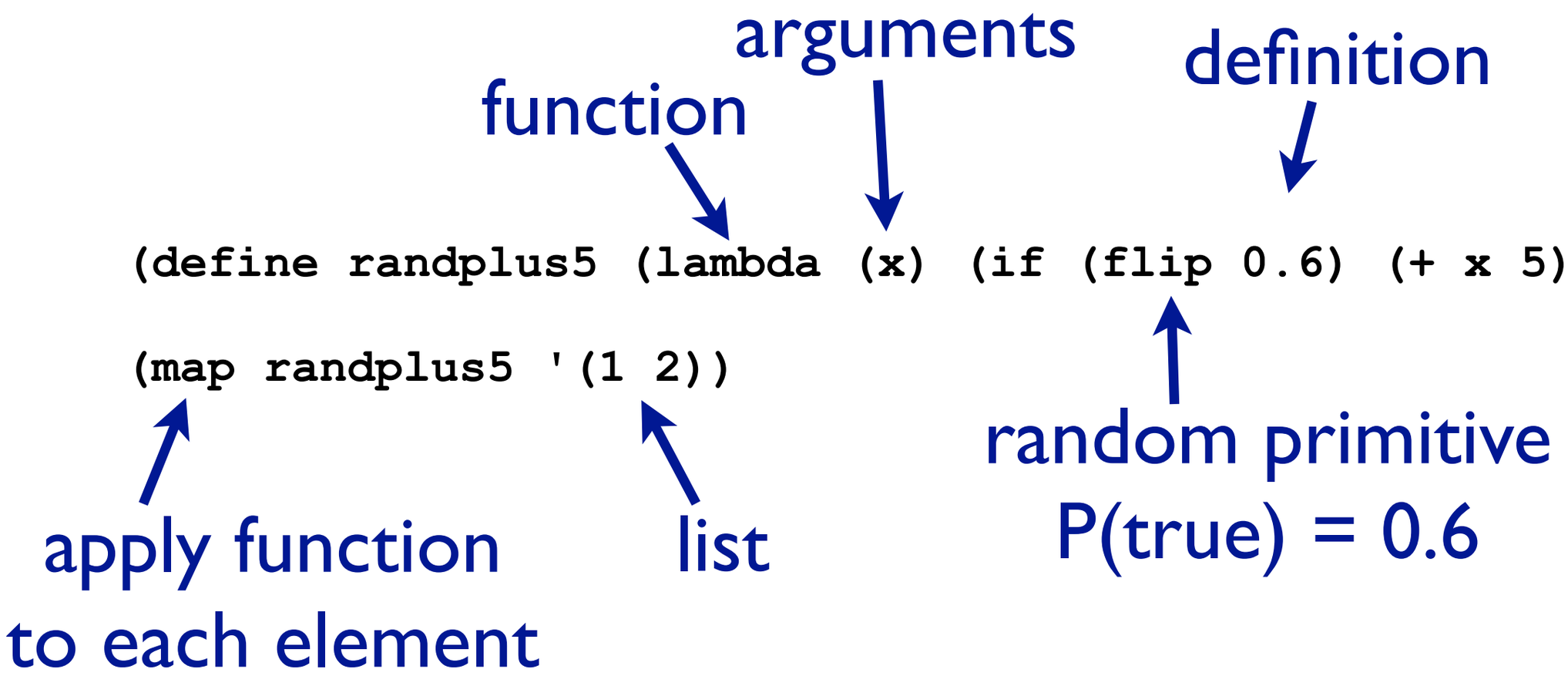
Church Example



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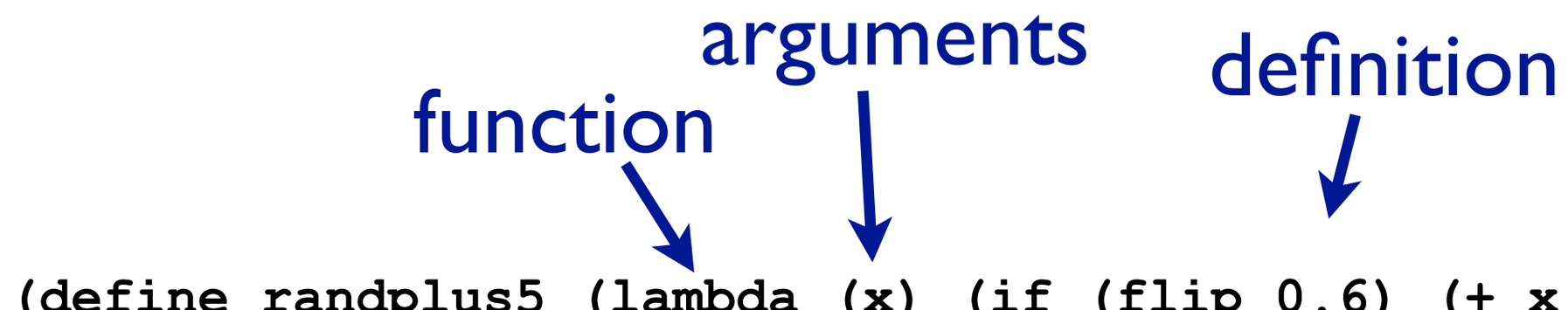


```
(map randplus5 '(1 2))
```

apply function to each element

list

result: (1 2) with 0.4×0.4
(1 7) with 0.4×0.6
(6 2) with 0.6×0.4
(6 7) with 0.6×0.6

in ProbLog?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

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lp5([],[]).
lp5([N|L],[M|K]) :-
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query(lp5([1,2],_)).
```

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```

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(1 6) with 0.4×0.6
(6 1) with 0.6×0.4
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stochastic memoization

in ProbLog?

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in ProbLog?

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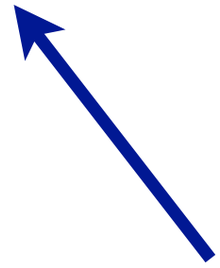
```

identifier distinguishes calls

Stochastic Memoization

```
(define randplus5 (mem (lambda (x) (if (flip 0.6) (+ x 5) x))))

(map randplus5 '(1 1))
```



remember first value &
reuse for all later calls

Stochastic Memoization

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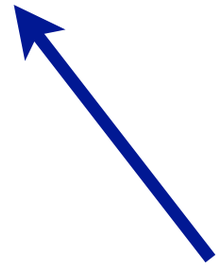

remember first value &
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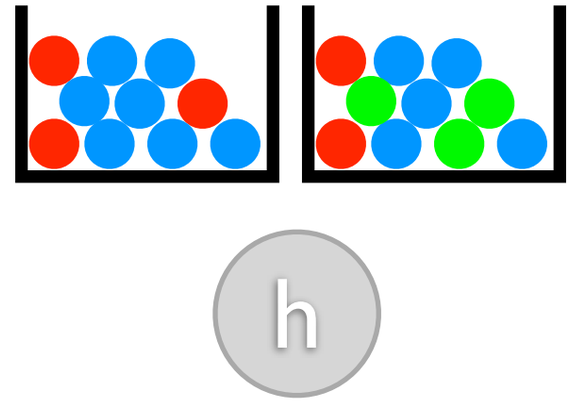
ProbLog always memoizes
PRISM never memoizes
Church allows fine-grained choice

Church by example:

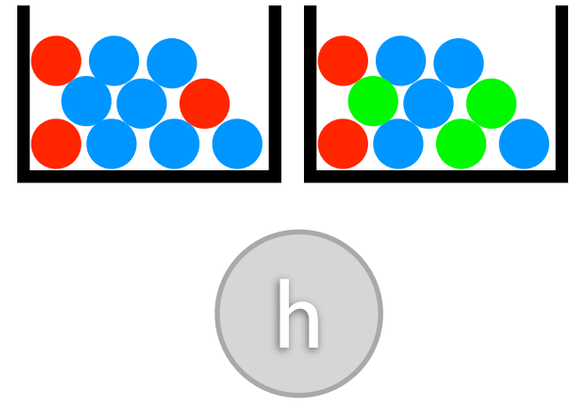
A bit of gambling



- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

Church by example:

A bit of gambling



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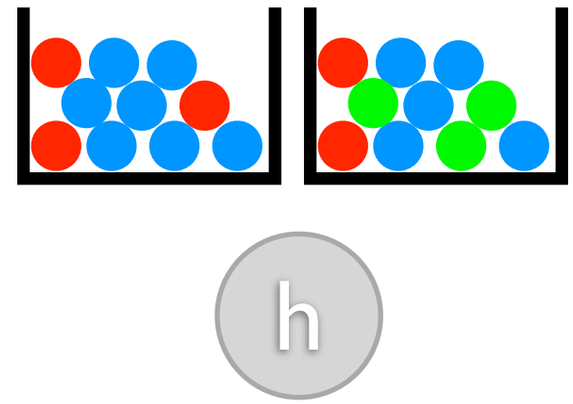

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```
(define heads (mem (lambda () (flip 0.4))))
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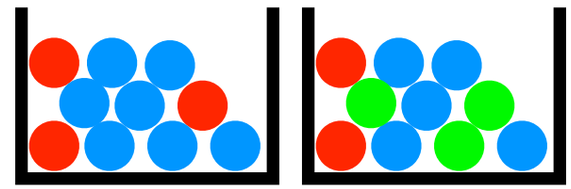
A bit of gambling



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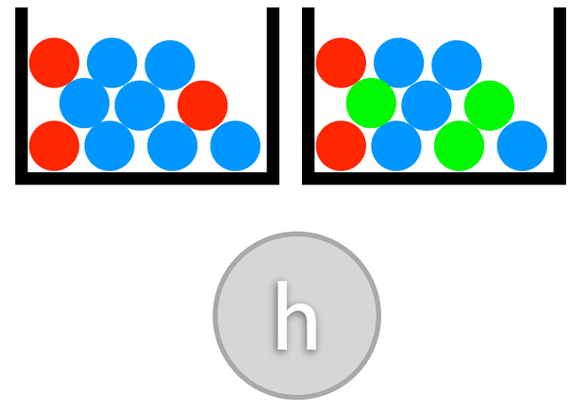

- toss (biased) coin & draw ball from each urn
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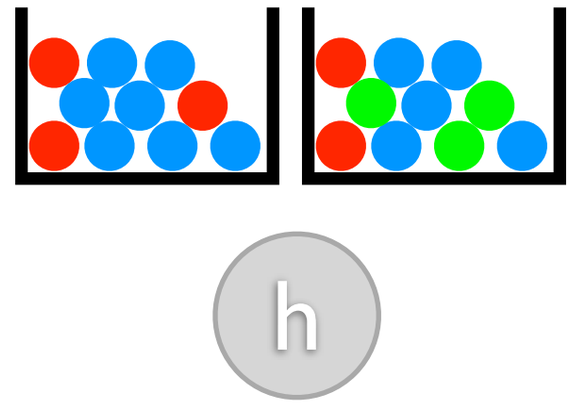
```
(define heads (mem (lambda () (flip 0.4))))

(define color1 (mem (lambda () (if (flip 0.3) 'red 'blue))))
```

Church by example:

A bit of gambling



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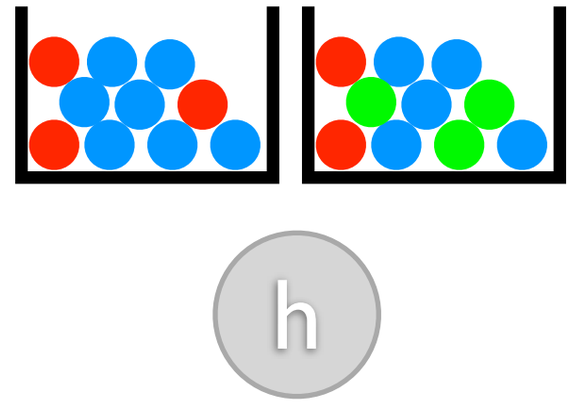

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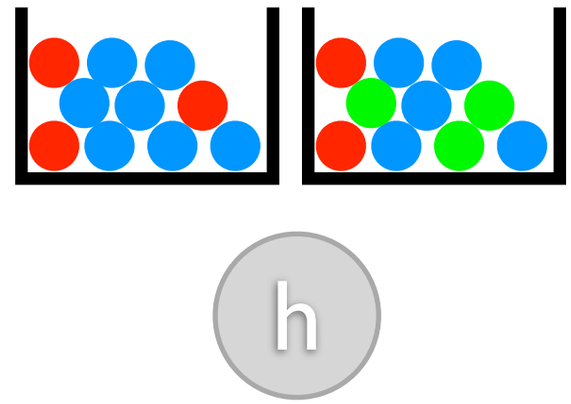

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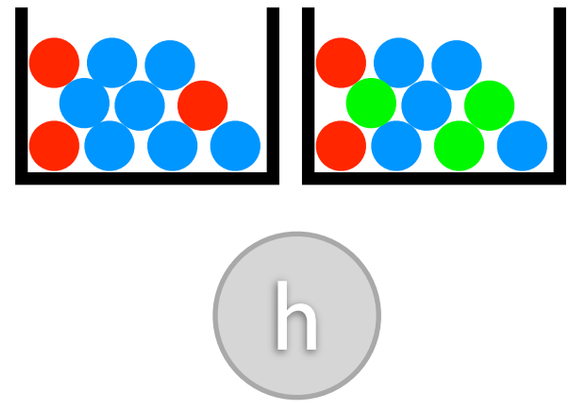

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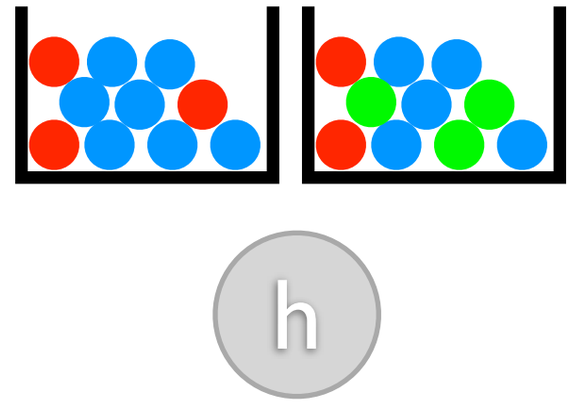

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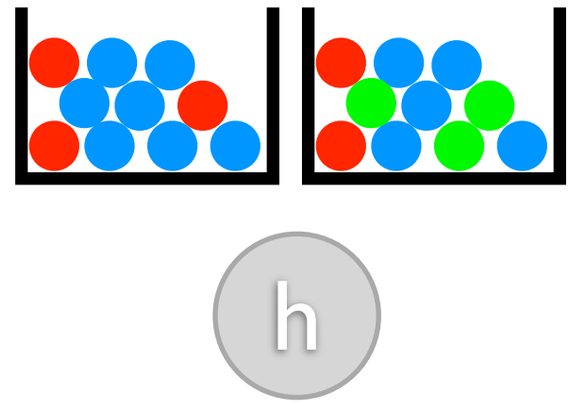

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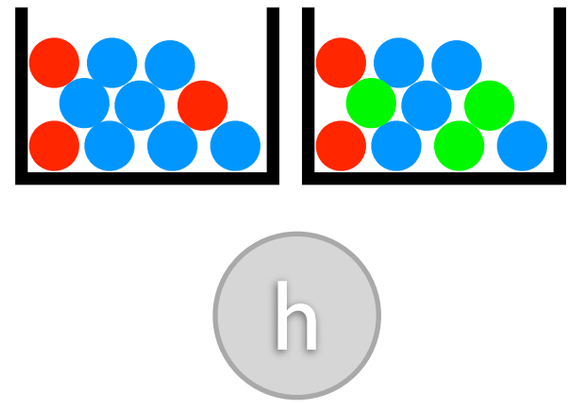

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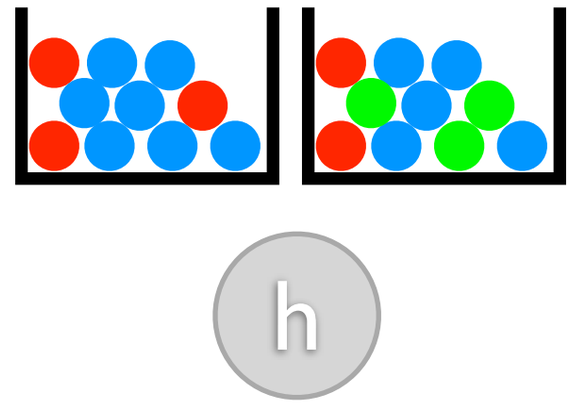

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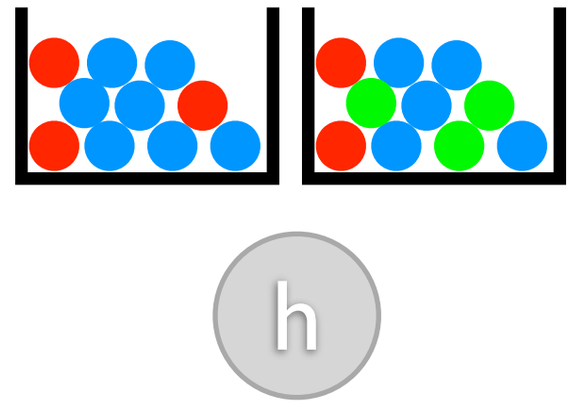

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Sampling execution

```
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(define redball (or (equal? (color1) 'red) (equal? (color2) 'red)))

(define win1 (and (heads) redball))

(define win2 (equal? (color1) (color2)))

(define win (or win1 win2))
```

```
win
```

query

Marginals via enumeration

```
(enumeration-query
  (define heads (mem (lambda () (flip 0.4))))
  (define color1 (mem (lambda () (if (flip 0.3) 'red 'blue))))
  (define color2 (mem (lambda ()
                        (multinomial '(red green blue) '(0.2 0.3 0.5)))))
(define redball (or (equal? (color1) 'red) (equal? (color2) 'red)))
(define win1 (and (heads) redball))
(define win2 (equal? (color1) (color2)))
(define win (or win1 win2))

win
true )
```

query

evidence

Histogram via sampling

```
(repeat 1000 (lambda ()
           (rejection-query

  (define heads (mem (lambda () (flip 0.4))))

  (define color1 (mem (lambda () (if (flip 0.3) 'red 'blue))))

  (define color2 (mem (lambda ()
                   (multinomial '(red green blue) '(0.2 0.3 0.5)))))

(define redball (or (equal? (color1) 'red) (equal? (color2) 'red)))

(define win1 (and (heads) redball))

(define win2 (equal? (color1) (color2)))

(define win (or win1 win2))

win

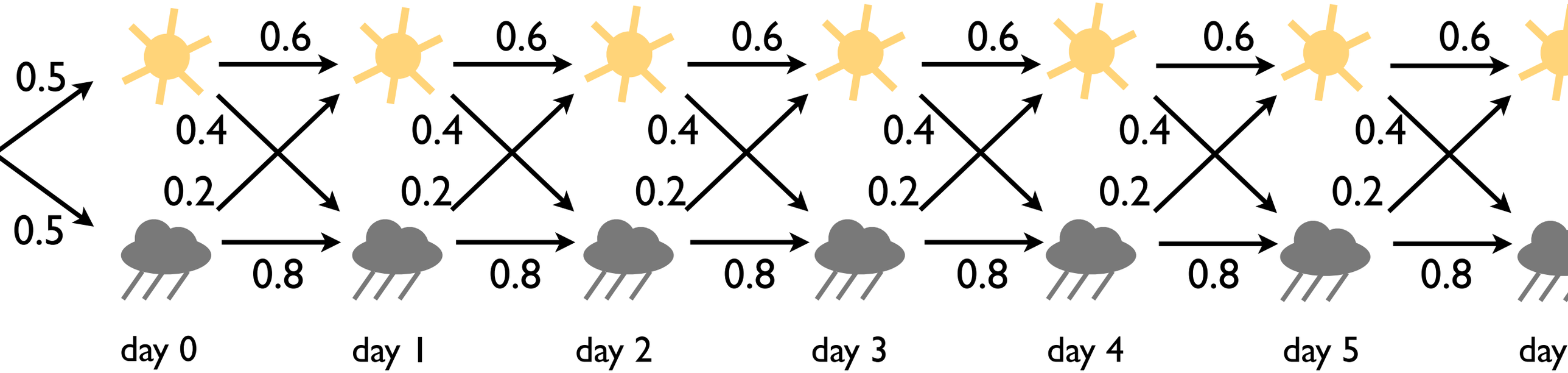
true ))))
```

query

evidence

Church by example:

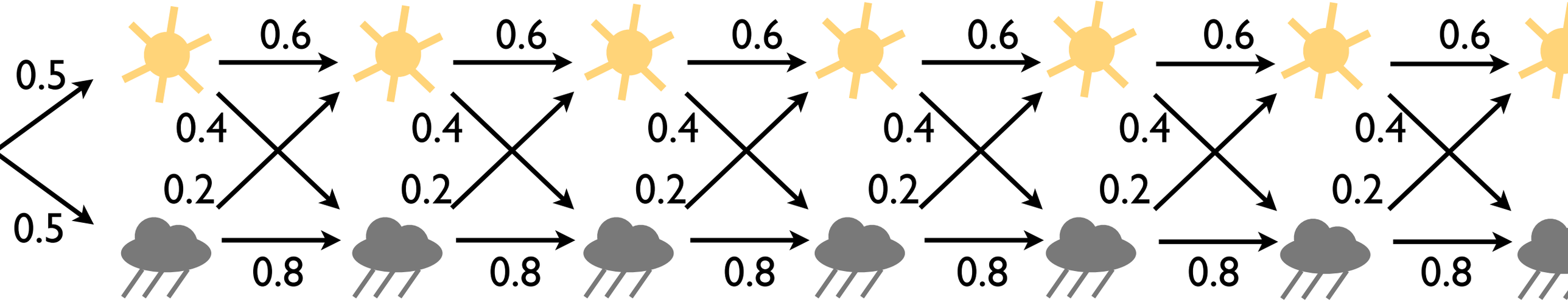
Rain or sun?




```
(define weather (mem (lambda (day) (if (equal? day 0)
                                       (weather0)
                                       (weatherN day (- day 1))))))
(define weather0 (lambda () (if (flip 0.5) 'sun 'rain)))
(define weatherN (lambda (today yesterday)
                   (if (equal? (weather yesterday) 'rain)
                       (if (flip 0.2) 'sun 'rain)
                       (if (flip 0.6) 'sun 'rain)))))
```

Church by example:

Rain or sun?



day 0 day 1 day 2 day 3 day 4 day 5 day

```
(define weather (mem (lambda (day) (if (equal? day 0)
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(list (weather 0) (weather 1) (weather 2))
```

Probabilistic Programming Summary

- Church: functional programming + random primitives
- probabilistic generative model
- stochastic memoization
- sampling
- increasing number of probabilistic programming languages using various underlying paradigms

ProbLog	**PRISM**	**Church**
probabilistic facts & choices	probabilistic choices	random primitives
all RVs memoized	no RVs memoized	user-defined per RV
Prolog	Prolog with mutually exclusive derivations	λ-calculus functions
distribution over worlds	distribution over derivations / answers	distribution over computations / answers

Roadmap

- Modeling
- Reasoning
- Language extensions
- Advanced topics

... with some detours on the way

Reasoning

- Exact inference with knowledge compilation
  - using proofs
  - using models
  - in PRISM
- Approximate inference

Answering Questions

Given:

program

queries

evidence

?

Find:

marginal probabilities

conditional probabilities

MPE state

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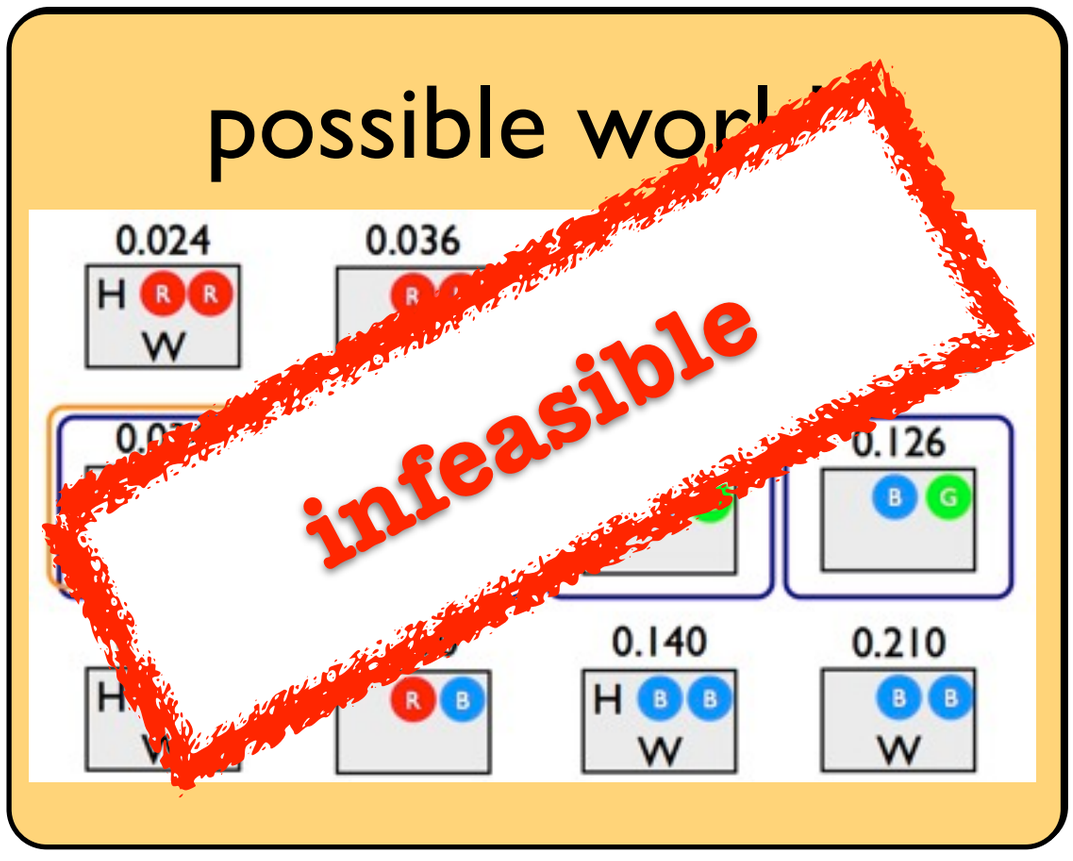

Find:

marginal probabilities

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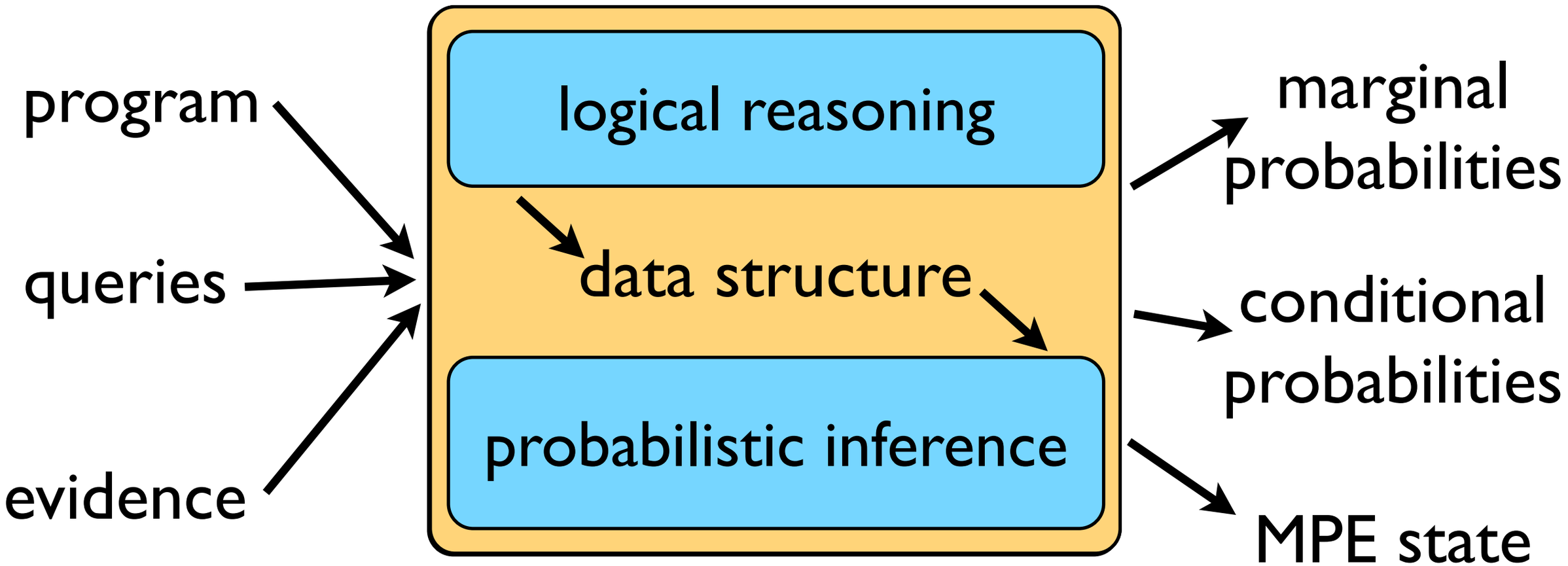
Given:

Find:

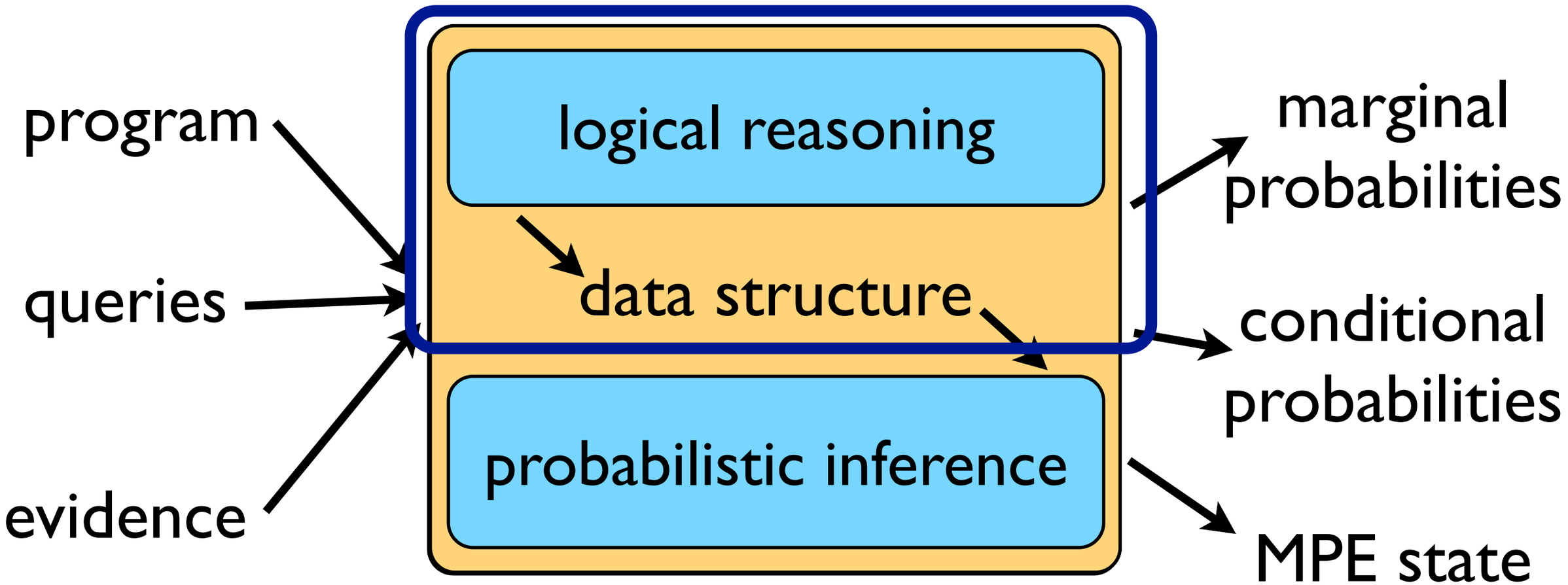


Answering Questions

Given:

knowledge compilation

Find:

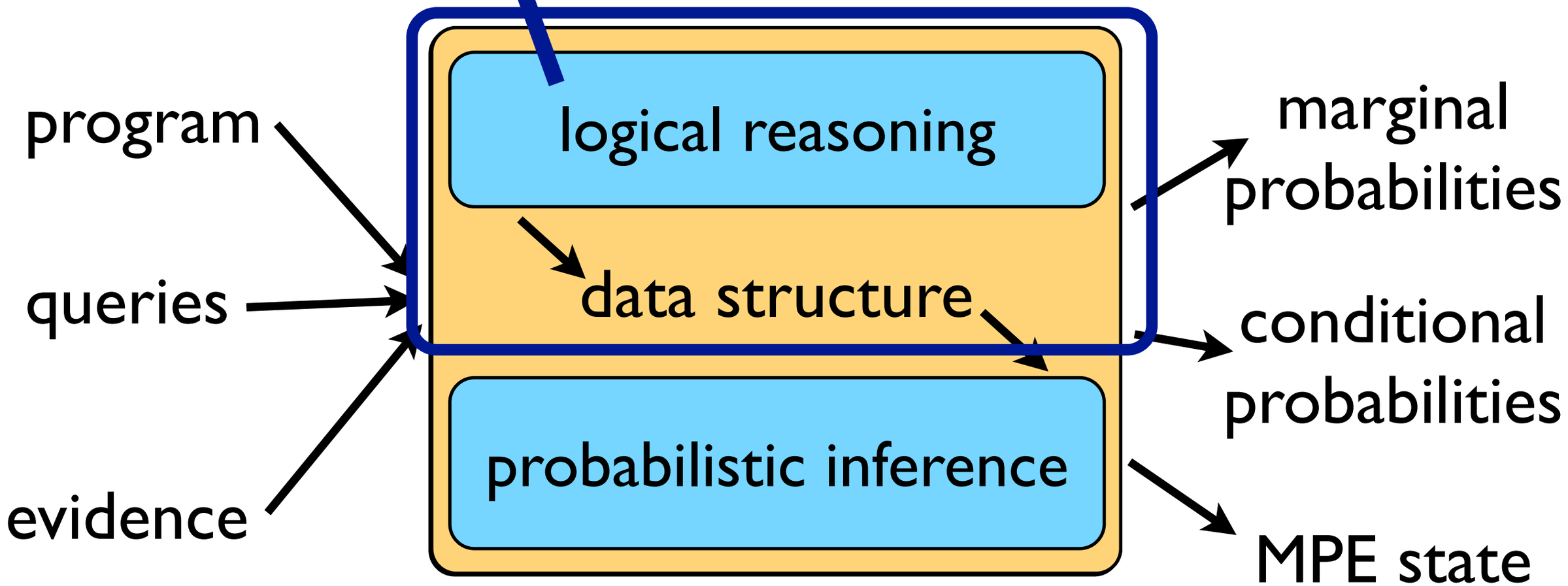


Answering Questions

1. using proofs
2. using models

knowledge compilation

Given:



Find:

Logical Reasoning: Proofs in Prolog

```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
      influences(Y,X),
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```

Logical Reasoning: Proofs in Prolog

```
?- smokes(carl).
```

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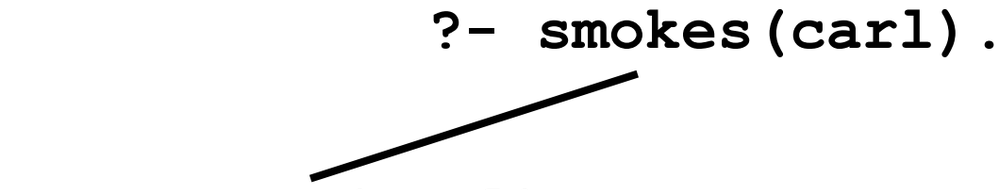

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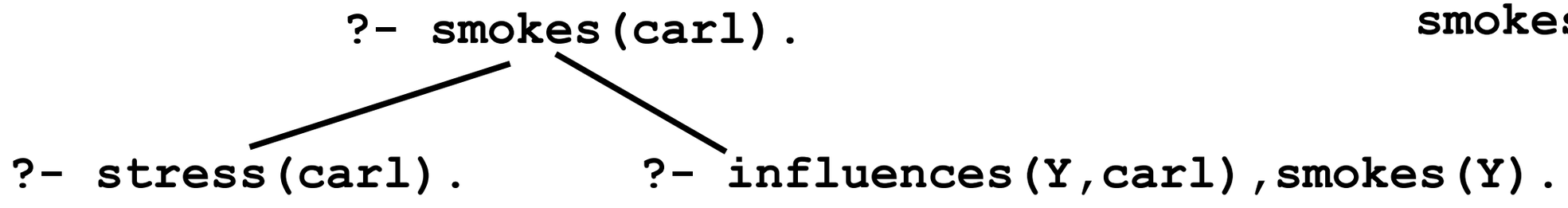

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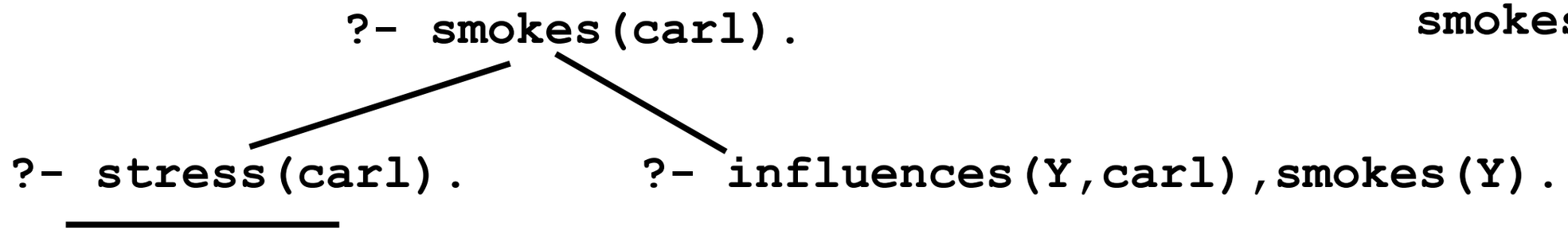

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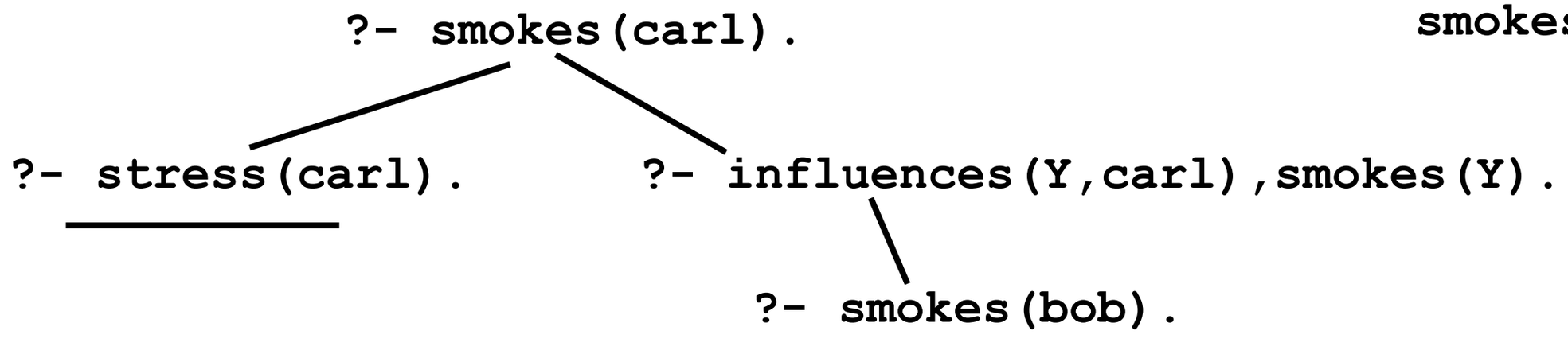

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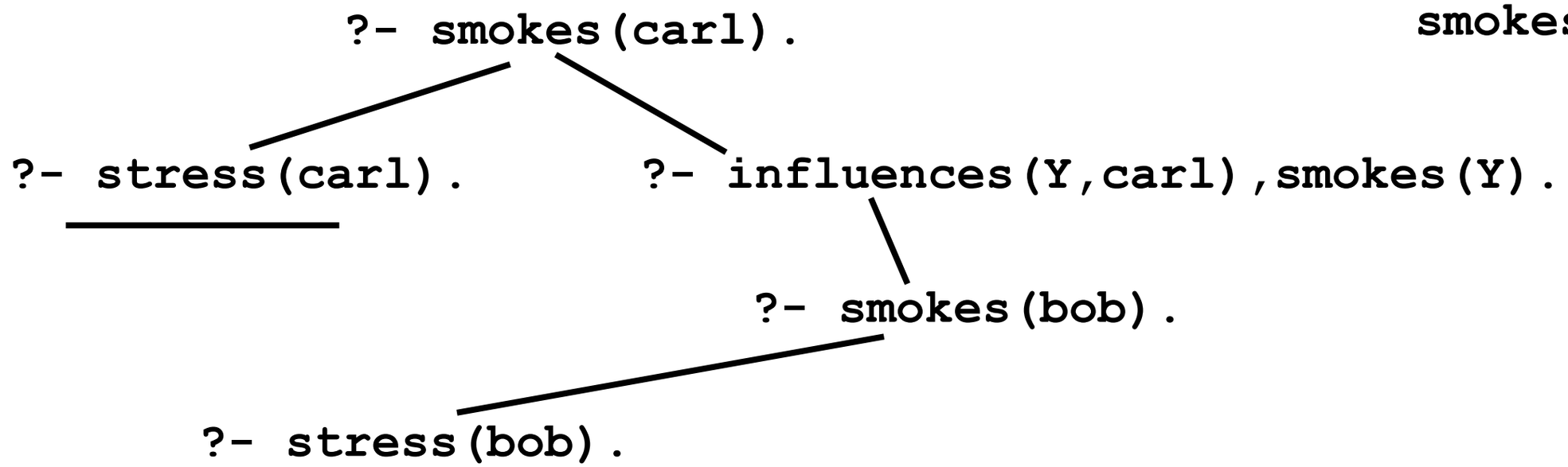

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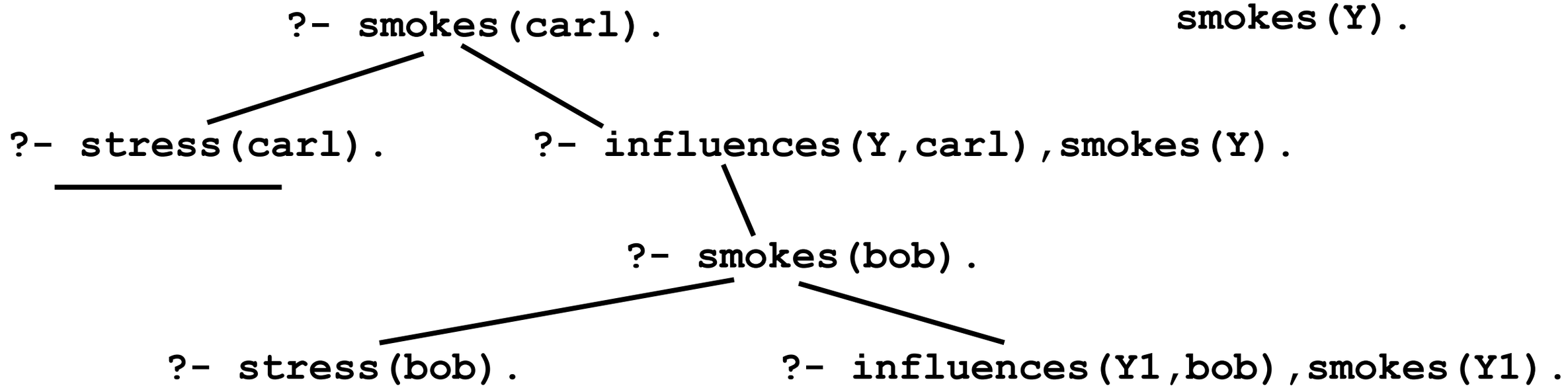

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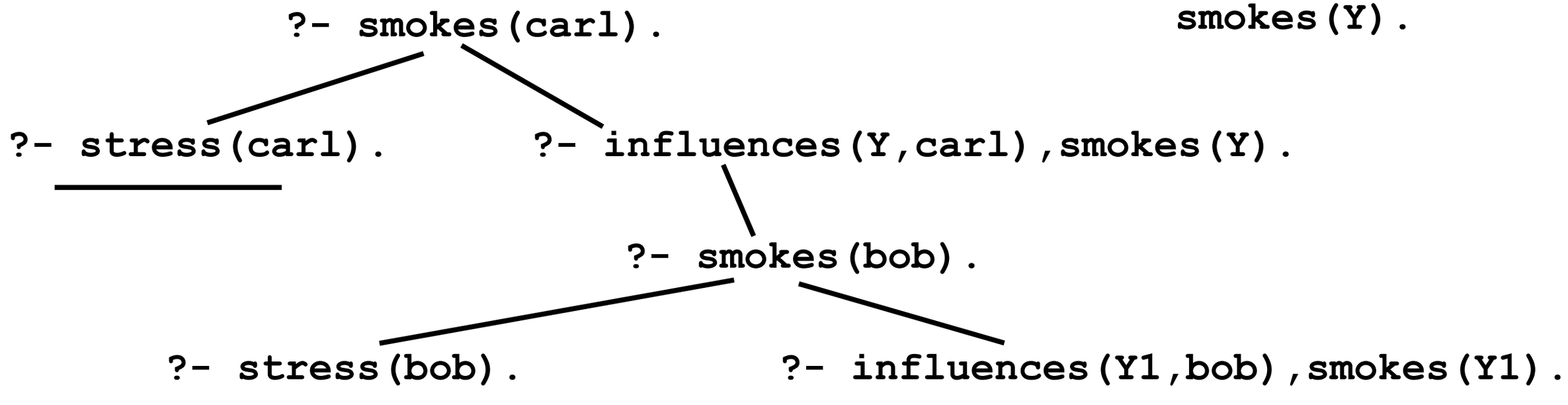

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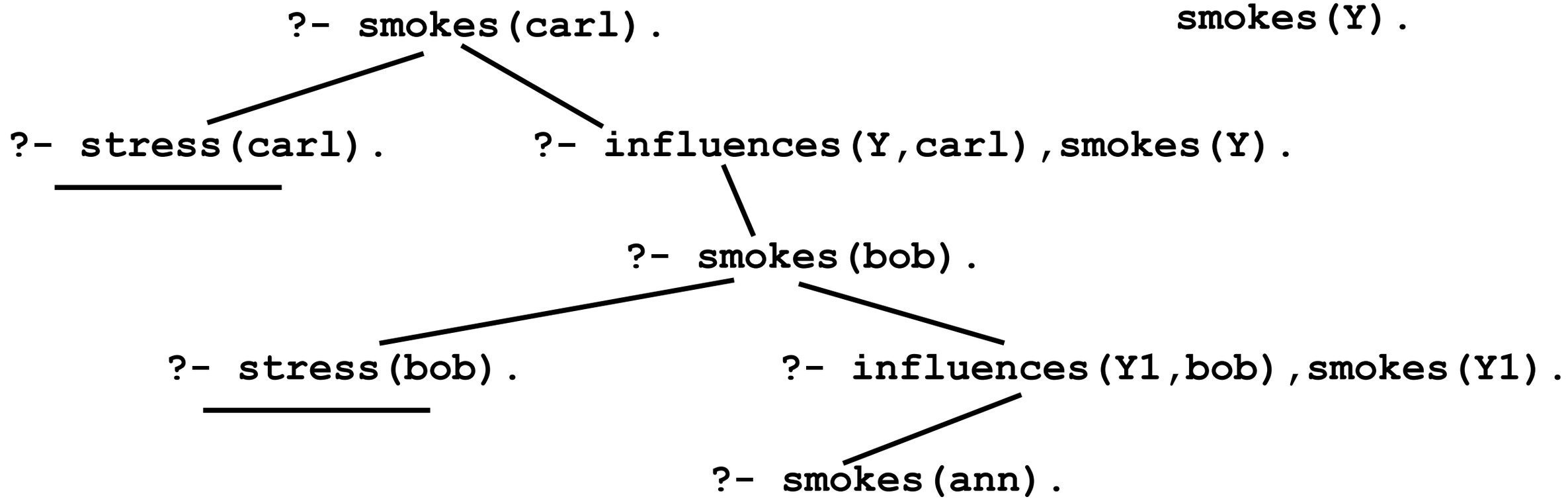

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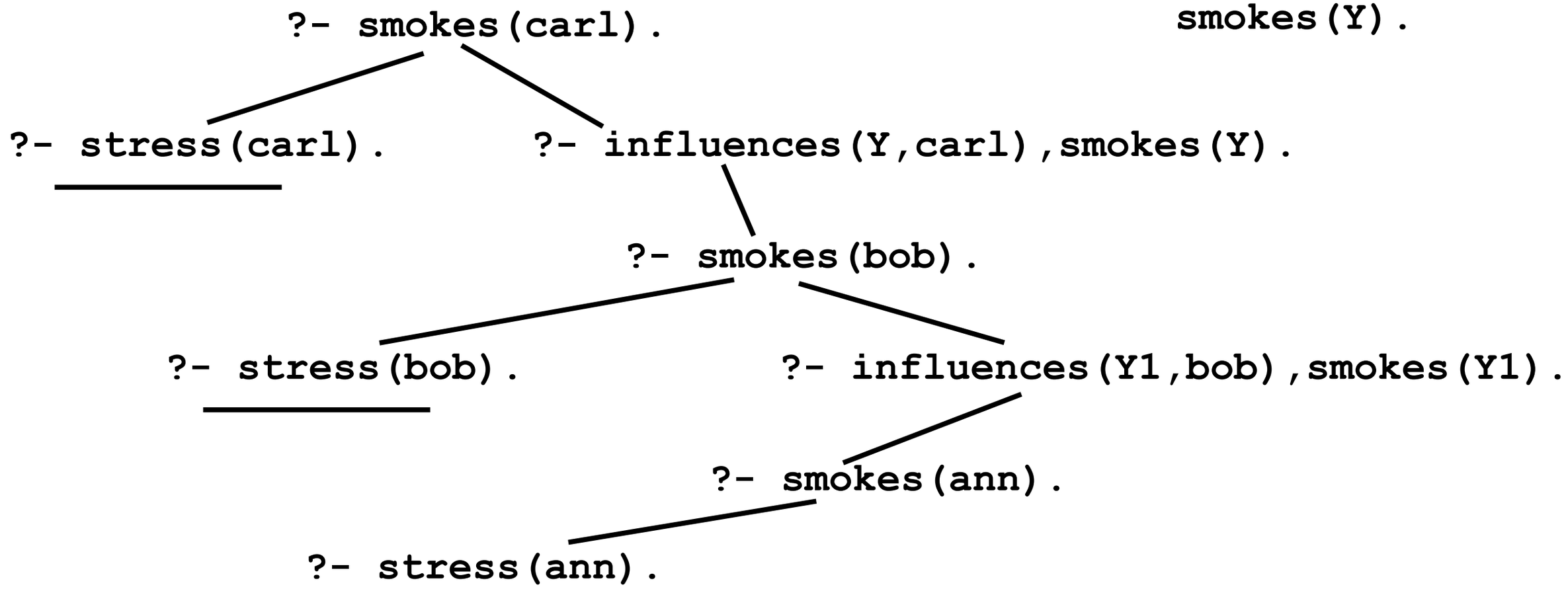

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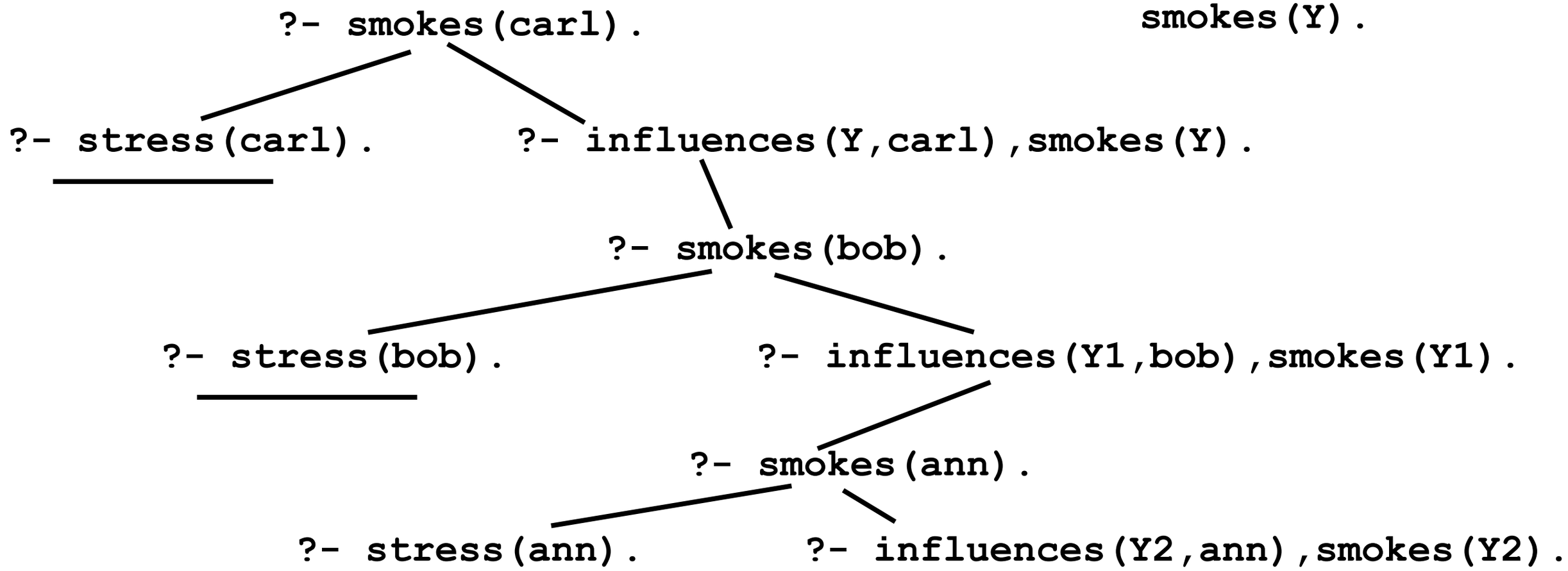

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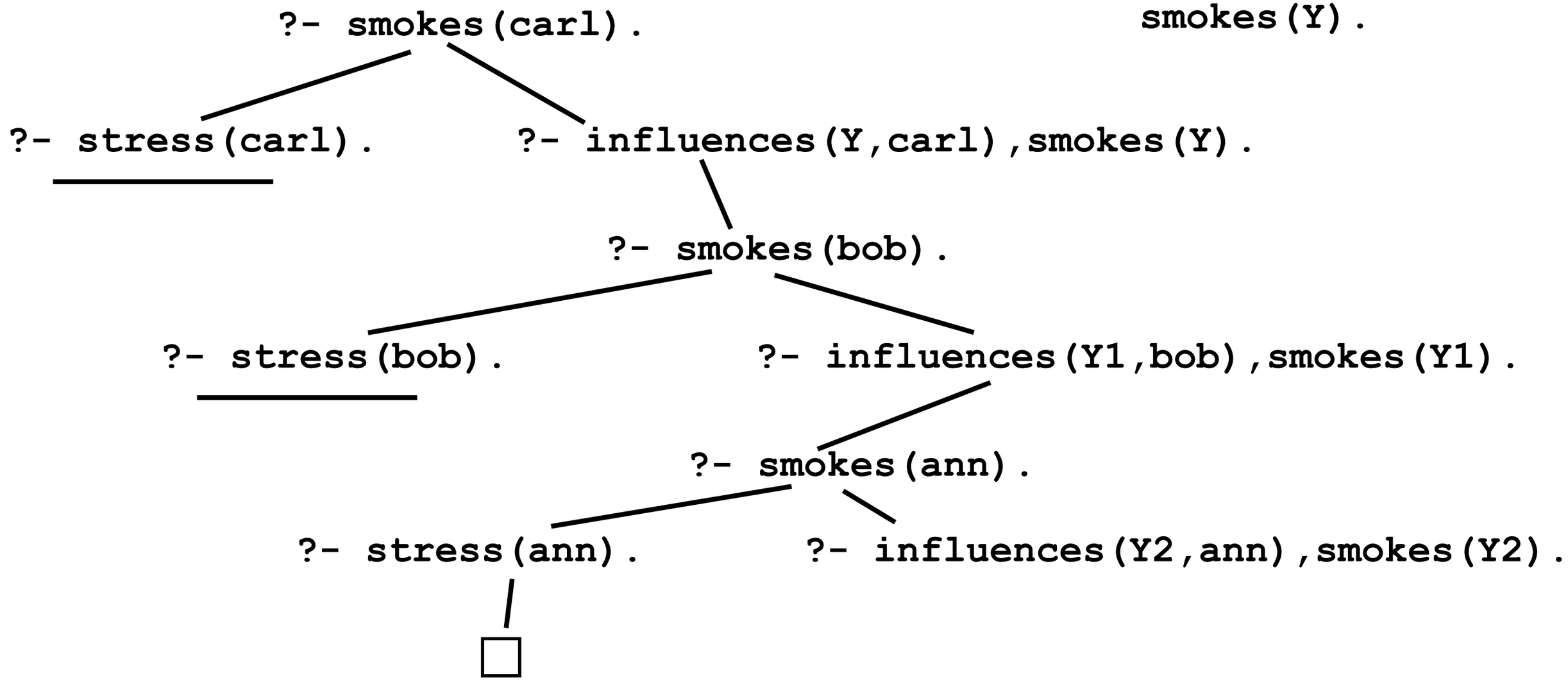

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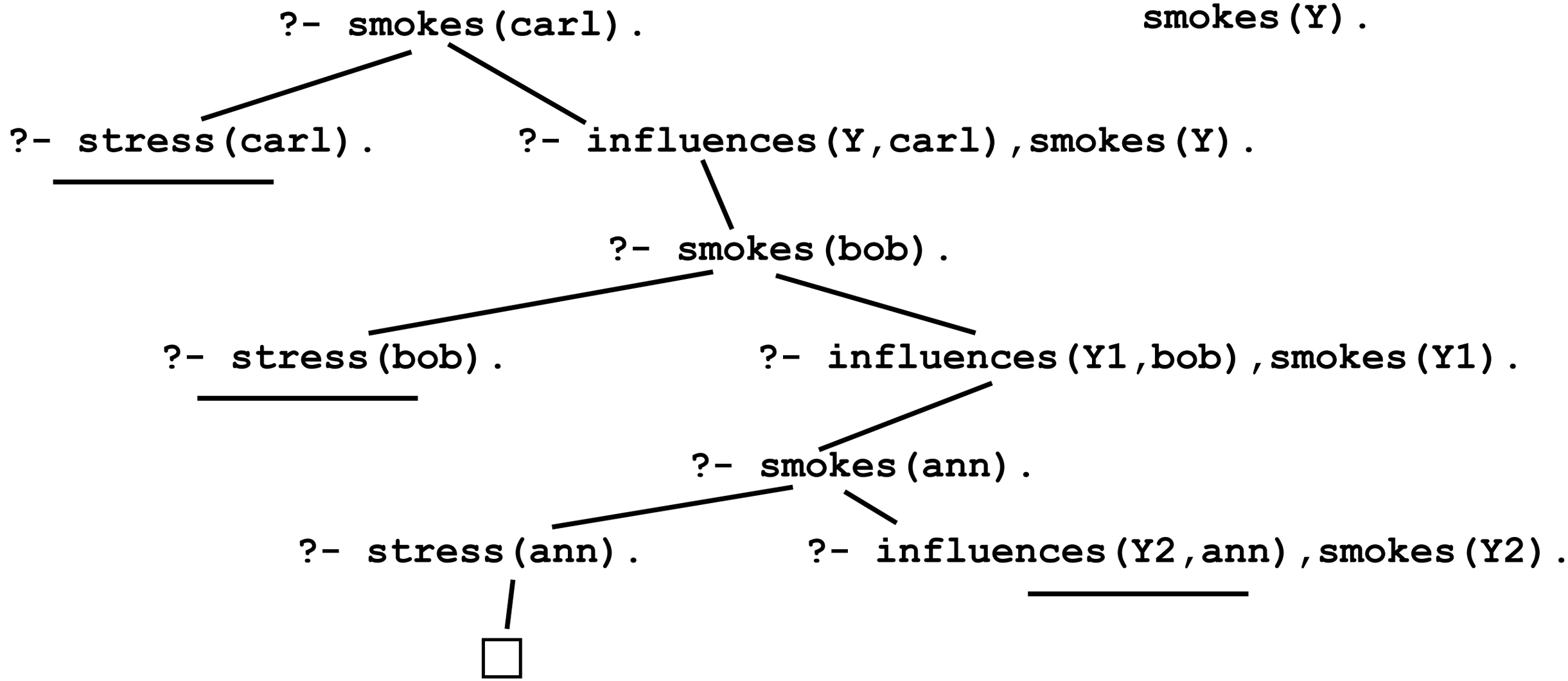

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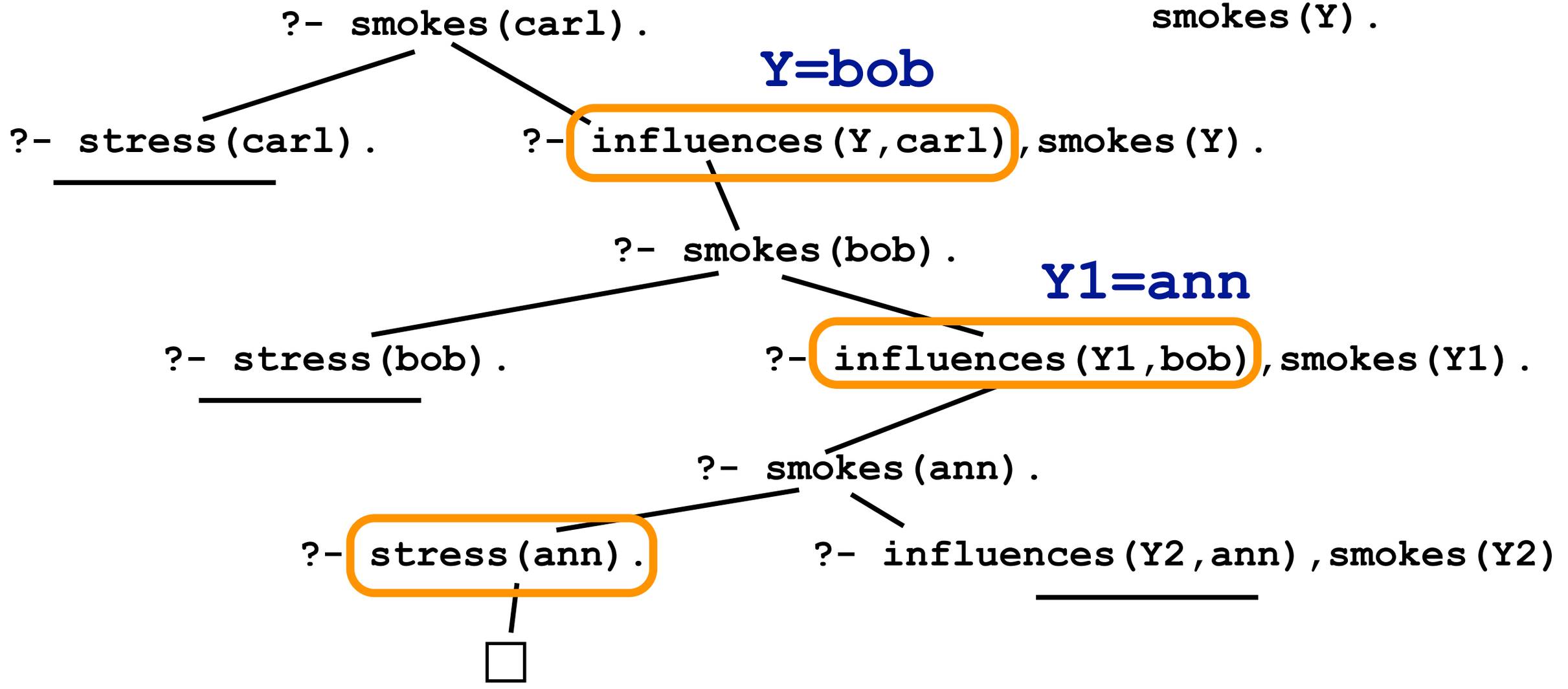


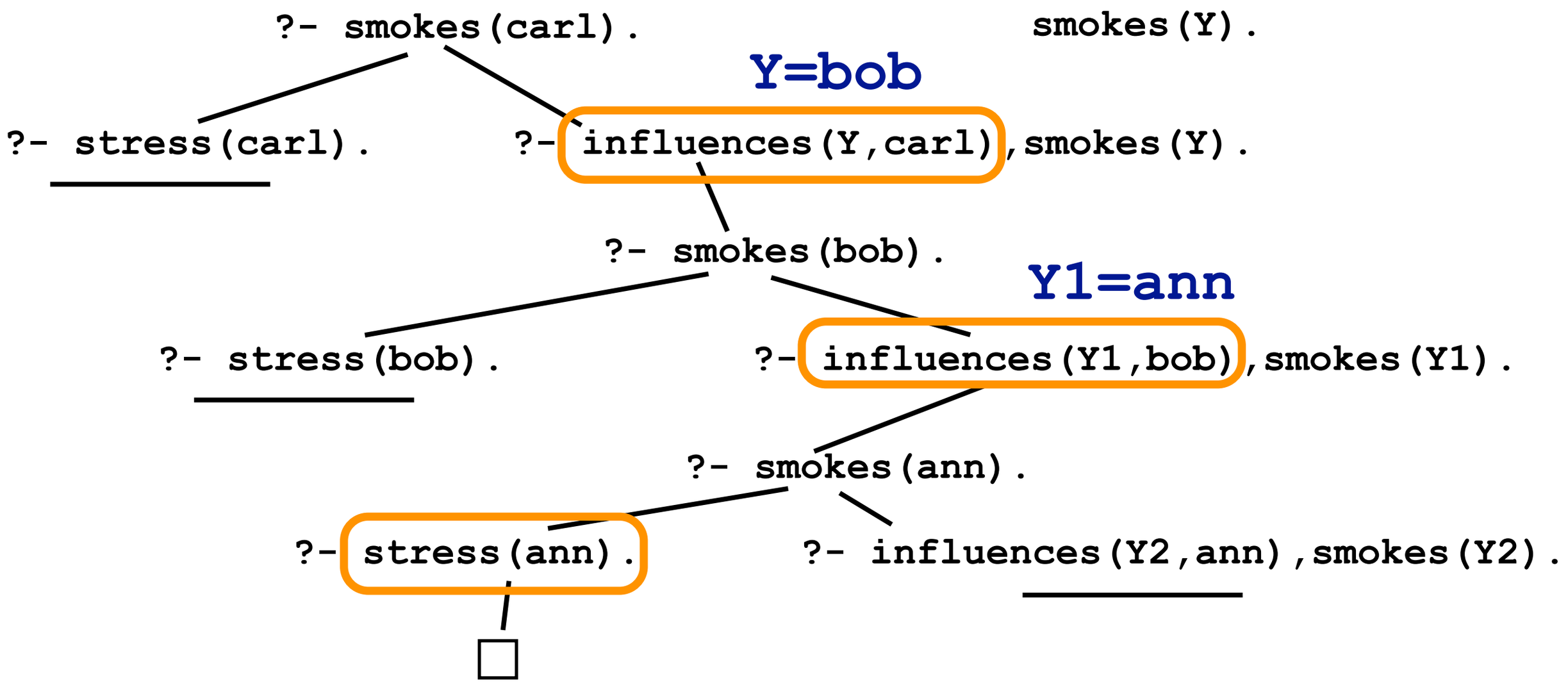
proof = facts used in successful derivation:

`influences(bob,carl) &influences(ann,bob) &stress(ann)`

Proofs in ProbLog

```
0.8::stress(ann).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X),
     smokes(Y).
```



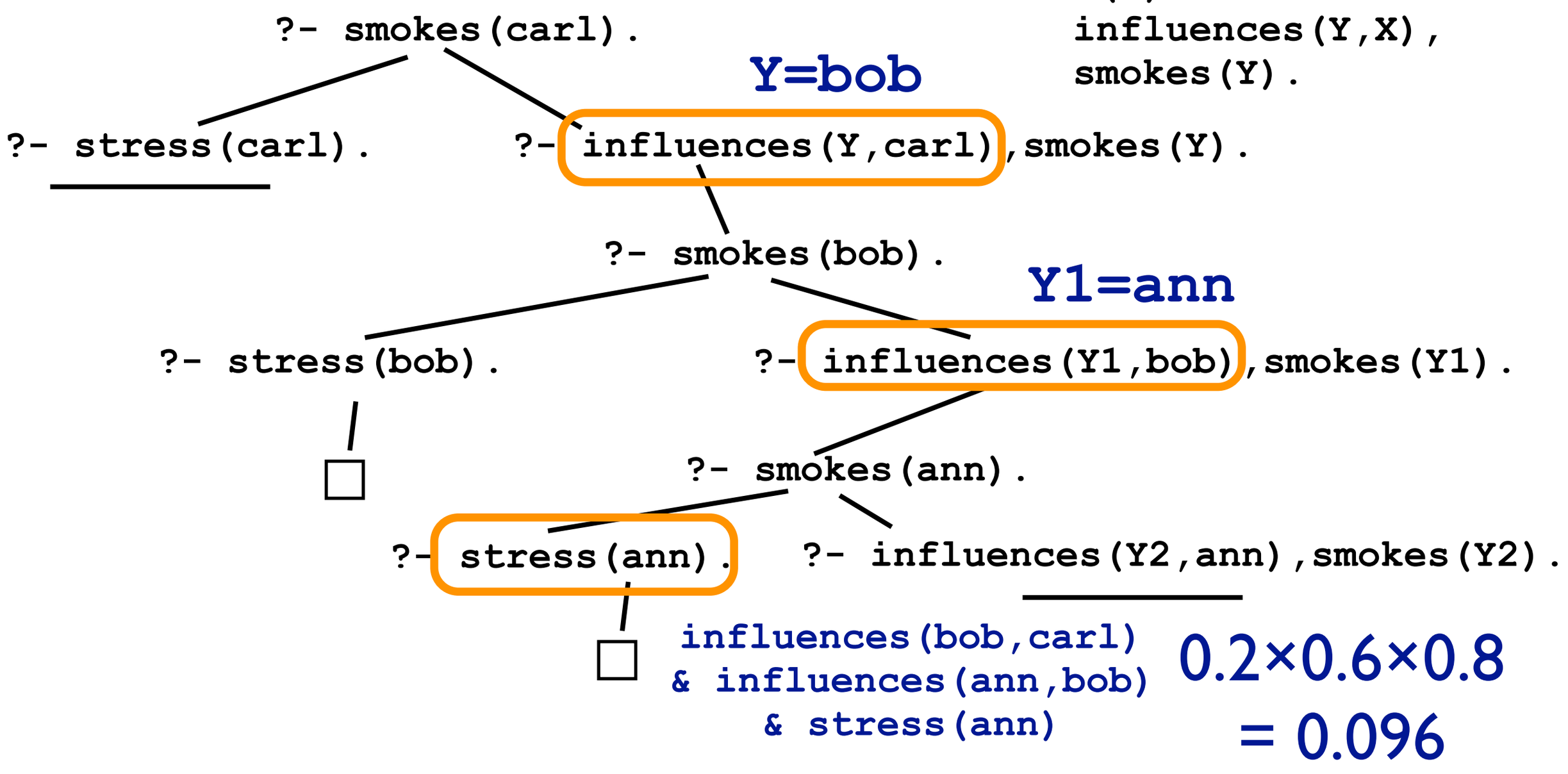
`influences(bob,carl)&influences(ann,bob)&stress(ann)`

probability of proof = 0.2 × 0.6 × 0.8 = 0.096

Proofs in ProbLog

```
0.8::stress(ann).
0.4::stress(bob).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

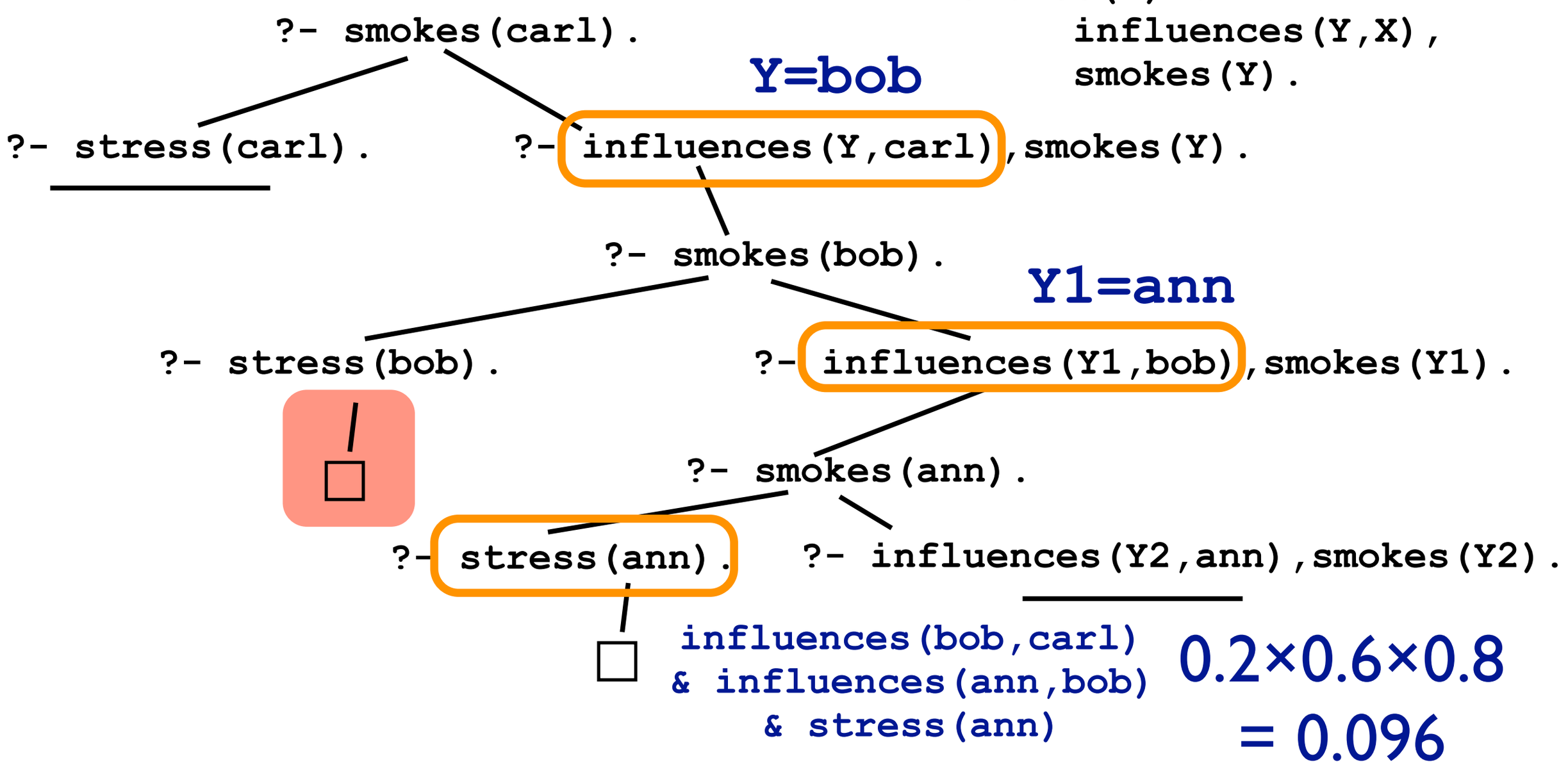
smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X),
     smokes(Y).
```



Proofs in ProbLog

```
0.8::stress(ann).
0.4::stress(bob).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

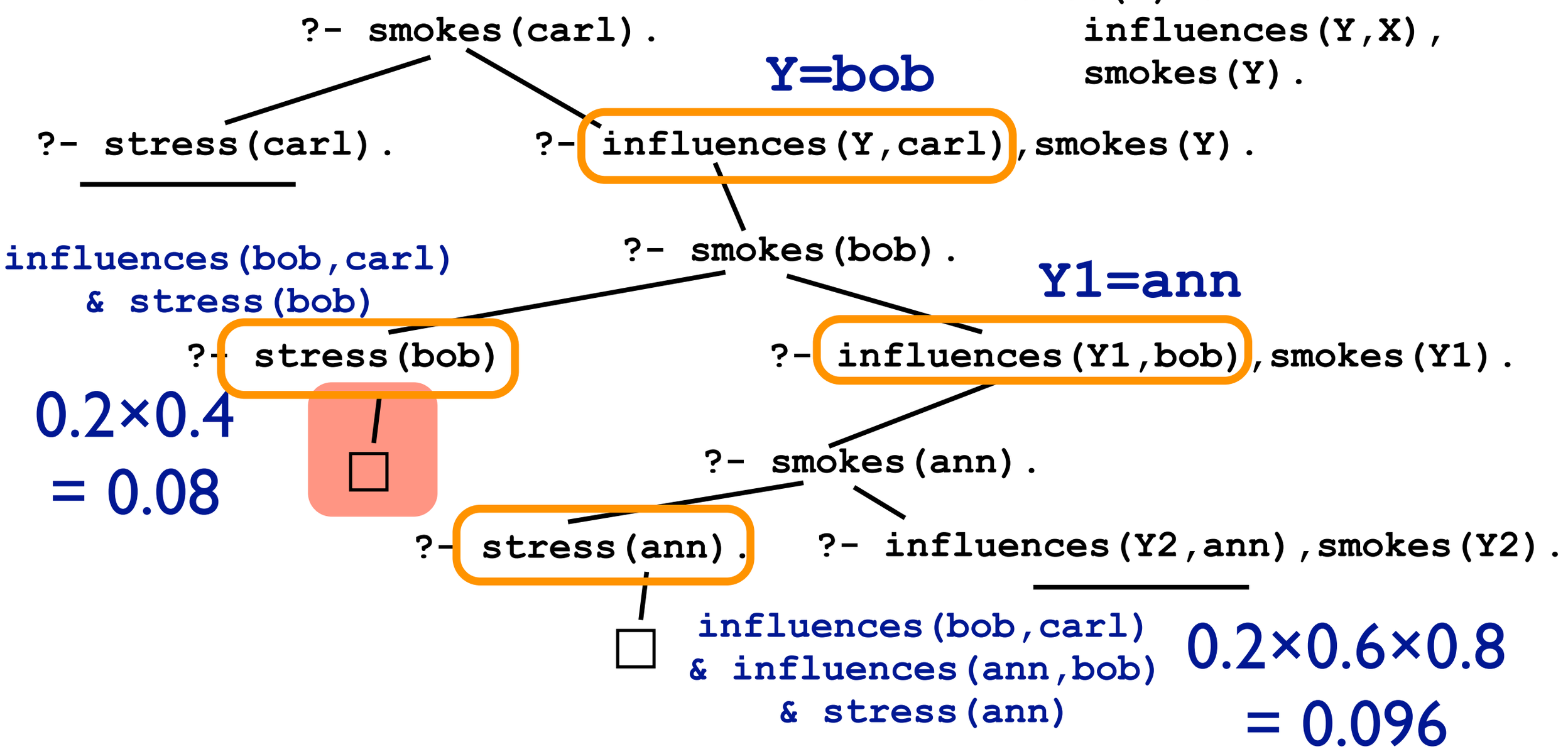
smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X),
     smokes(Y).
```



Proofs in ProbLog

```
0.8::stress(ann).
0.4::stress(bob).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

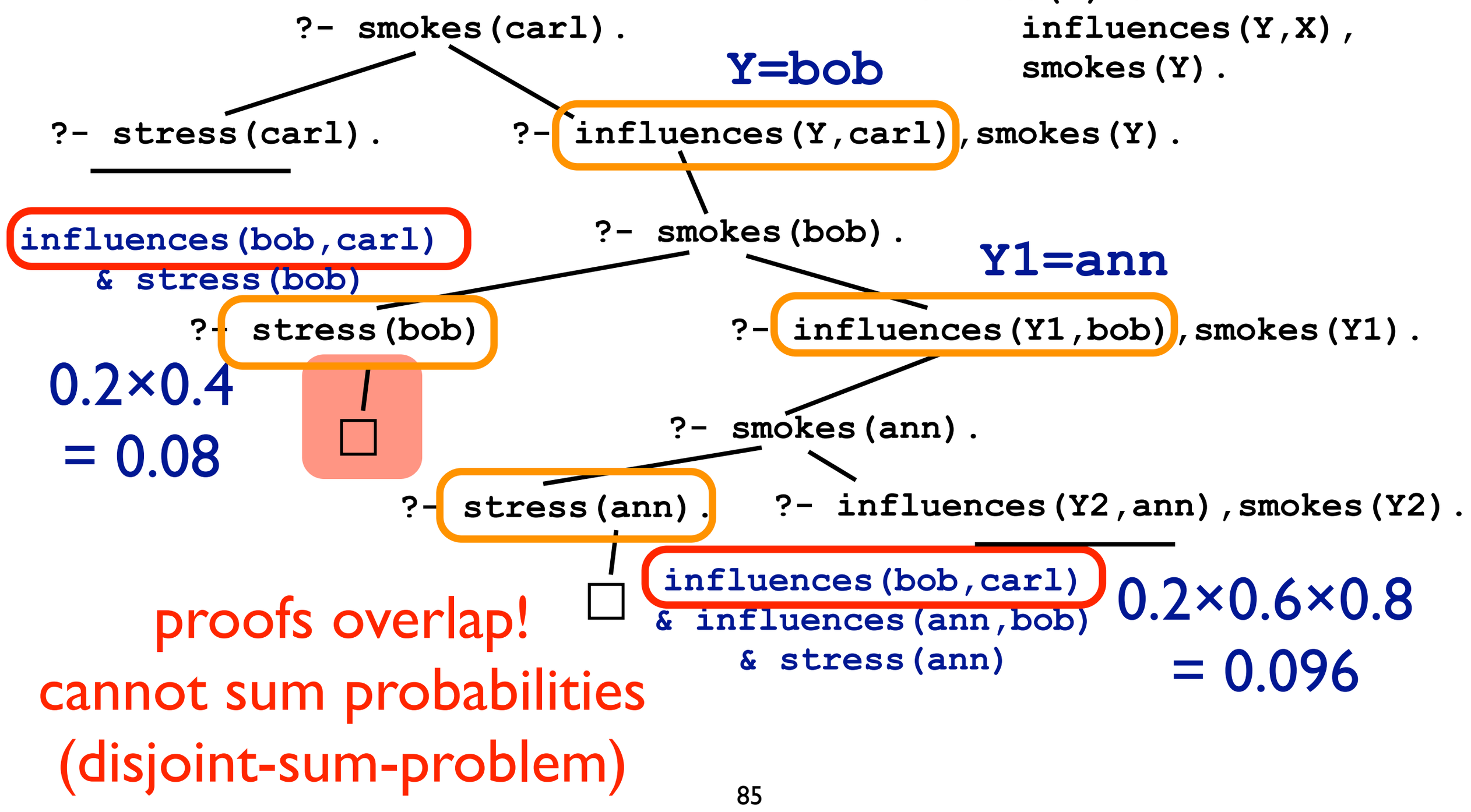
smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X),
     smokes(Y).
```



Proofs in ProbLog

```
0.8::stress(ann).
0.4::stress(bob).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X),
     smokes(Y).
```



Disjoint-Sum-Problem

possible worlds

```
infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)
infl(bob,carl) &   infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)
...
```

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

```
infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)

infl(bob,carl) &   infl(ann,bob) &   st(ann) &   st(bob)

infl(bob,carl) & \+infl(ann,bob) &   st(ann) &   st(bob)

infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)

infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)

...
```

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

infl(bob,carl) & infl(ann,bob) & st(ann) & \+st(bob)

infl(bob,carl) & infl(ann,bob) & st(ann) & st(bob)

infl(bob,carl) & \+infl(ann,bob) & st(ann) & st(bob)

infl(bob,carl) & infl(ann,bob) & \+st(ann) & st(bob)

infl(bob,carl) & \+infl(ann,bob) & \+st(ann) & st(bob)

... **influences(bob,carl) & stress(bob)**

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

```
infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)
infl(bob,carl) &   infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)
```

... **influences(bob,carl) & stress(bob)**

sum of proof probabilities: 0.096+0.08 = 0.1760

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

infl(bob,carl) & infl(ann,bob) & st(ann) & \+st(bob)	0.0576
infl(bob,carl) & infl(ann,bob) & st(ann) & st(bob)	0.0384
infl(bob,carl) & \+infl(ann,bob) & st(ann) & st(bob)	0.0256
infl(bob,carl) & infl(ann,bob) & \+st(ann) & st(bob)	0.0096
infl(bob,carl) & \+infl(ann,bob) & \+st(ann) & st(bob)	0.0064
... **influences(bob,carl) & stress(bob)**	∑ = 0.1376

sum of proof probabilities: 0.096+0.08 = 0.1760

Disjoint-Sum-Problem

possible worlds

solution: knowledge compilation

`infl(bob,carl) & infl(ann,bob) & st(ann) & \+st(bob)`	0.0576
`infl(bob,carl) & infl(ann,bob) & st(ann) & st(bob)`	0.0384
`infl(bob,carl) & \+infl(ann,bob) & st(ann) & st(bob)`	0.0256
`infl(bob,carl) & infl(ann,bob) & \+st(ann) & st(bob)`	0.0096
`infl(bob,carl) & \+infl(ann,bob) & \+st(ann) & st(bob)`	0.0064
`...` `influences(bob,carl) & stress(bob)`	$\sum = 0.1376$

sum of proof probabilities: 0.096+0.08 = 0.1760

Binary Decision Diagrams [Bryant 86]

- compact graphical representation of Boolean formula
- popular in many branches of CS
- automatically disjoins proofs
  → efficient probability computation

Binary Decision Diagrams
[Bryant 86]

$X \vee Y \vee Z$

Binary Decision Diagrams

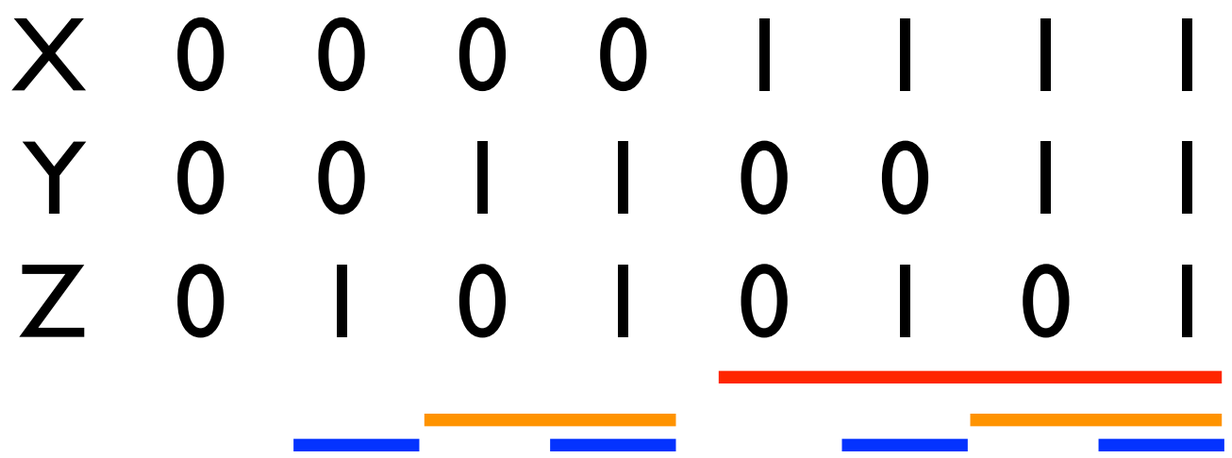
[Bryant 86]

$X \vee Y \vee Z$

X	0	0	0	0	I	I	I	I
Y	0	0	I	I	0	0	I	I
Z	0	I	0	I	0	I	0	I

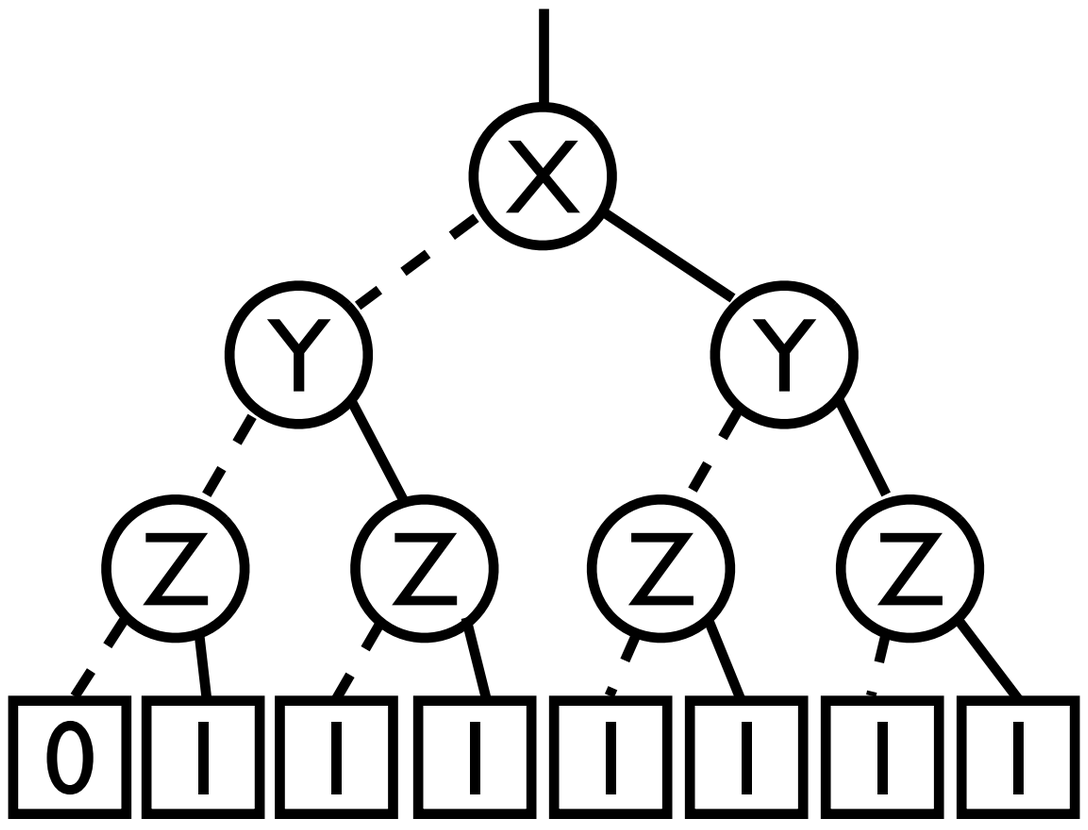
Binary Decision Diagrams
[Bryant 86]

$X \vee Y \vee Z$



Binary Decision Diagrams

[Bryant 86]

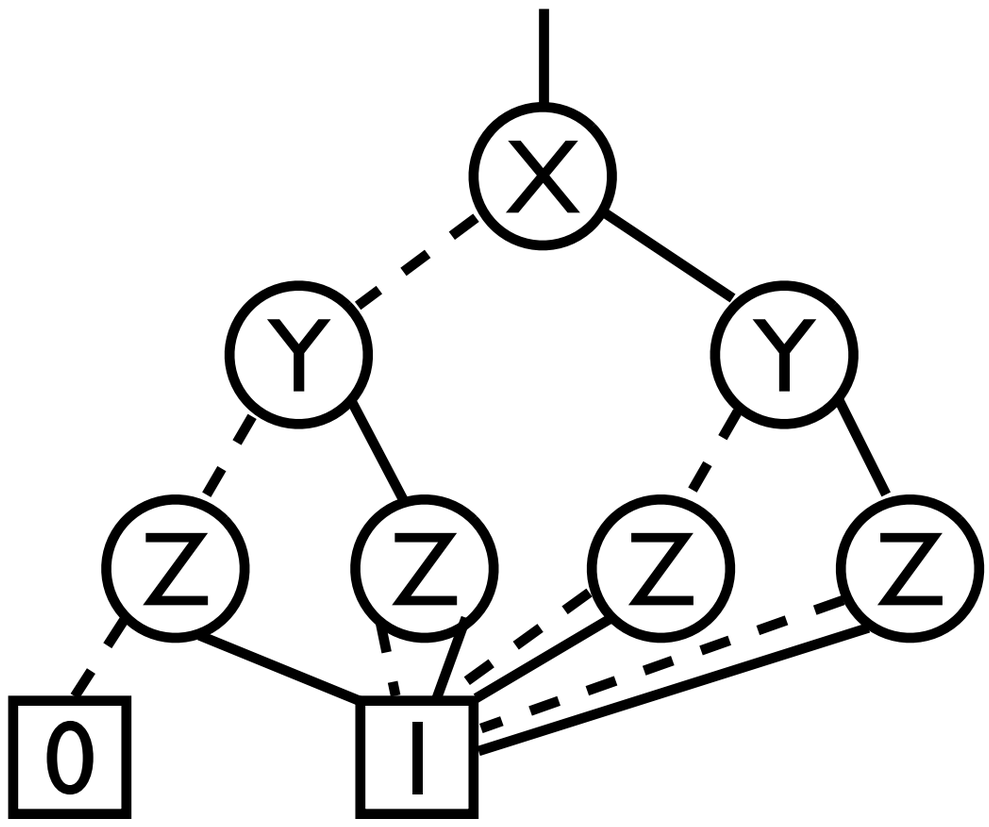


$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams

[Bryant 86]

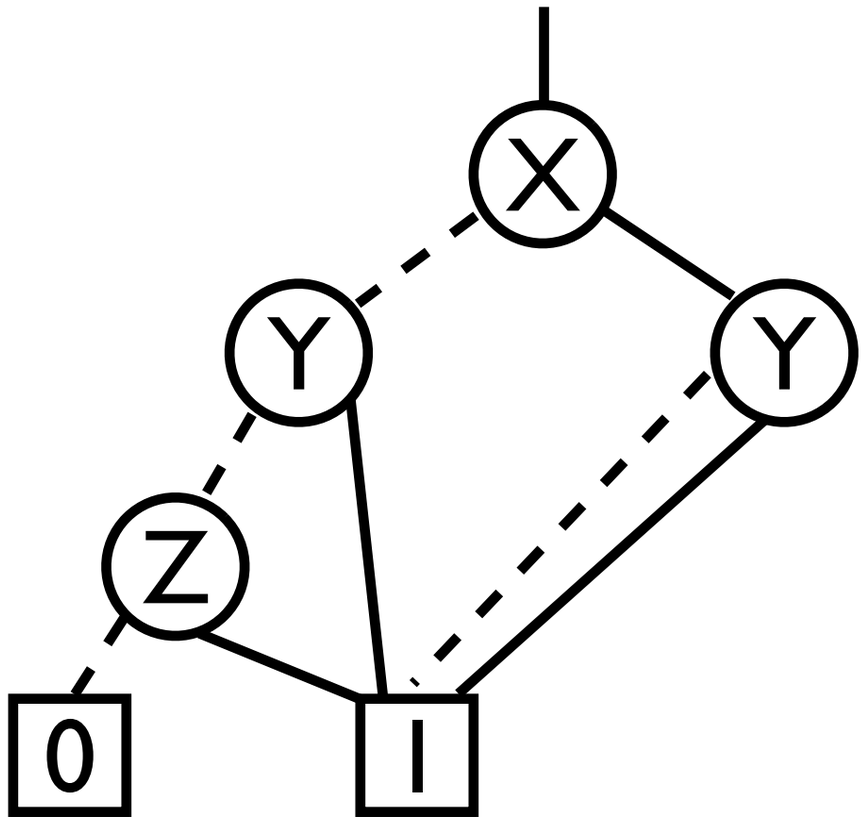


$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams

[Bryant 86]

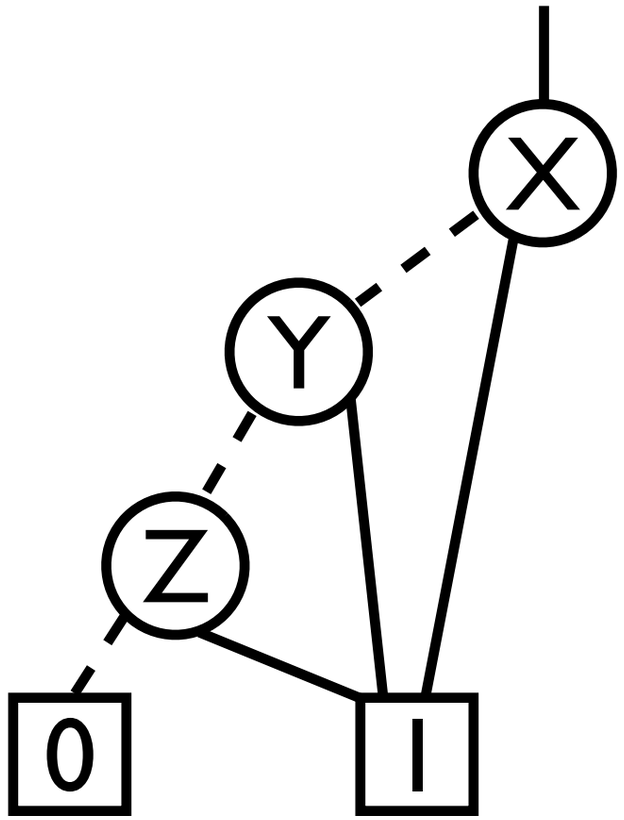


$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams

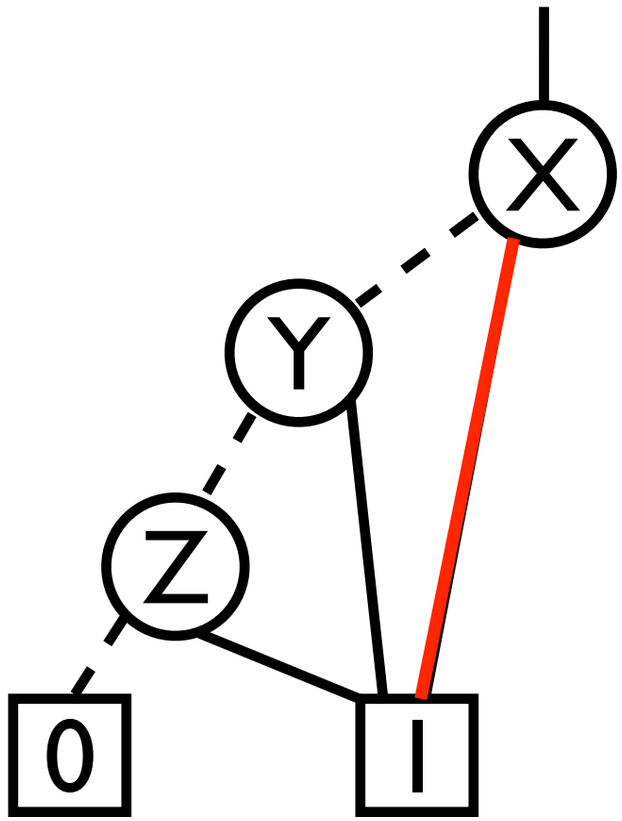
[Bryant 86]



$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]

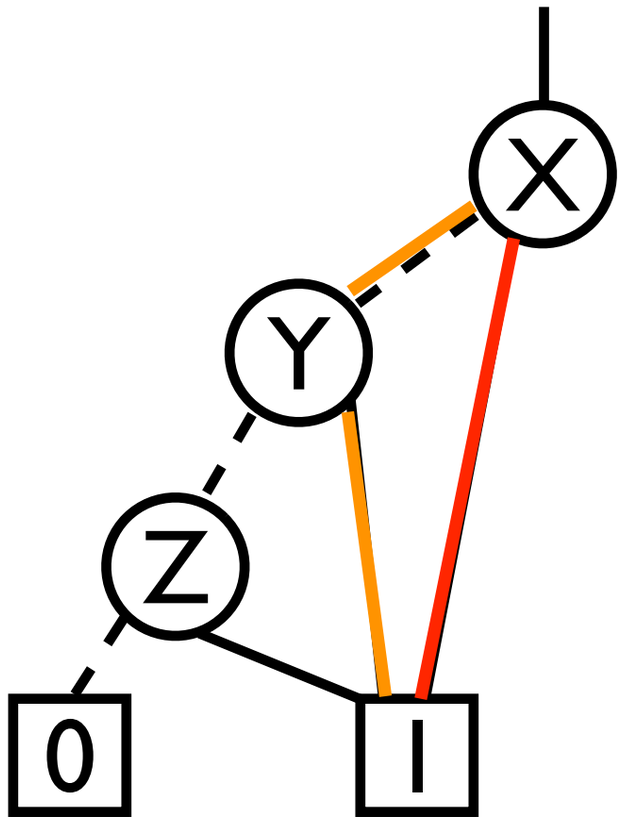


$X \vee Y \vee Z$

$X \vee (\neg X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z)$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]



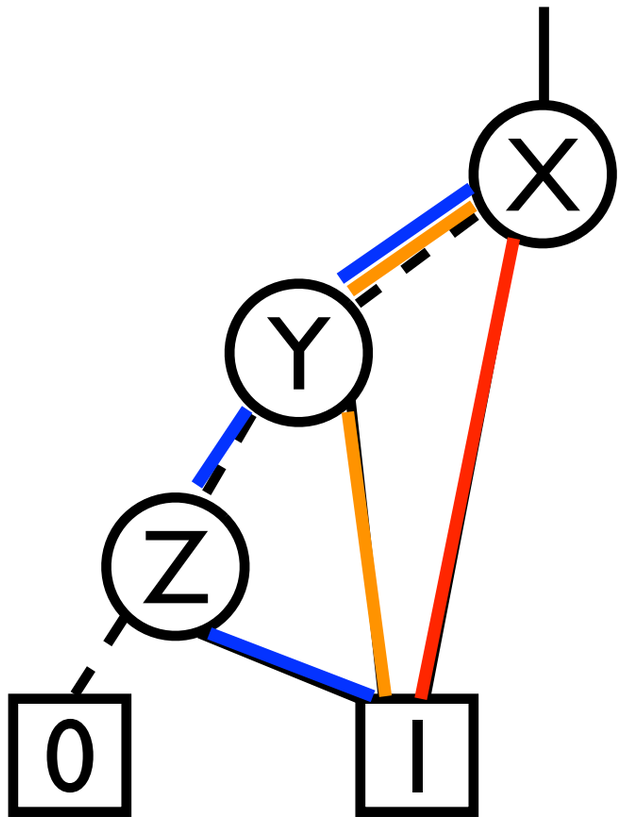
$$X \vee Y \vee Z$$

$$X \vee (\neg X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z)$$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams

[Bryant 86]

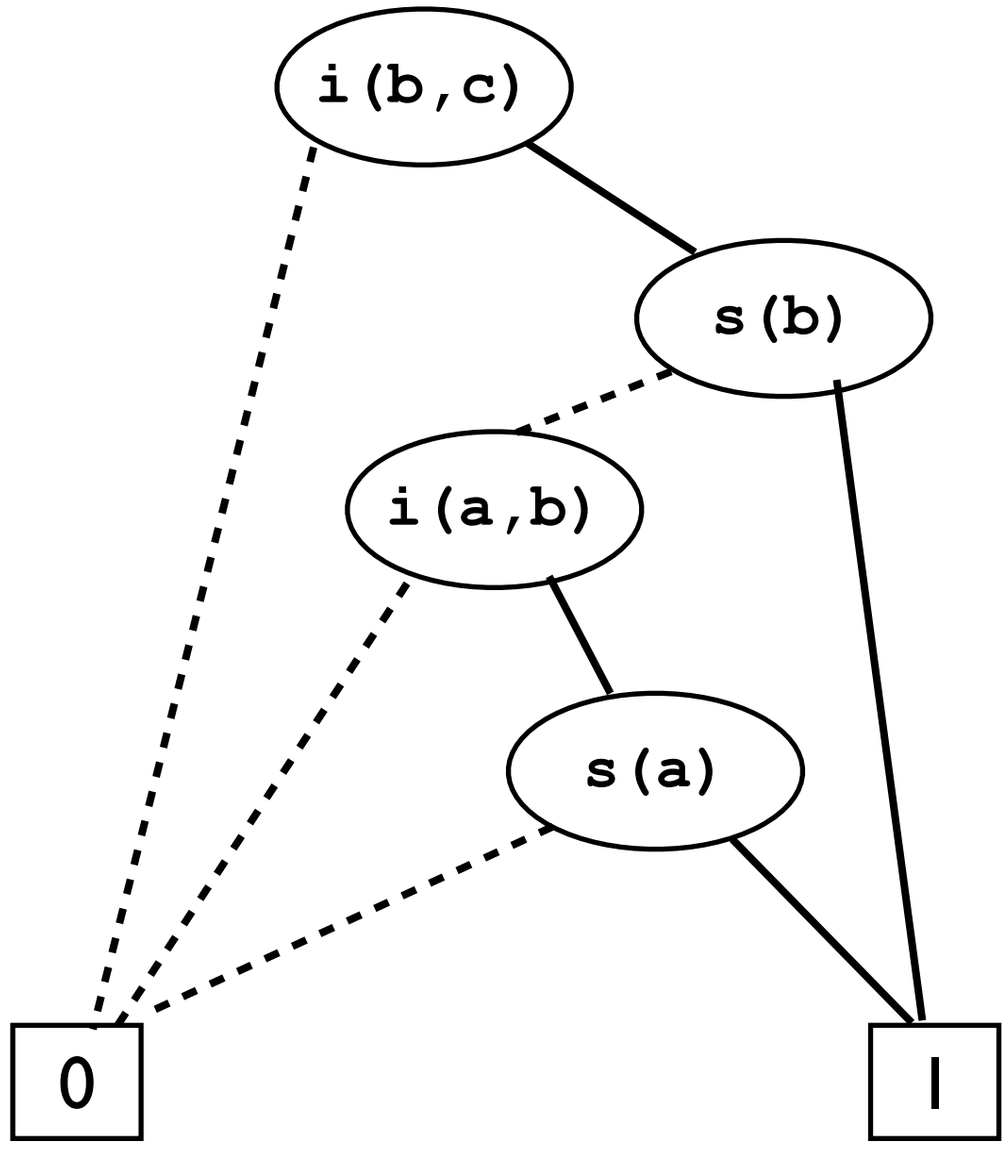


$$X \vee Y \vee Z$$

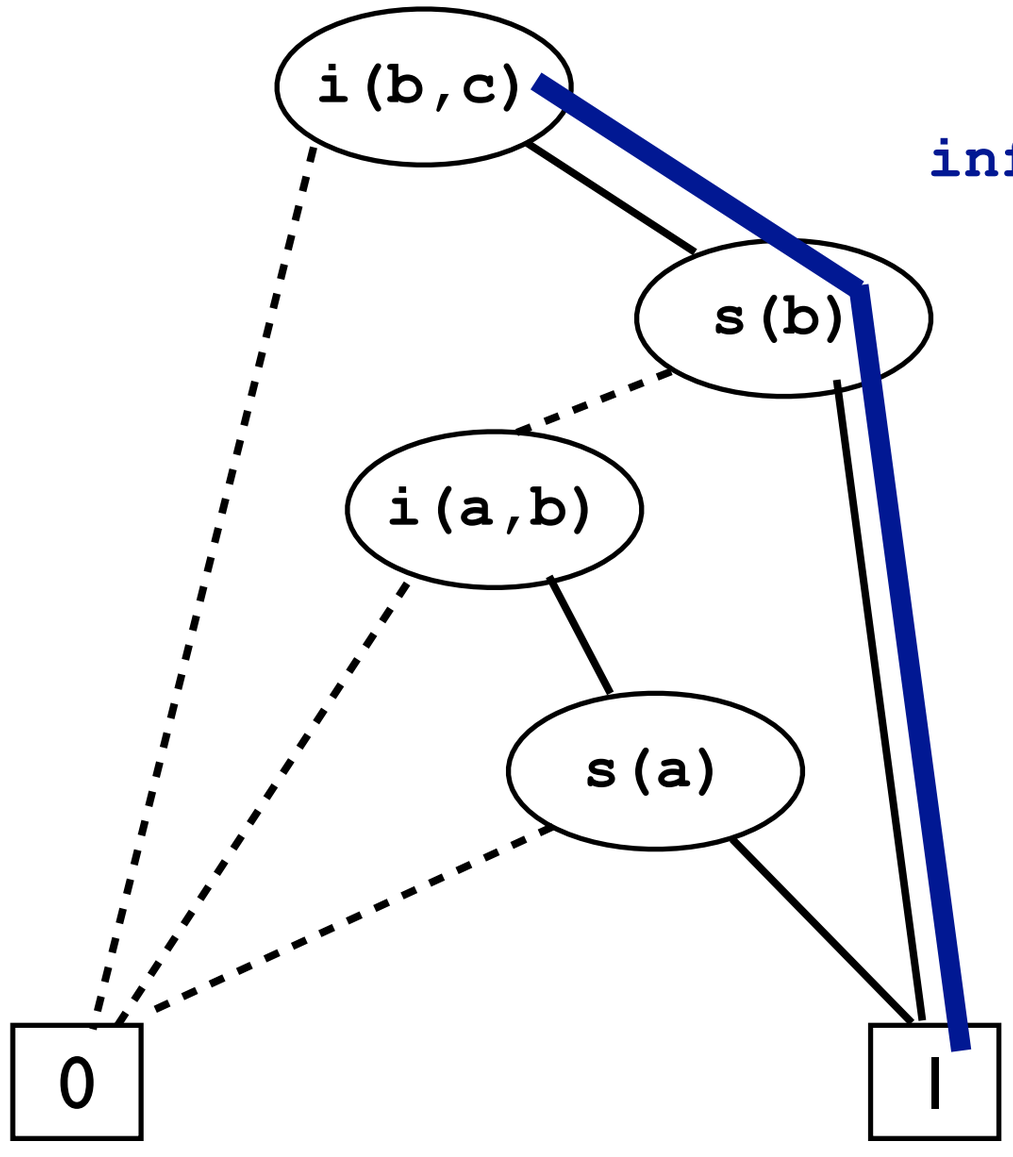
$$X \vee (\neg X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z)$$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]



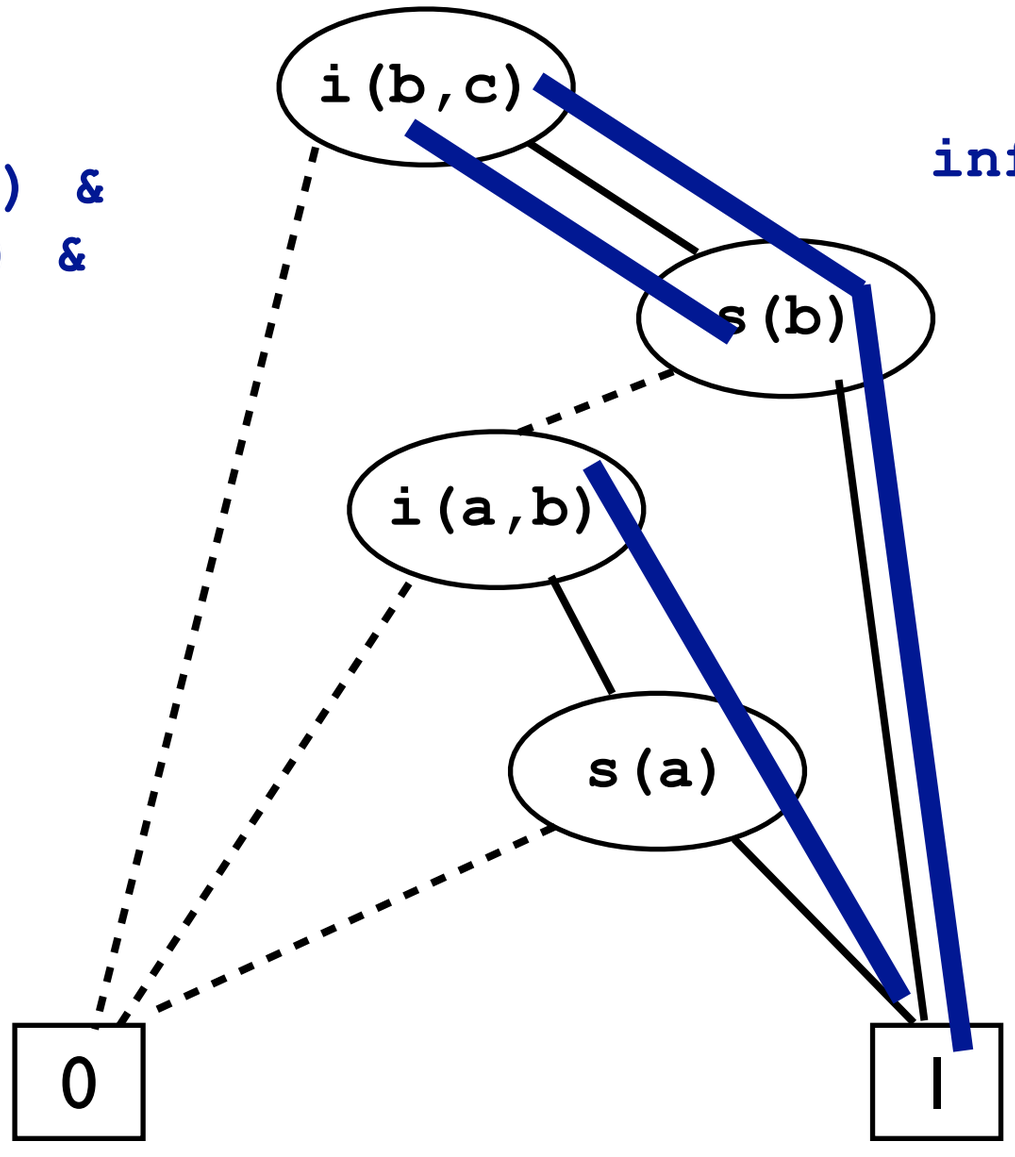
Binary Decision Diagrams
[Bryant 86]



influences(bob,carl) & stress(bob)

Binary Decision Diagrams
[Bryant 86]

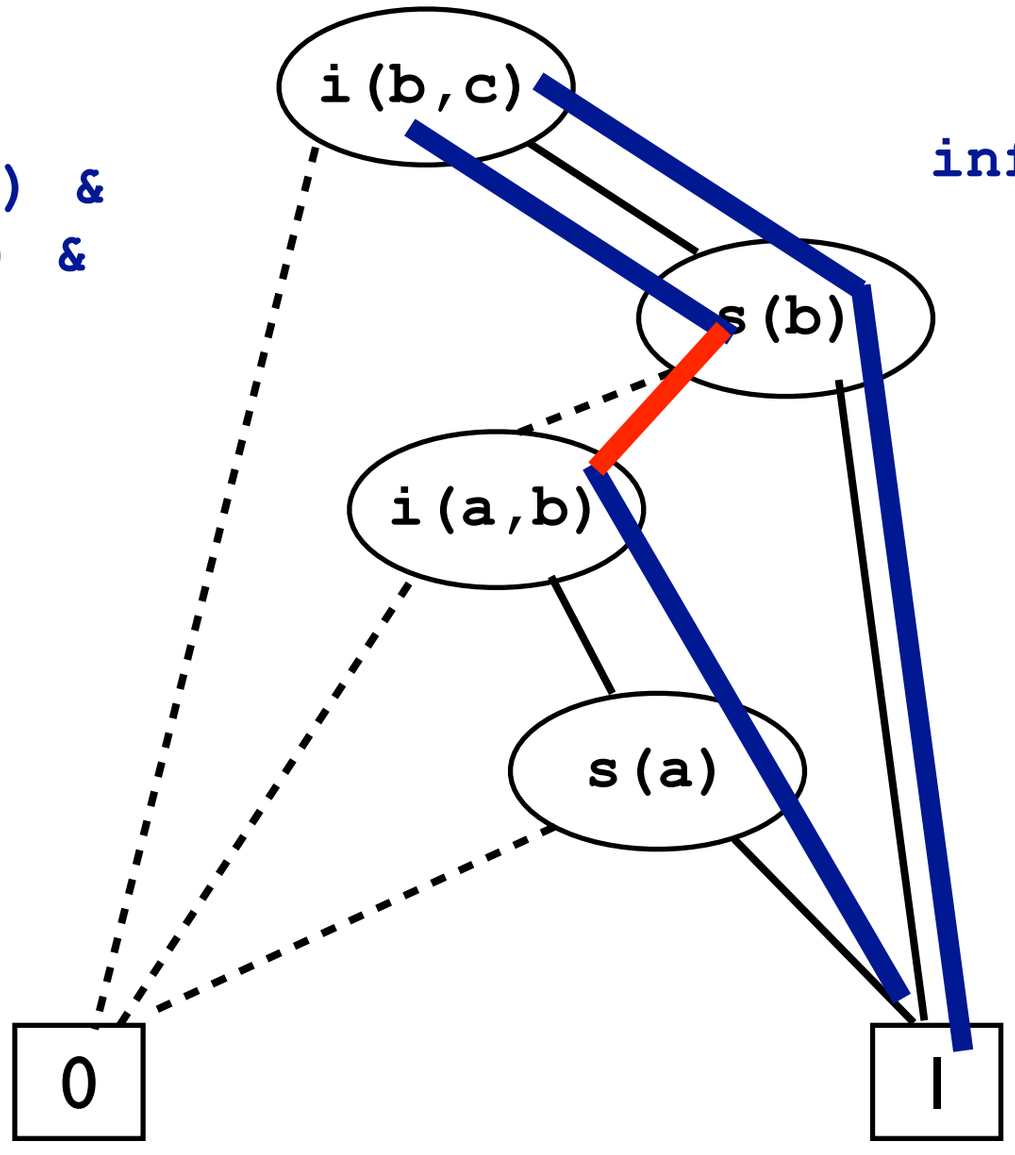
influences(bob,carl) & influences(ann,bob) & stress(ann)



influences(bob,carl) & stress(bob)

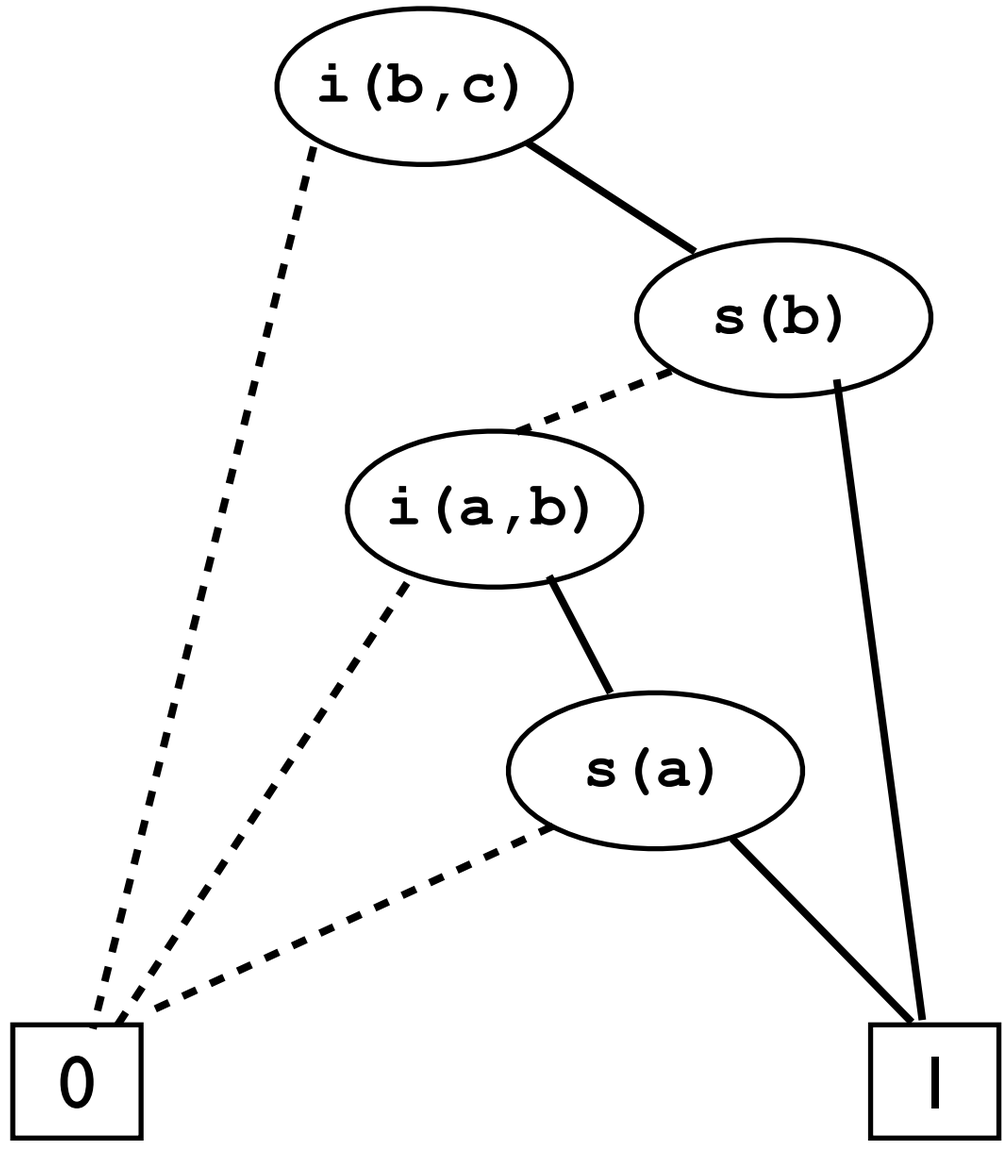
Binary Decision Diagrams
[Bryant 86]

`influences(bob,carl) & influences(ann,bob) & stress(ann)`
`& not stress(bob)`

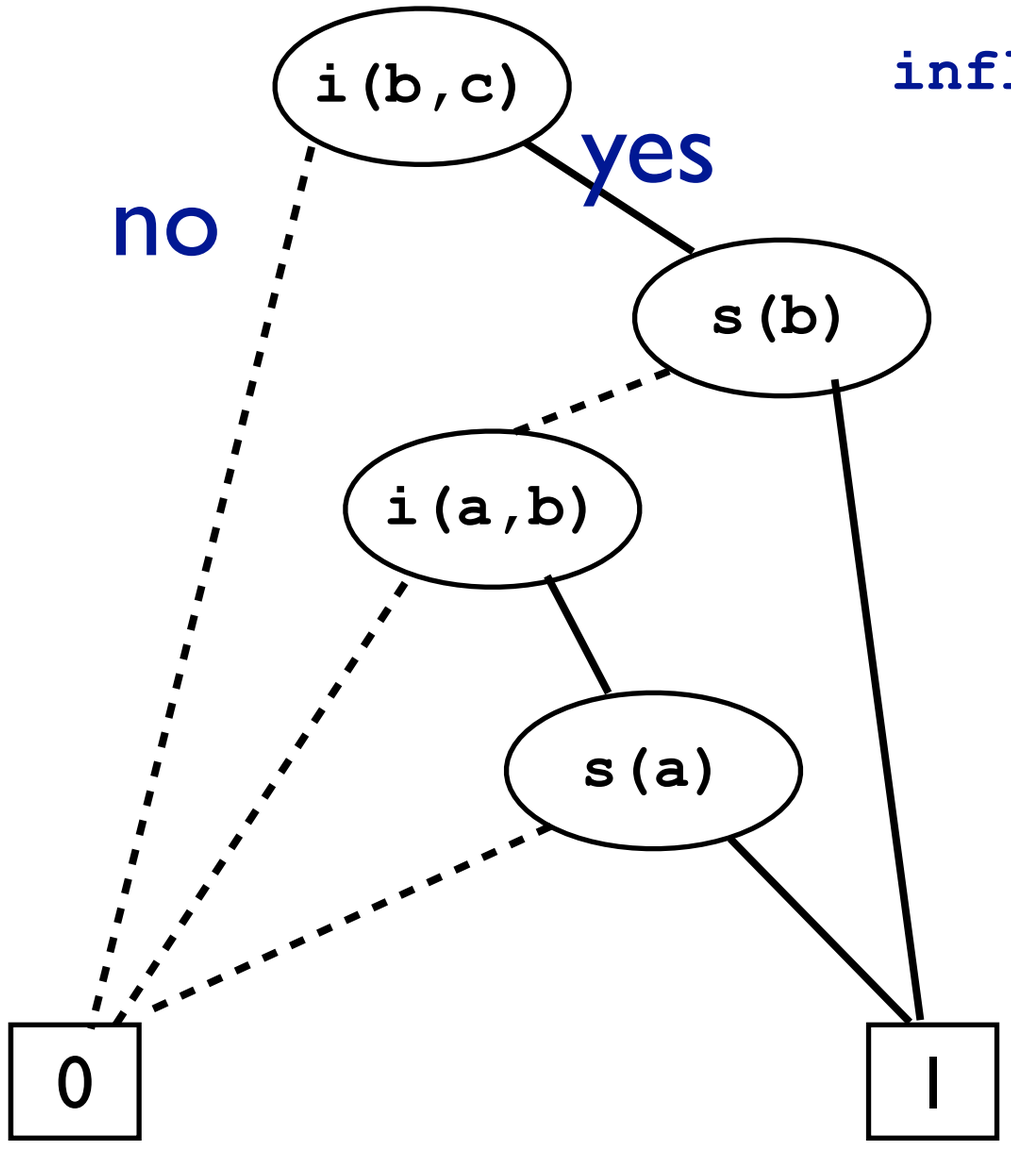


`influences(bob,carl) & stress(bob)`

Binary Decision Diagrams

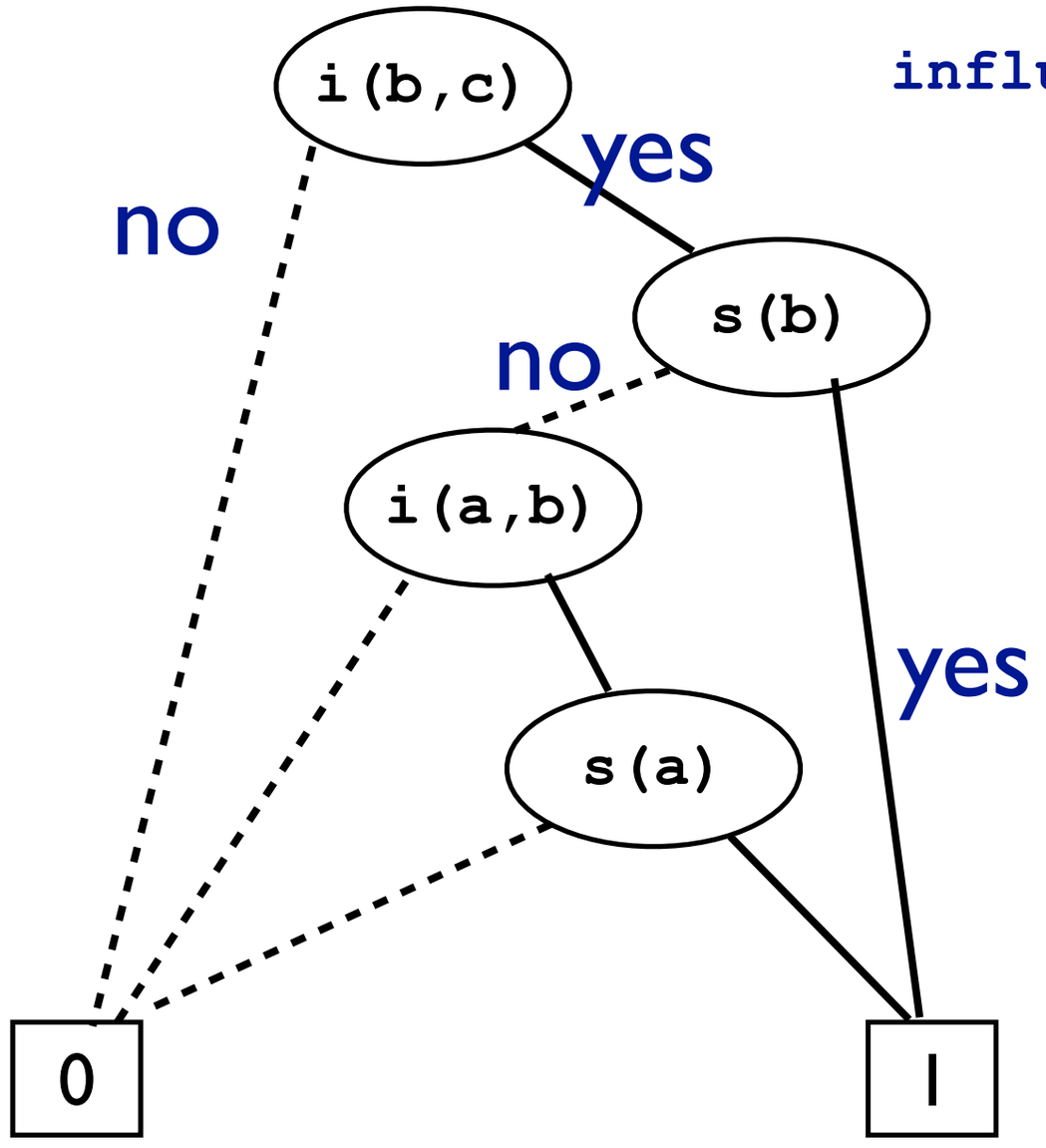


Binary Decision Diagrams



influences(bob,carl)?

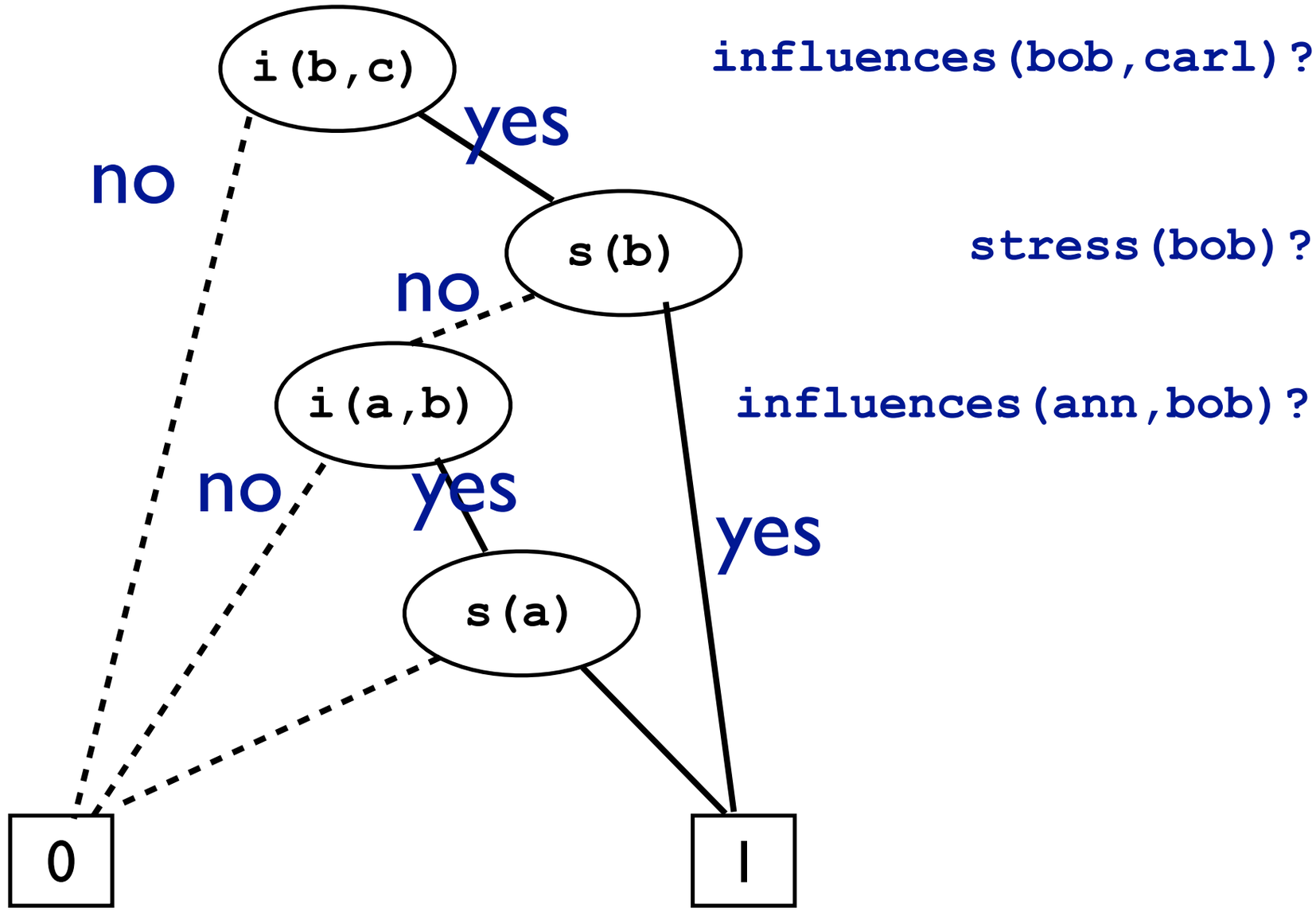
Binary Decision Diagrams



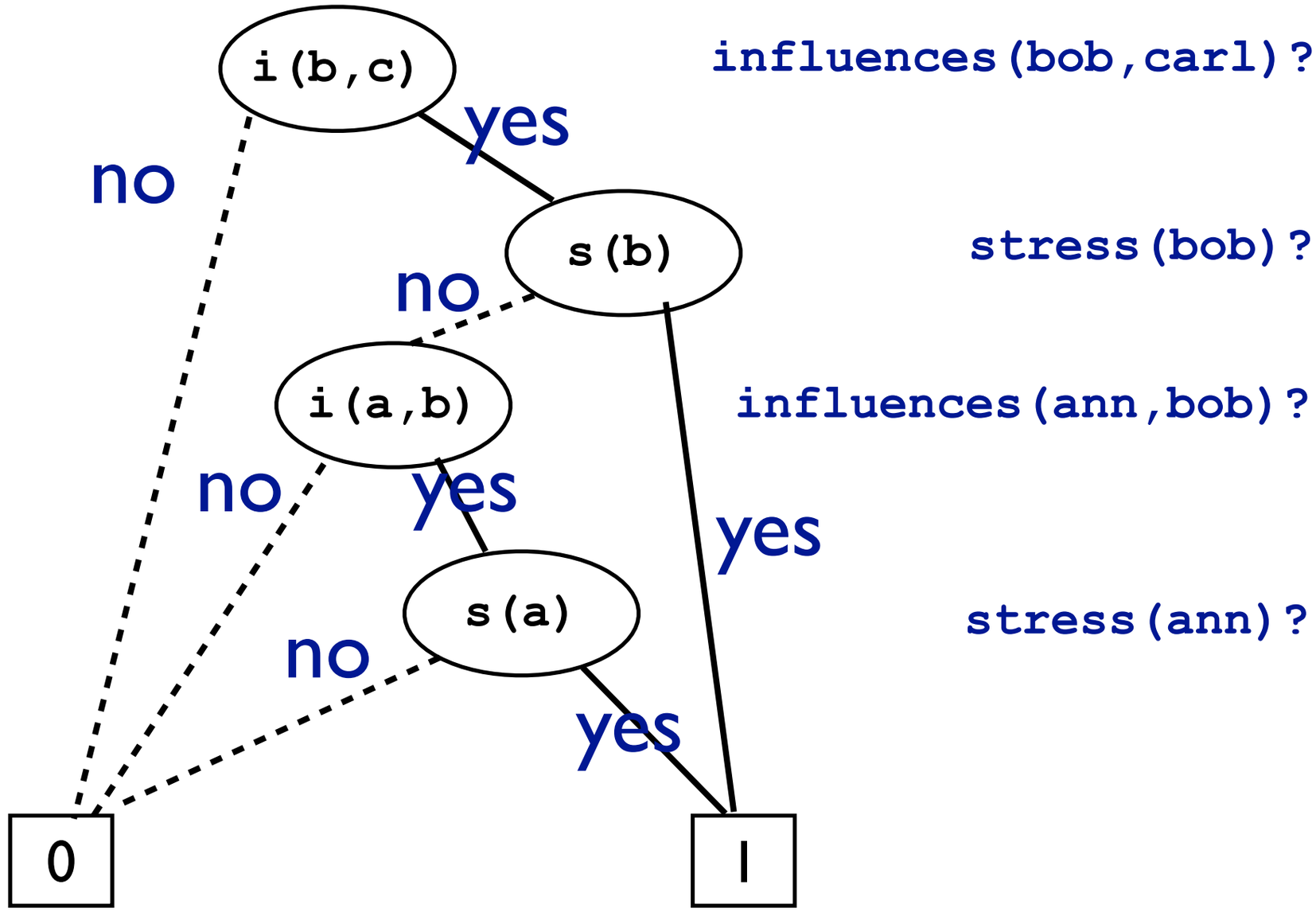
`influences(bob,carl)?`

`stress(bob)?`

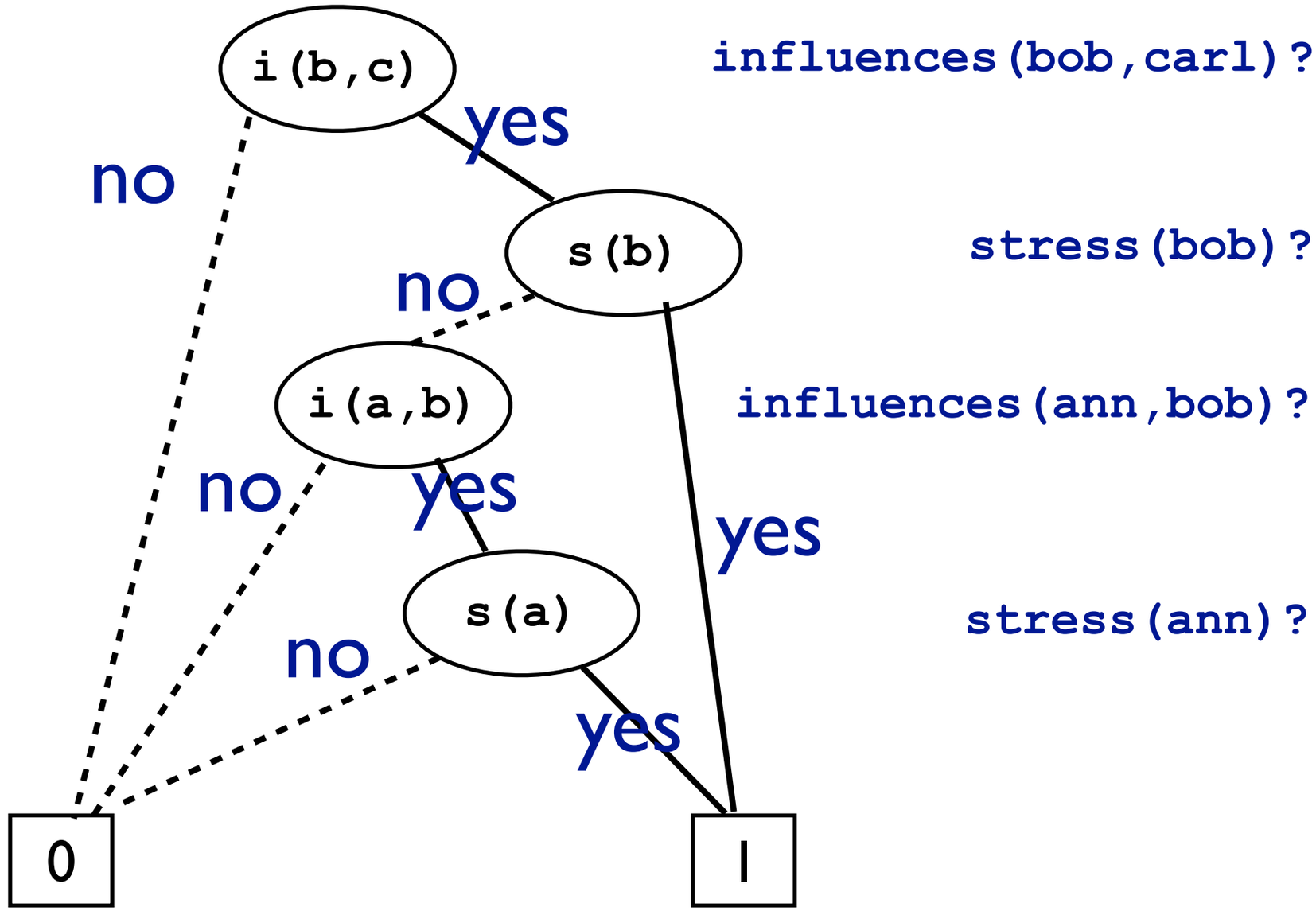
Binary Decision Diagrams



Binary Decision Diagrams

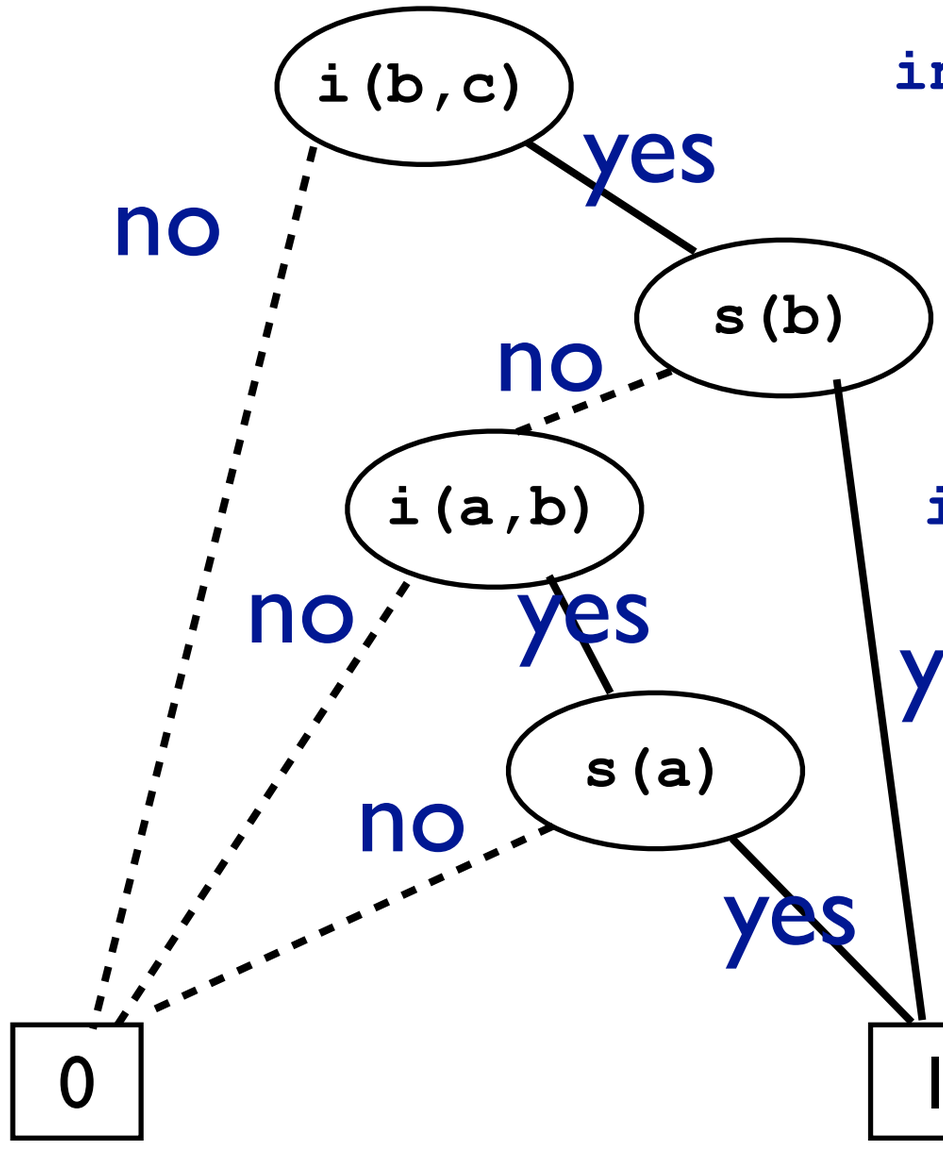


Binary Decision Diagrams



smokes(c) = i(b,c)&s(b) v
i(b,c)&i(a,b)&s(a)

Binary Decision Diagrams



influences(bob,carl)?

stress(bob)?

influences(ann,bob)?

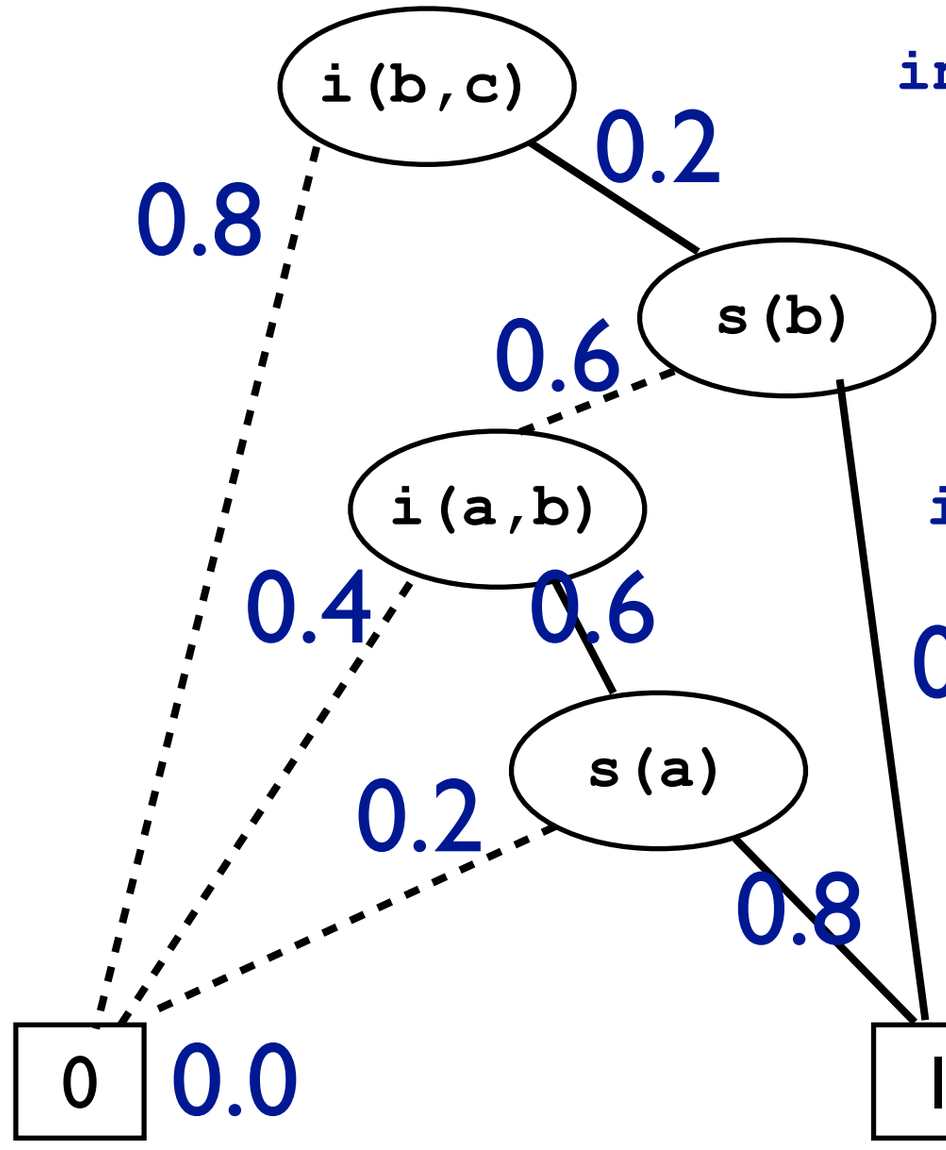
stress(ann)?

probability of

smokes(c)?

smokes(c) = i(b,c)&s(b) v
i(b,c)&i(a,b)&s(a)

Binary Decision Diagrams



`influences(bob,carl)?`

`stress(bob)?`

`influences(ann,bob)?`

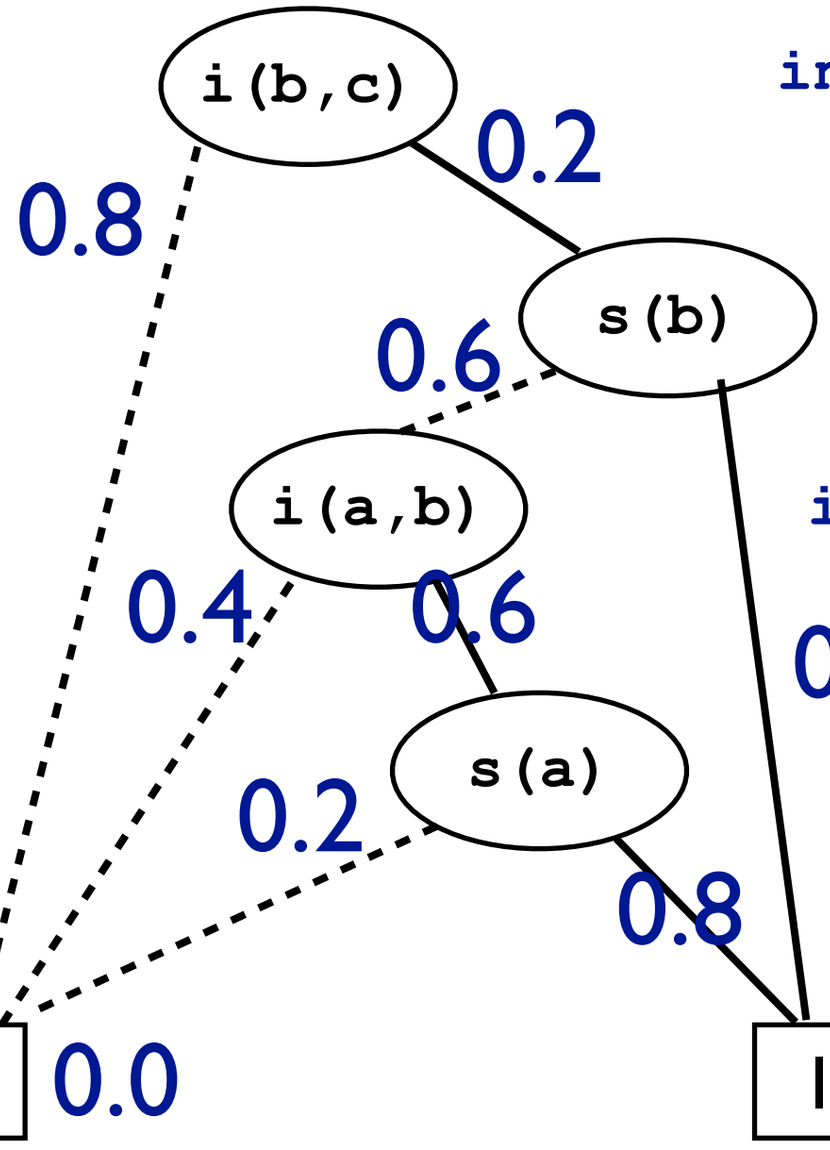
`stress(ann)?`

probability of

`smokes(c)?`

`smokes(c) = i(b,c)&s(b) v`
`i(b,c)&i(a,b)&s(a)`

Binary Decision Diagrams



`influences(bob,carl)?`

`stress(bob)?`

`influences(ann,bob)?`

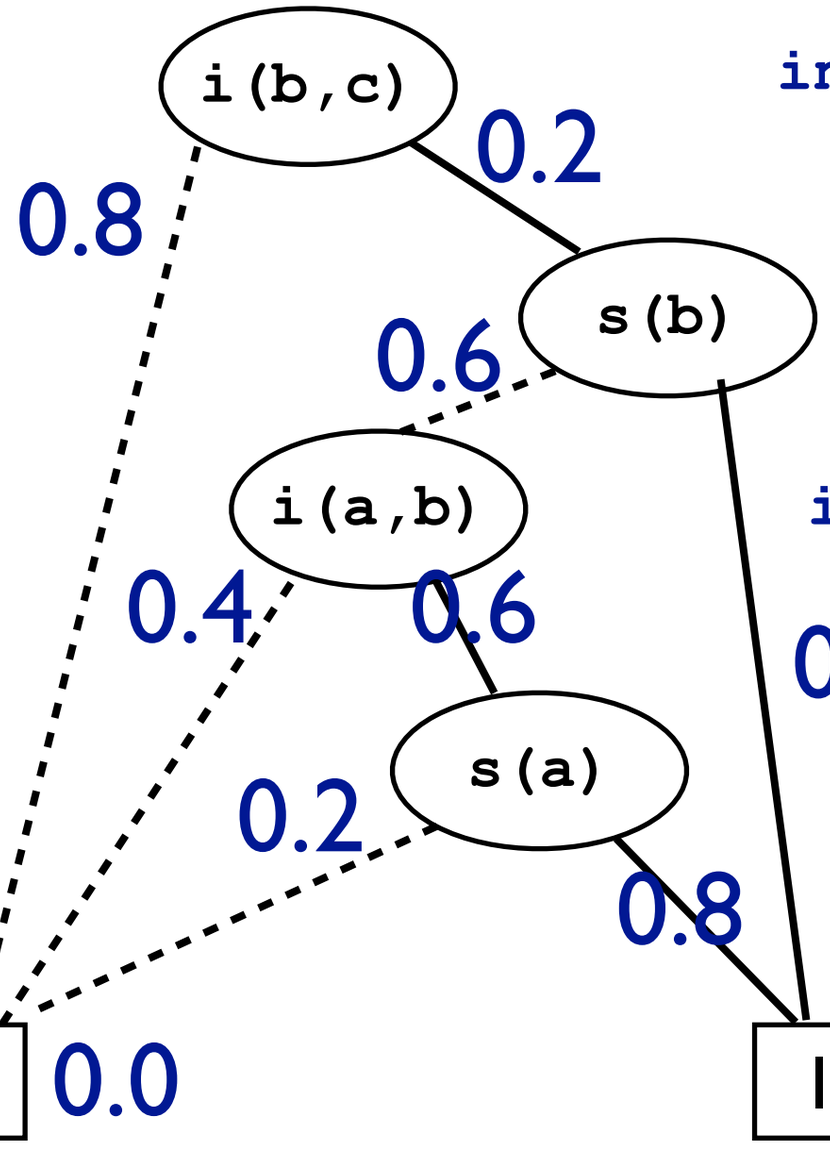
`stress(ann)?`

0.2×0.0+0.8×1.0=0.8

probability of `smokes(c)`?

`smokes(c) = i(b,c)&s(b) v i(b,c)&i(a,b)&s(a)`

Binary Decision Diagrams



influences(bob,carl)?

stress(bob)?

0.4×0.0+0.6×0.8=0.48

influences(ann,bob)?

0.2×0.0+0.8×1.0=0.8

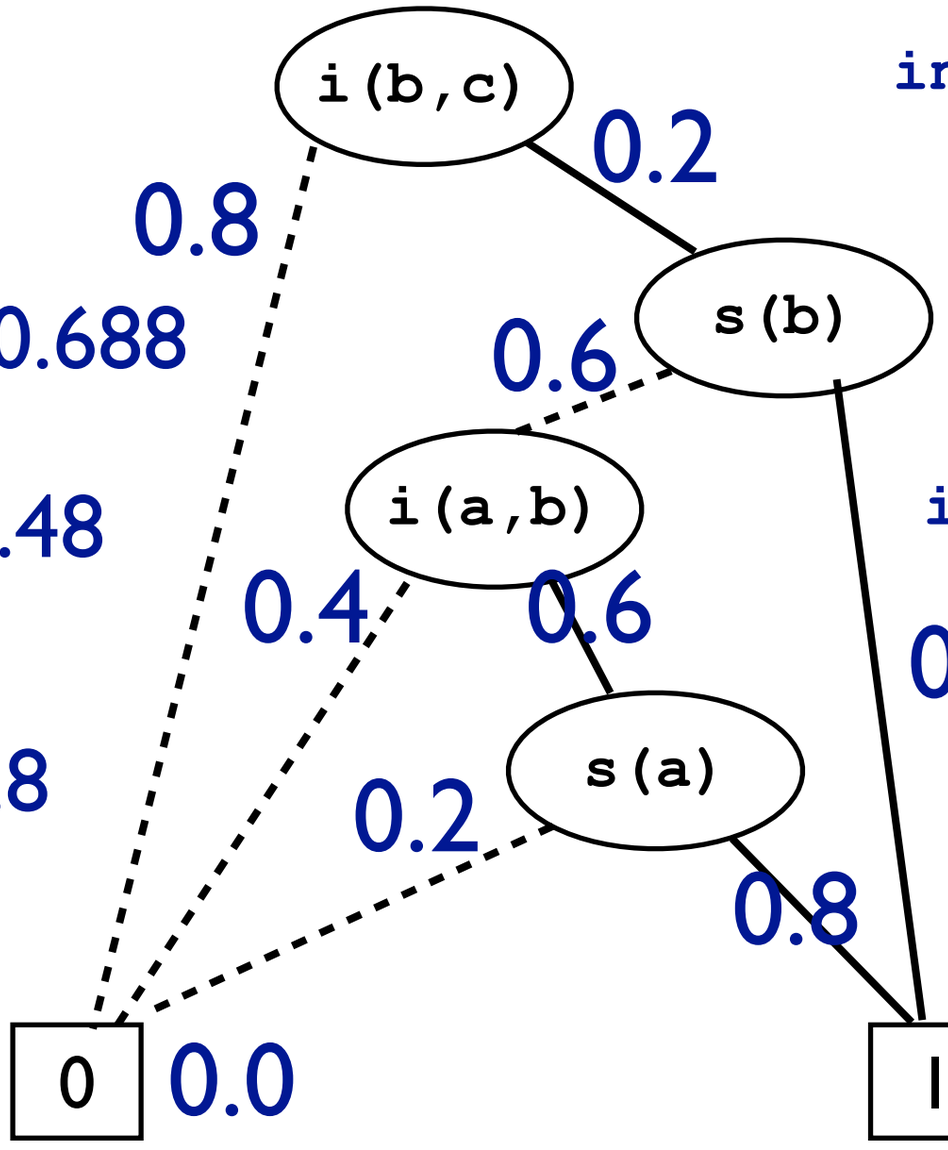
stress(ann)?

probability of

smokes(c)?

smokes(c) = i(b,c)&s(b) v
i(b,c)&i(a,b)&s(a)

Binary Decision Diagrams



influences(bob,carl)?

stress(bob)?

influences(ann,bob)?

stress(ann)?

0.6×0.48+0.4×1.0=0.688

0.4×0.0+0.6×0.8=0.48

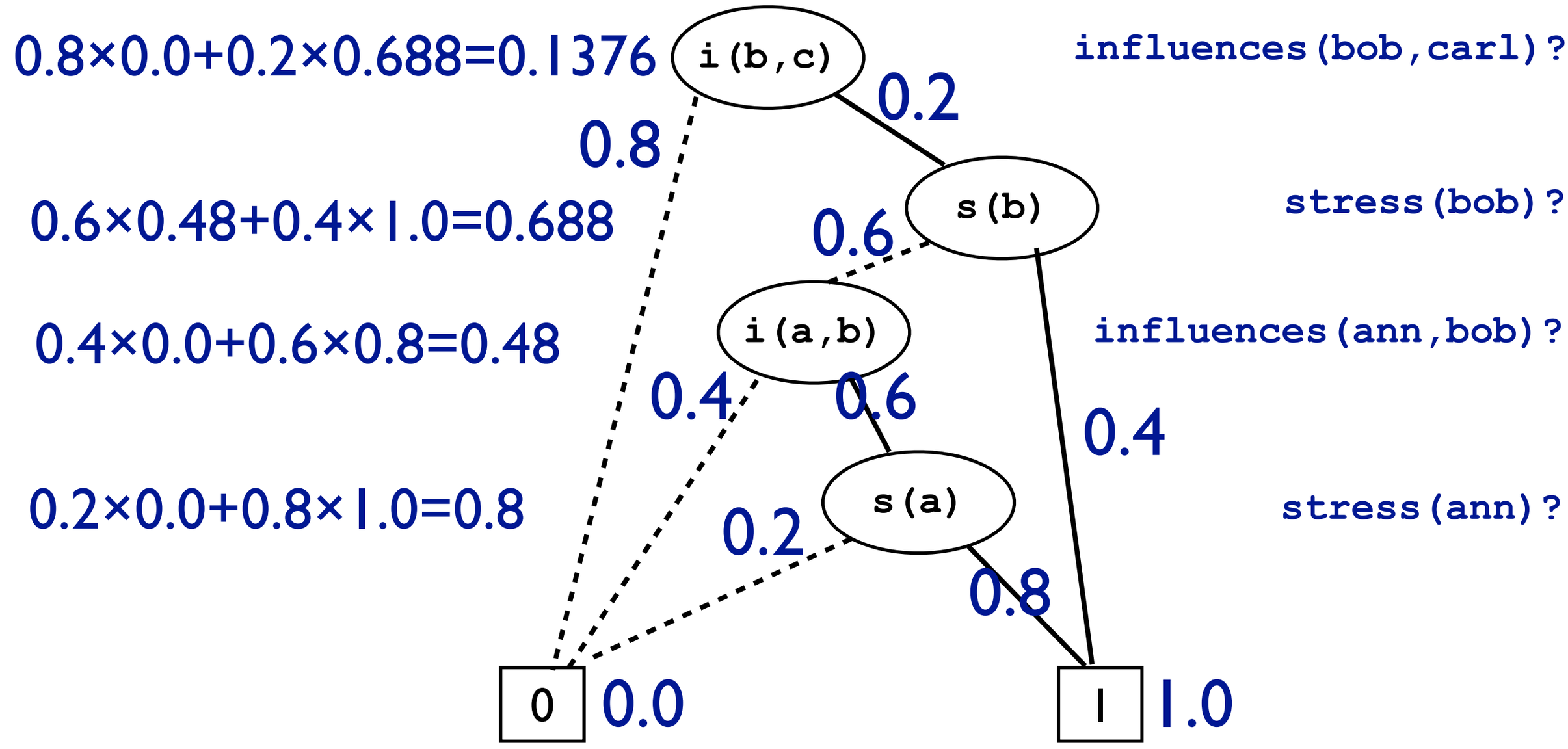
0.2×0.0+0.8×1.0=0.8

probability of

smokes(c)?

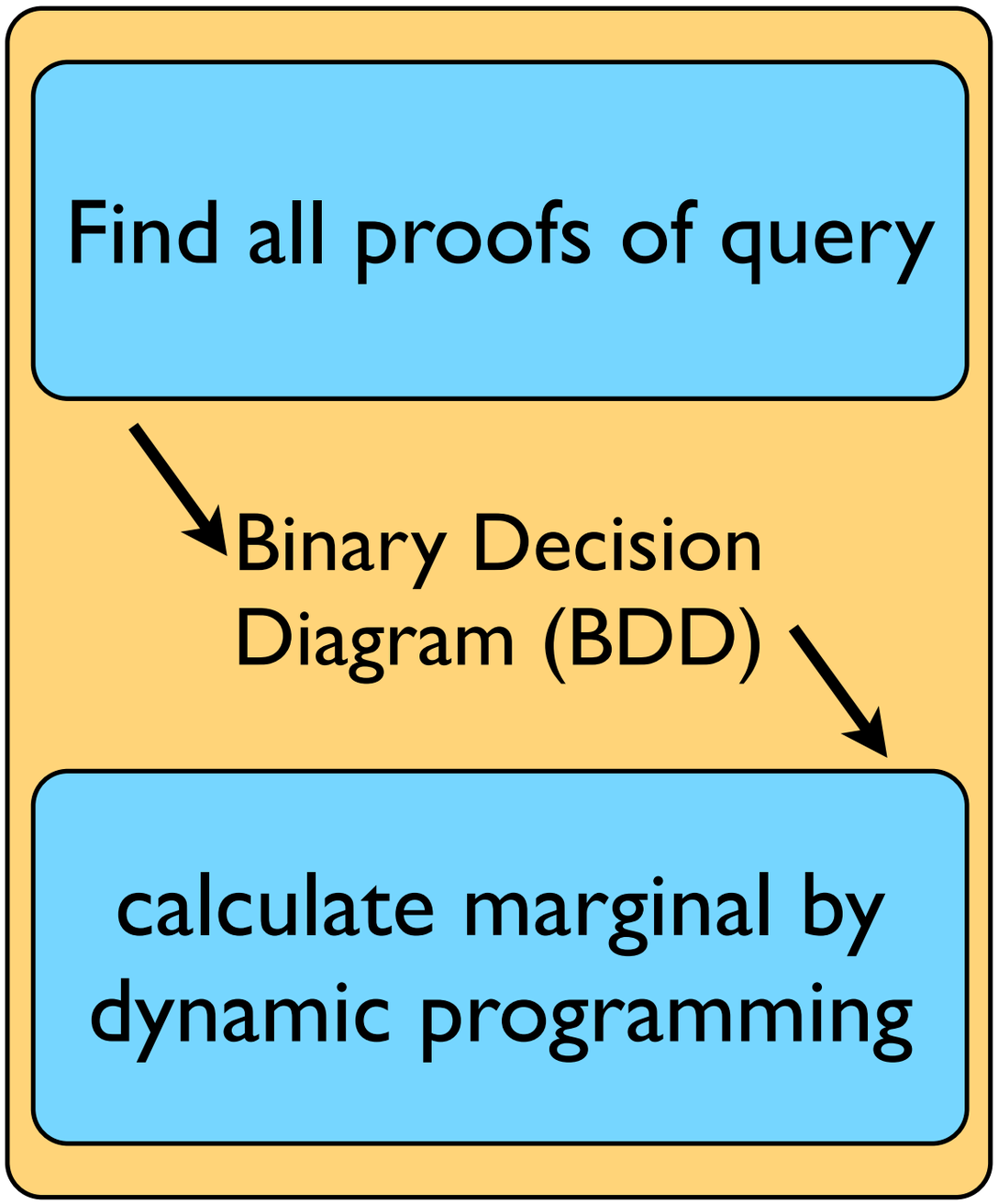
smokes(c) = i(b,c)&s(b) v
i(b,c)&i(a,b)&s(a)

Binary Decision Diagrams



Initial Approach

(ProbLog1 & others)

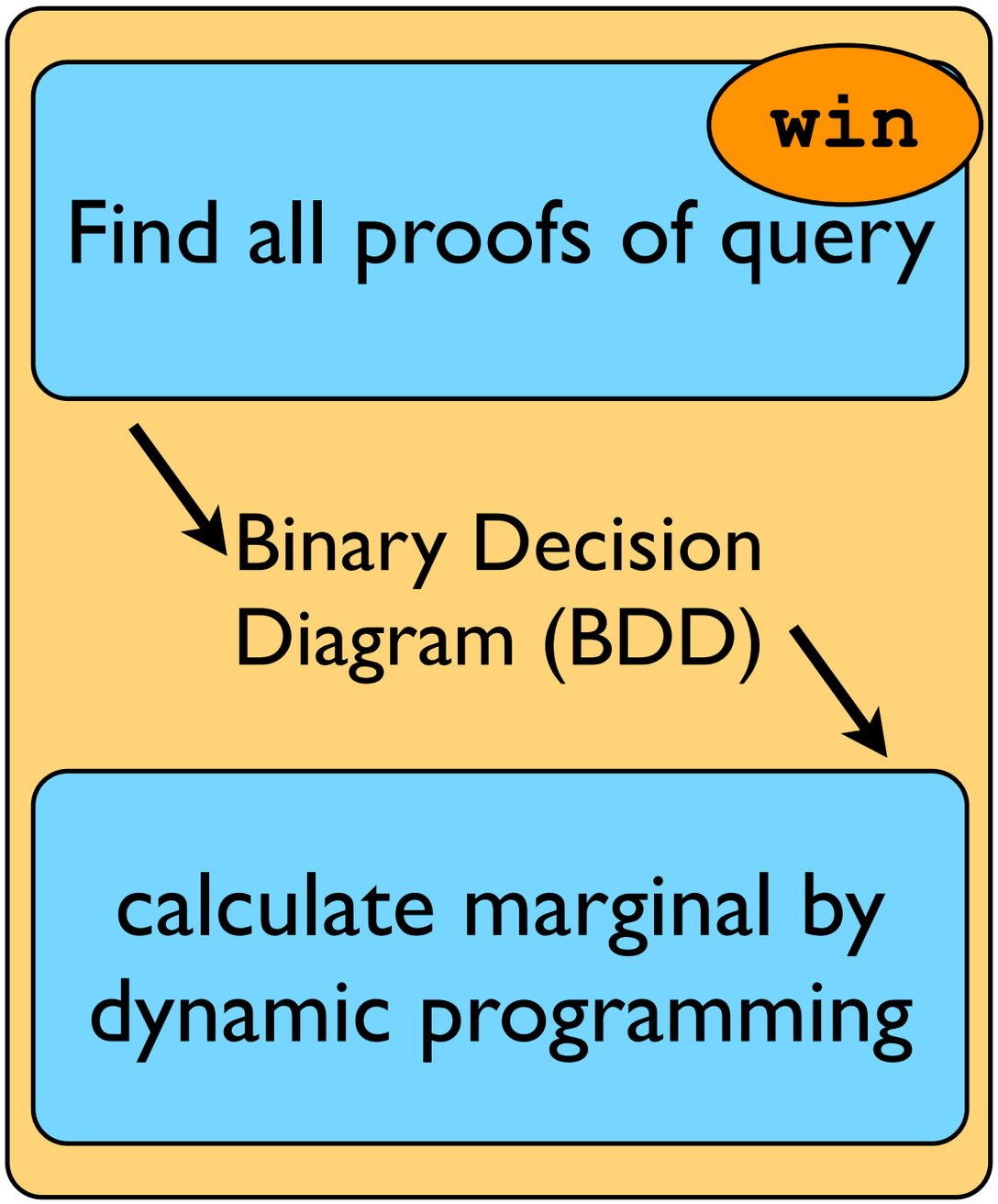


[De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Initial Approach

(ProbLog1 & others)

```
0.4::heads(1).
0.7::heads(2).
0.5::heads(3).
win :- heads(1).
win :- heads(2),heads(3).
```

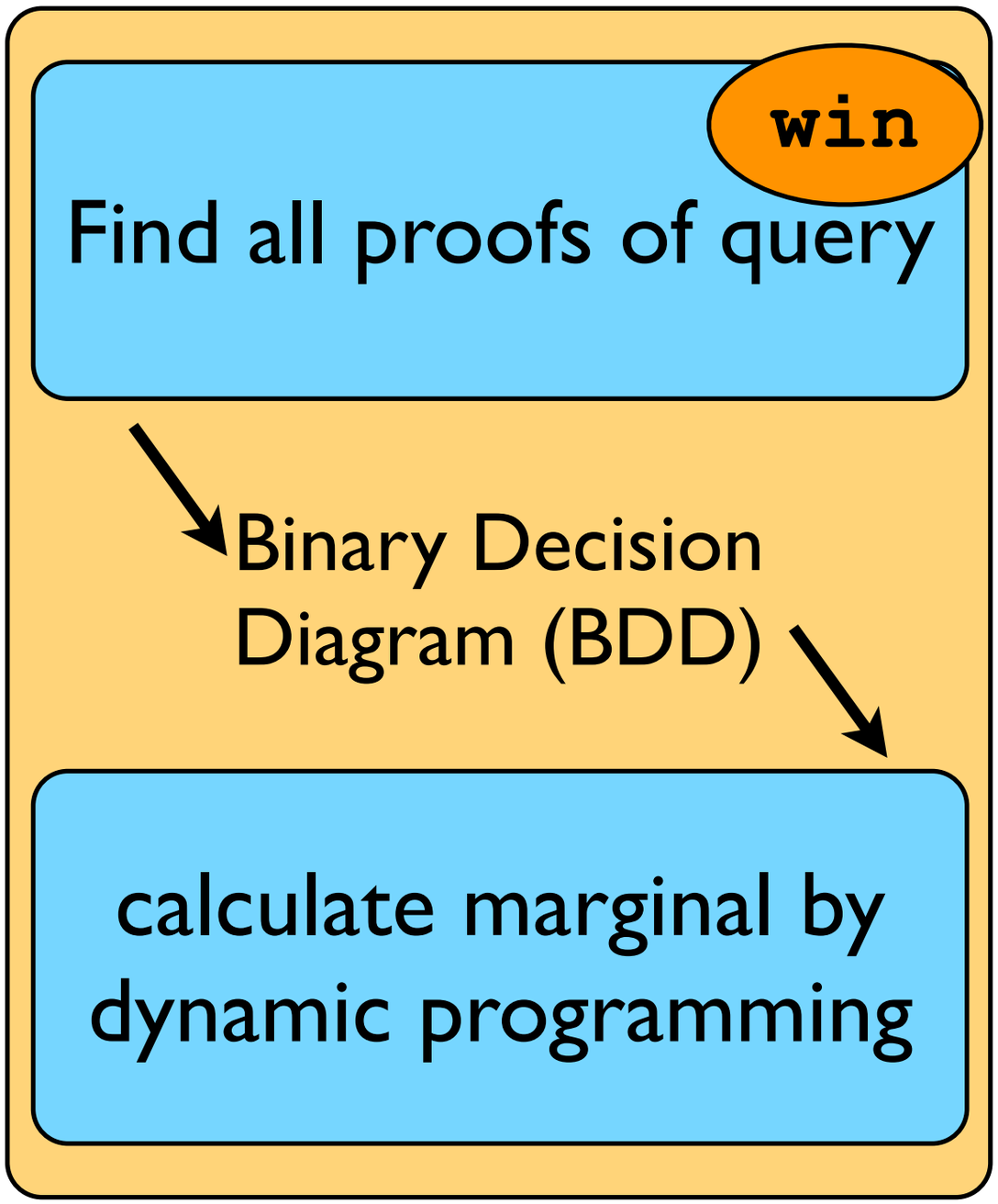


[De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Initial Approach

(ProbLog1 & others)

```
0.4::heads(1).
0.7::heads(2).
0.5::heads(3).
win :- heads(1).
win :- heads(2),heads(3).
```




```
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heads(2)  & heads(3)
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[De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

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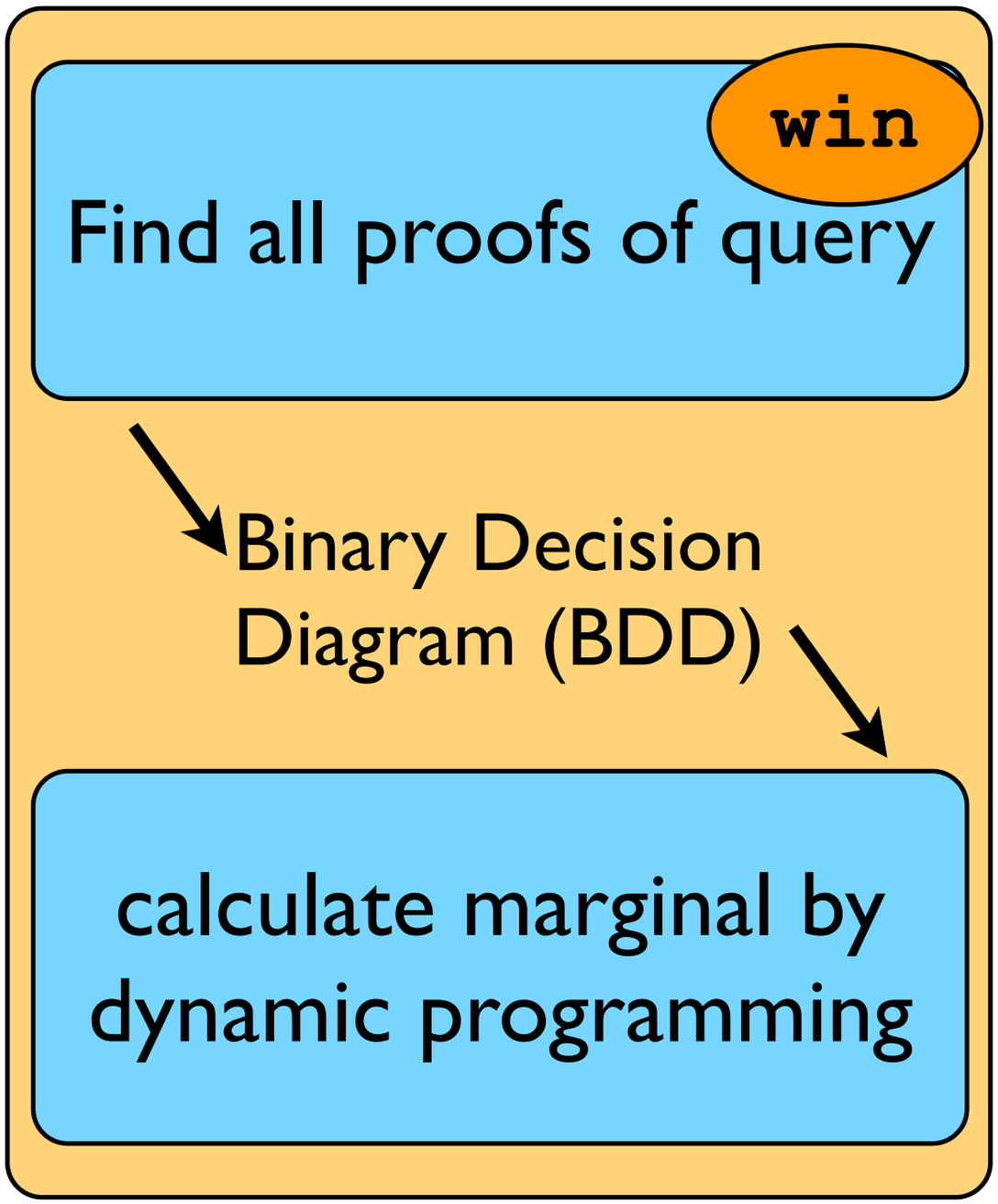


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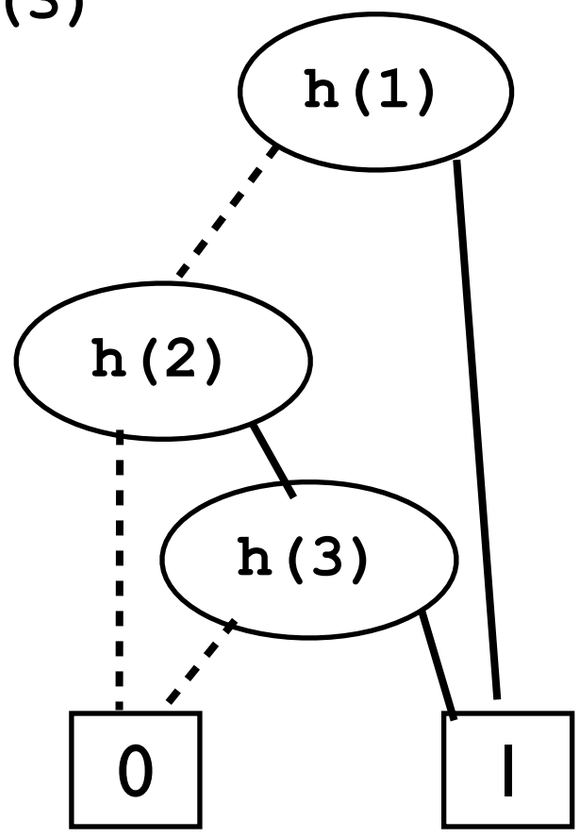

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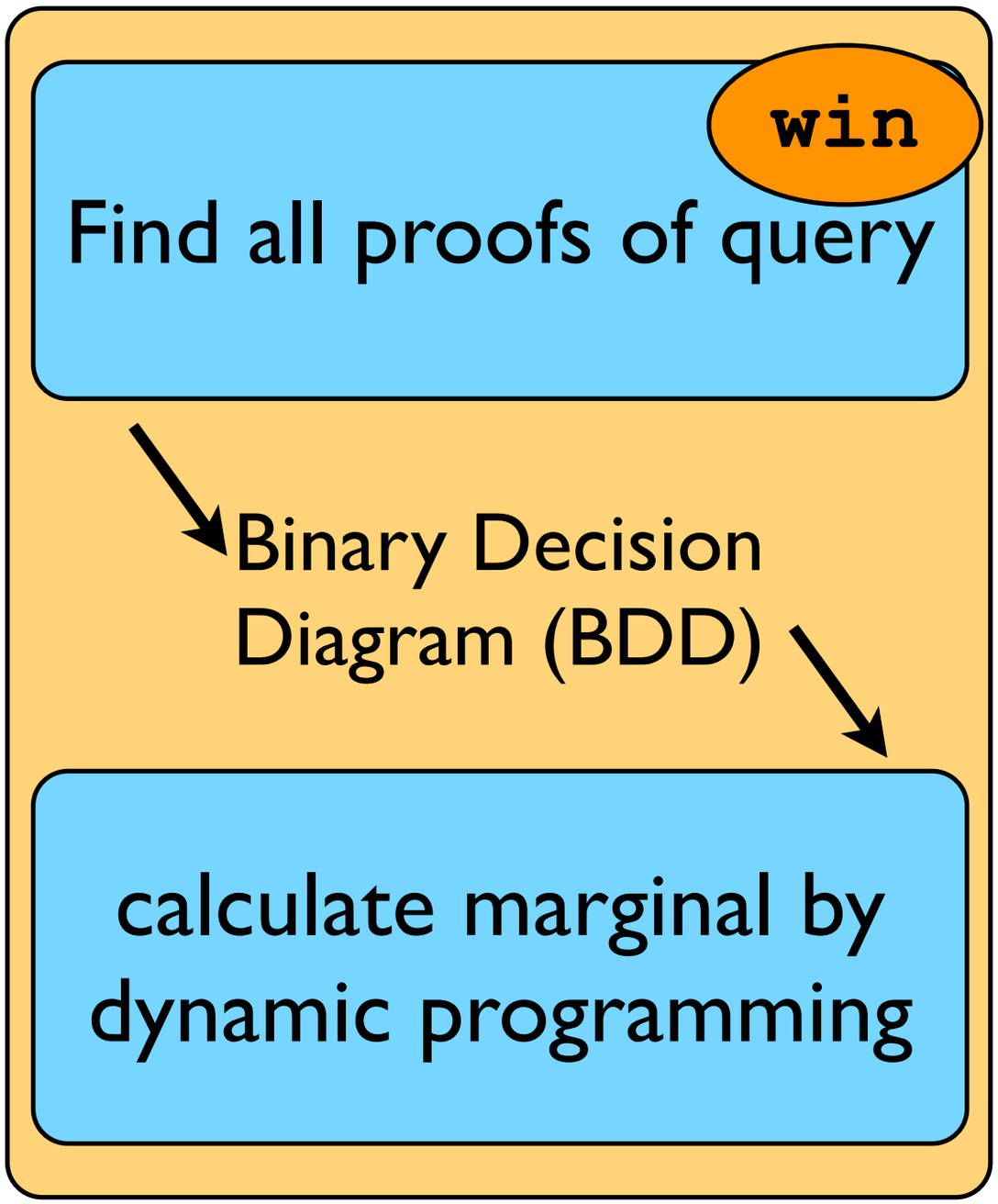


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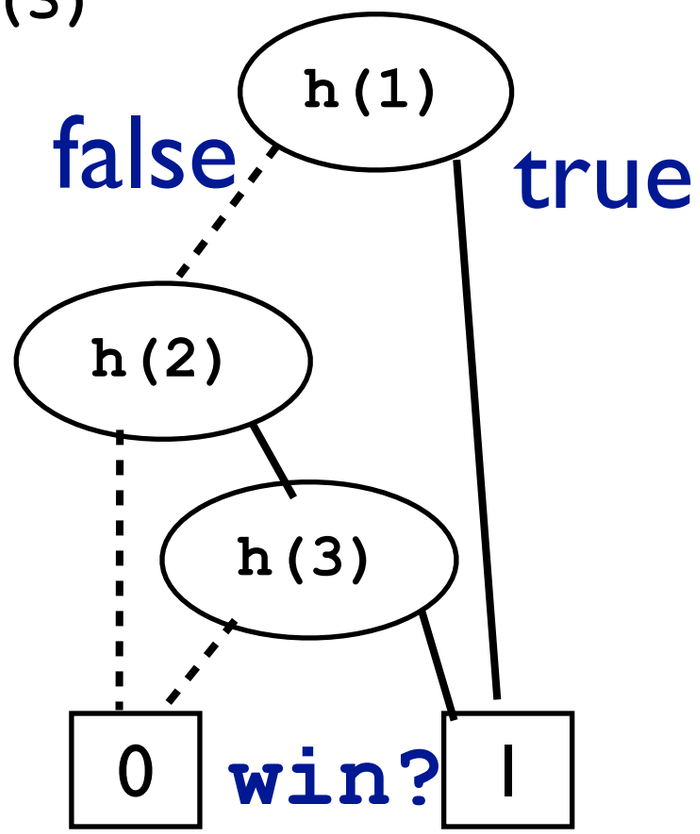

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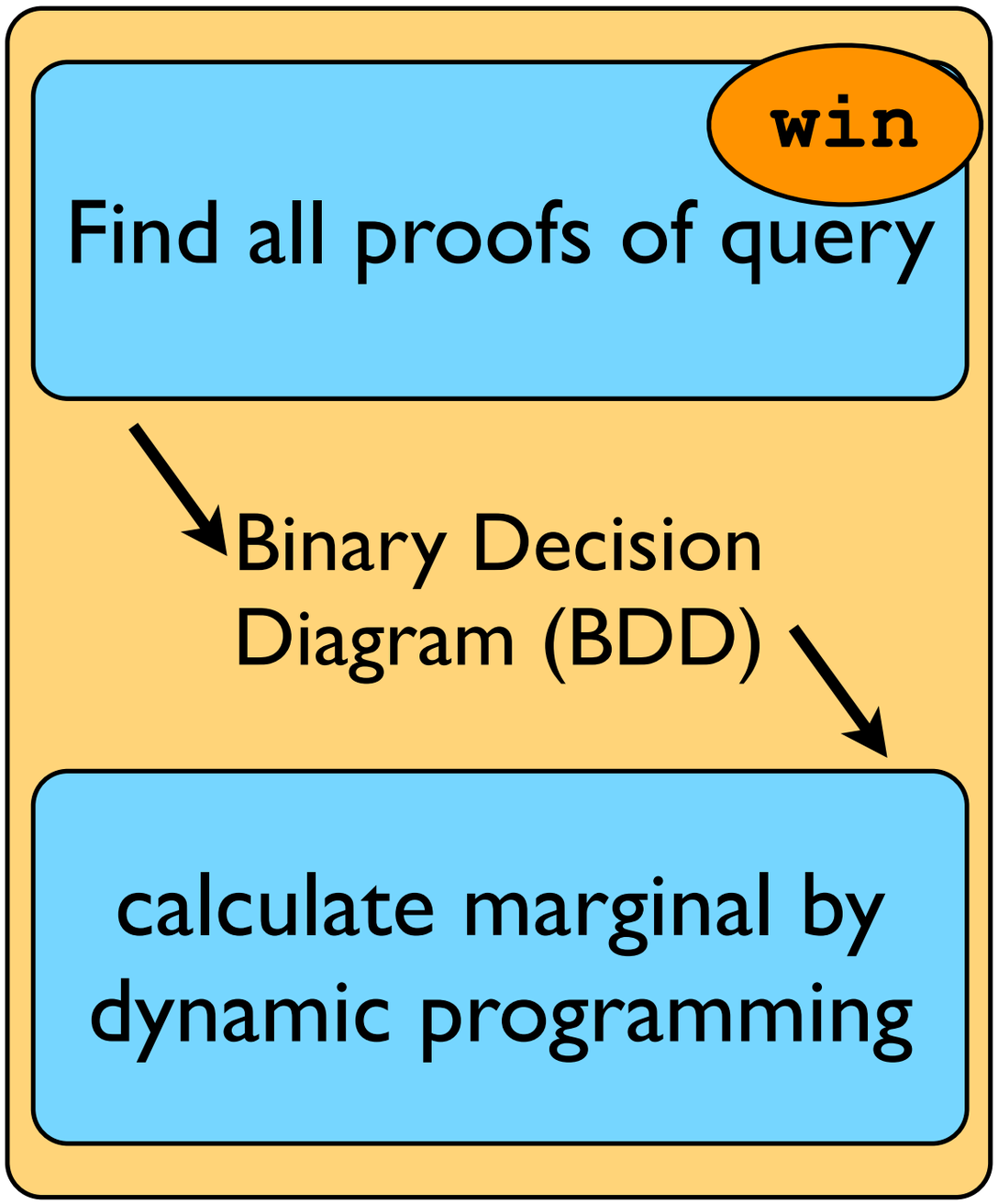


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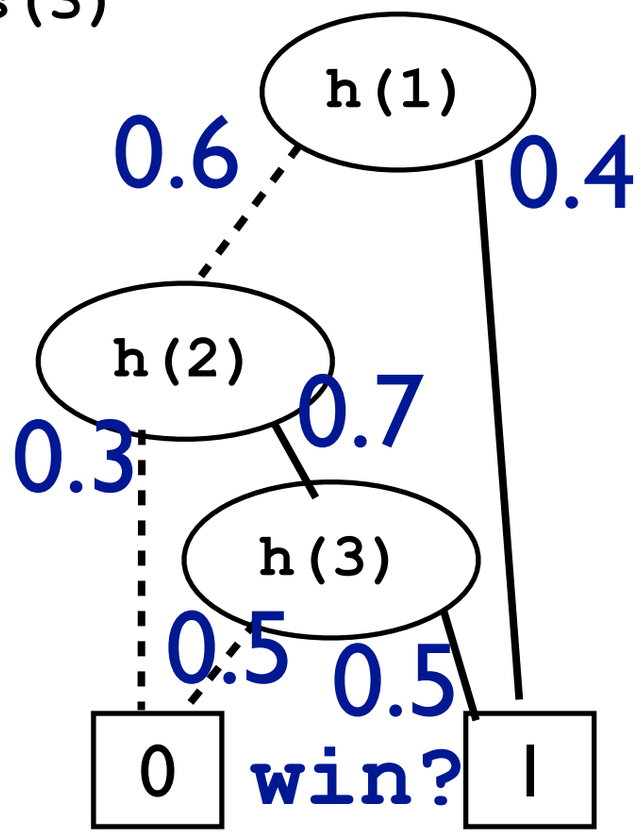


P(**win**) = probability of reaching 1-leaf

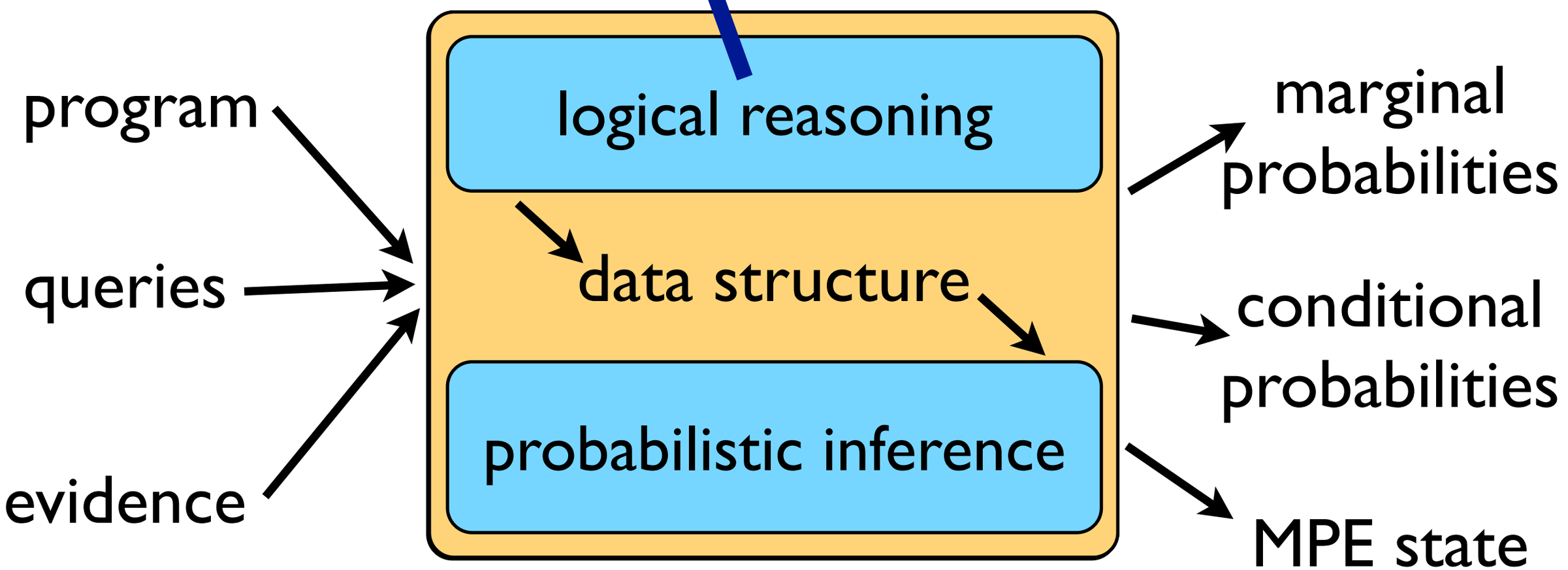
 [De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Answering Questions

1. using proofs
2. using models

Given:

Find:



Logical Reasoning: Models in Prolog

```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
      influences(Y,X),
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- Forward reasoning to construct unique model:

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→ weighted model counting

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$$WMC(\phi) = \sum_{I_V \models \phi} \prod_{l \in I_V} w(l)$$

Weighted Model Counting

propositional formula in conjunctive normal form (CNF)

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weight of literal

for p::f,
w(f) = p
w(not f) = 1−p

Weighte

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

propositional formula in conjunctive normal form (CNF)

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ProbLog → CNF

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0.8::stress(ann).
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- Find relevant ground rules by backward reasoning

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- Find relevant ground rules by backward reasoning

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smokes(carl) :- influences(bob,carl),smokes(bob).
smokes(bob) :- stress(bob).
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```

- Convert to propositional logic formula

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- Convert to propositional logic formula

may require loop-breaking

$$sm(c) \leftrightarrow (i(b,c) \wedge sm(b)) \wedge sm(b) \leftrightarrow (st(b) \vee (i(a,b) \wedge sm(a))) \wedge sm(a) \leftrightarrow st(a)$$

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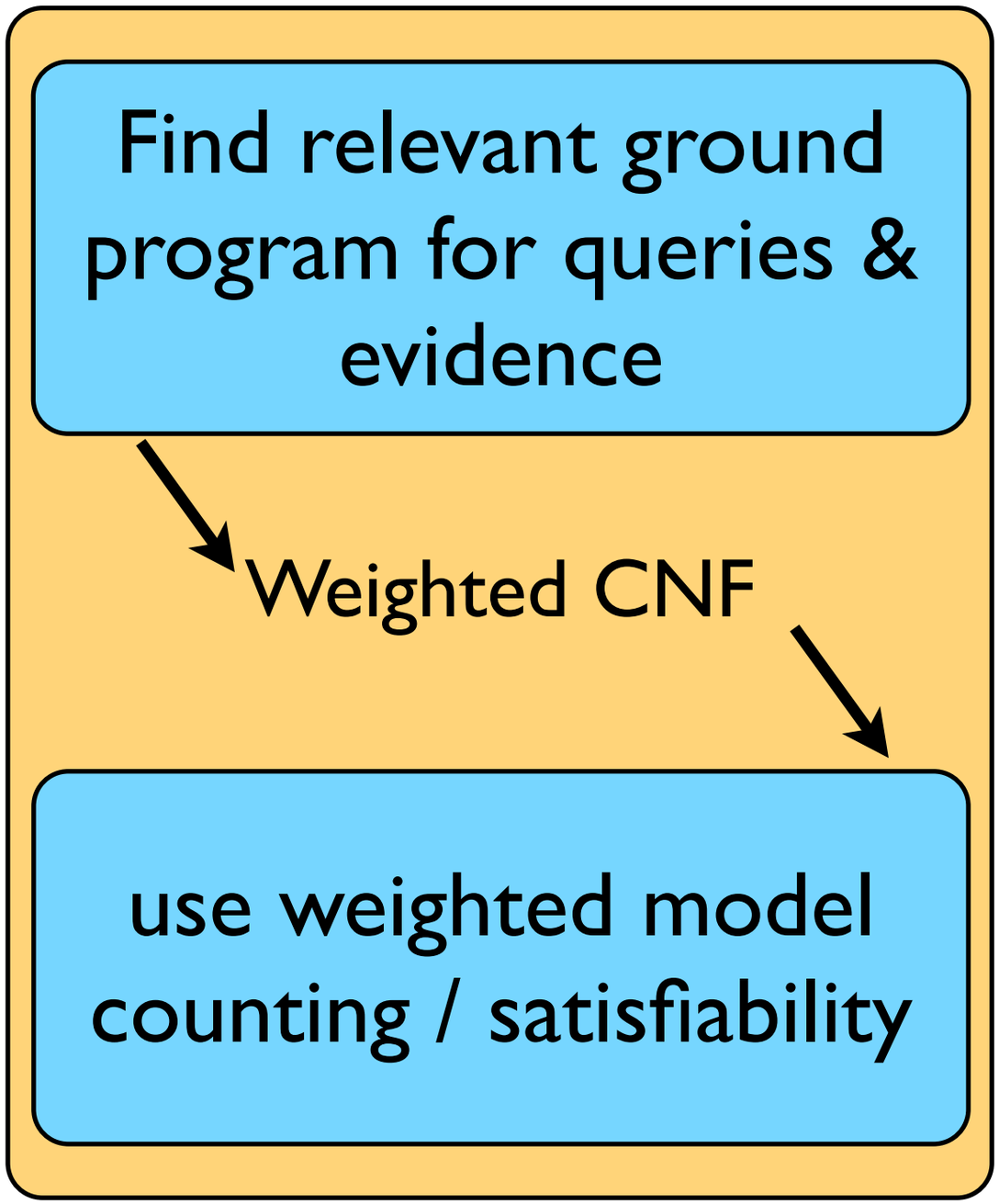
- Rewrite in CNF (as usual)

Current Approach

(ProbLog2)



[Fierens et al, TPLP 14]

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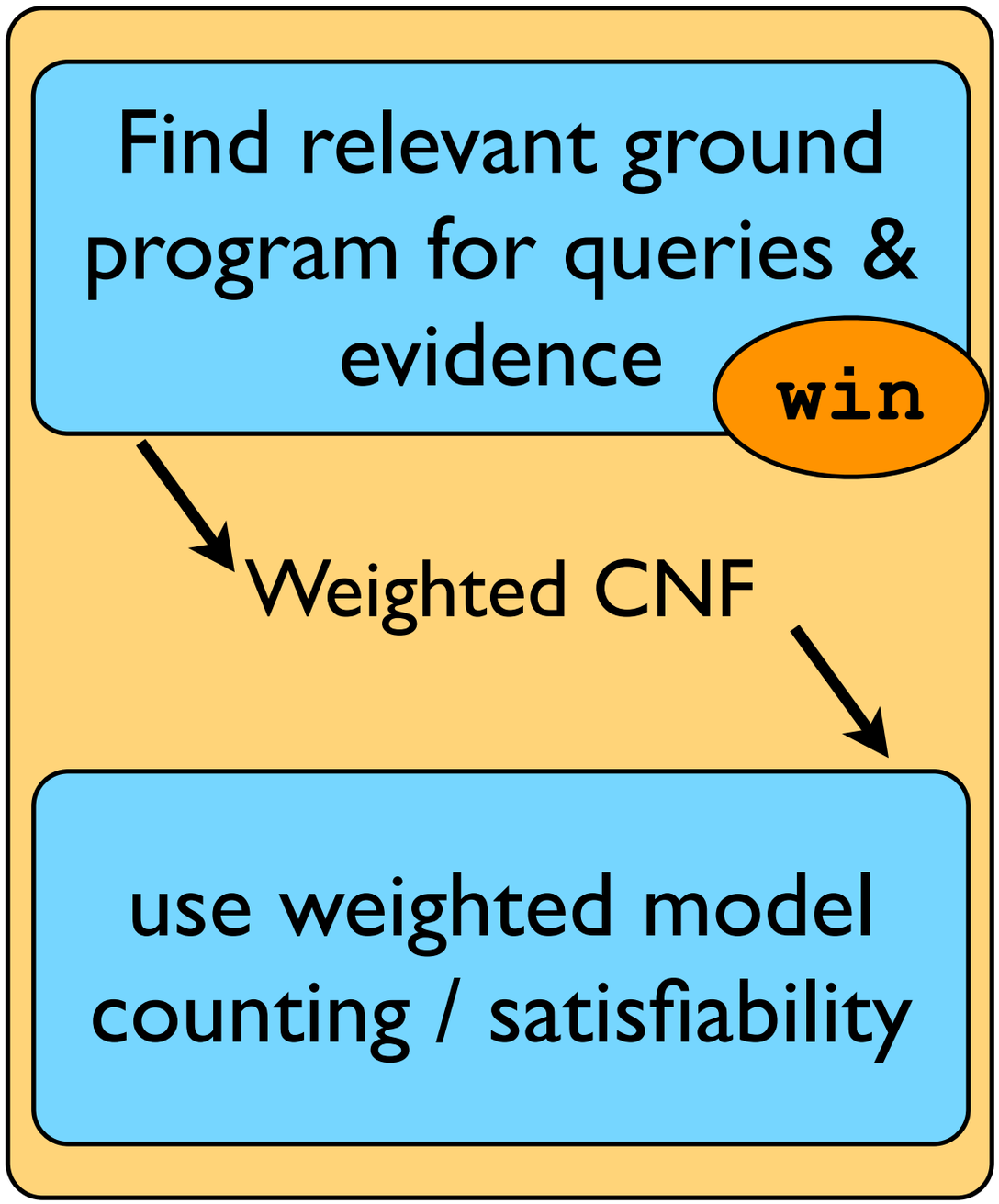

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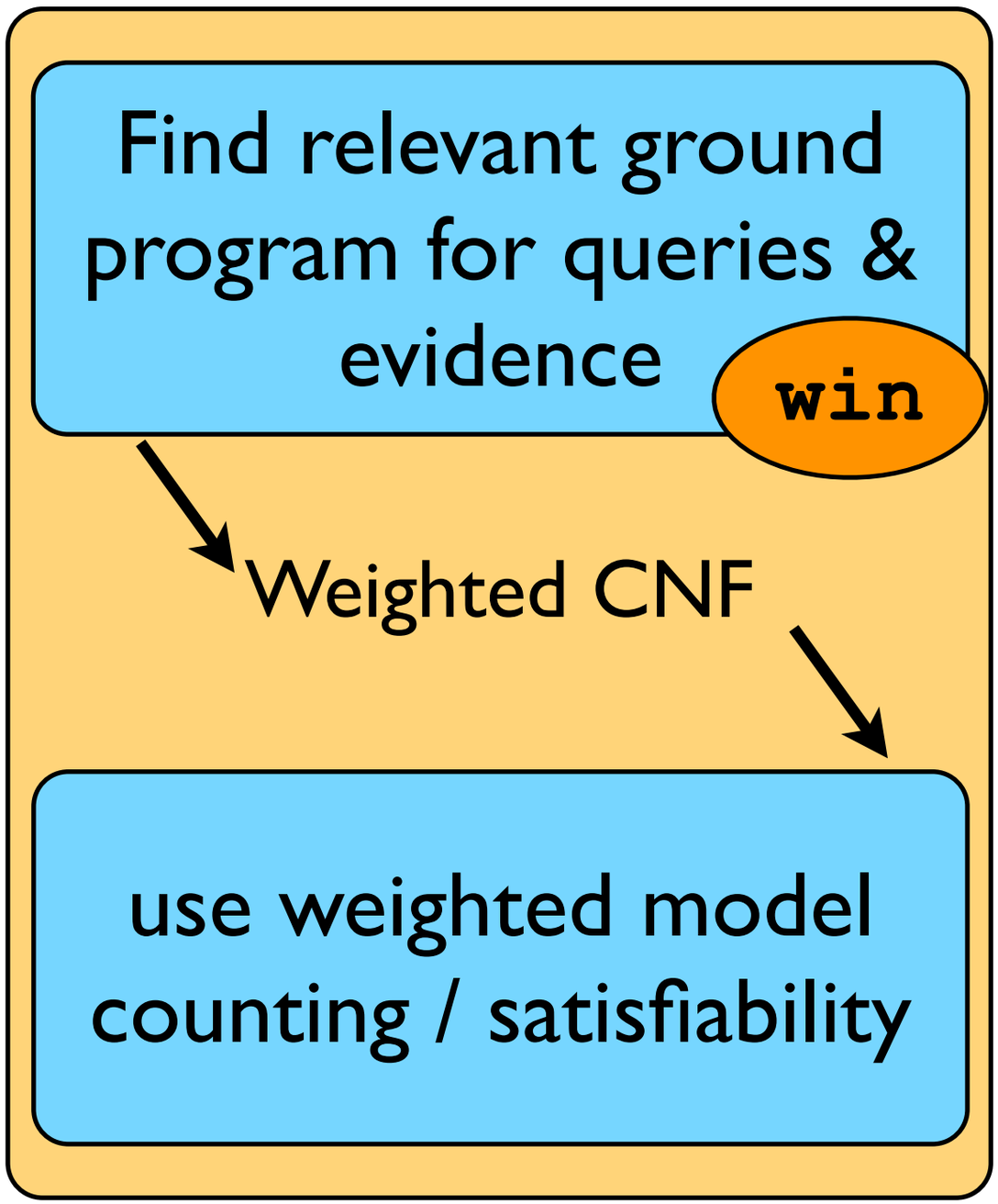


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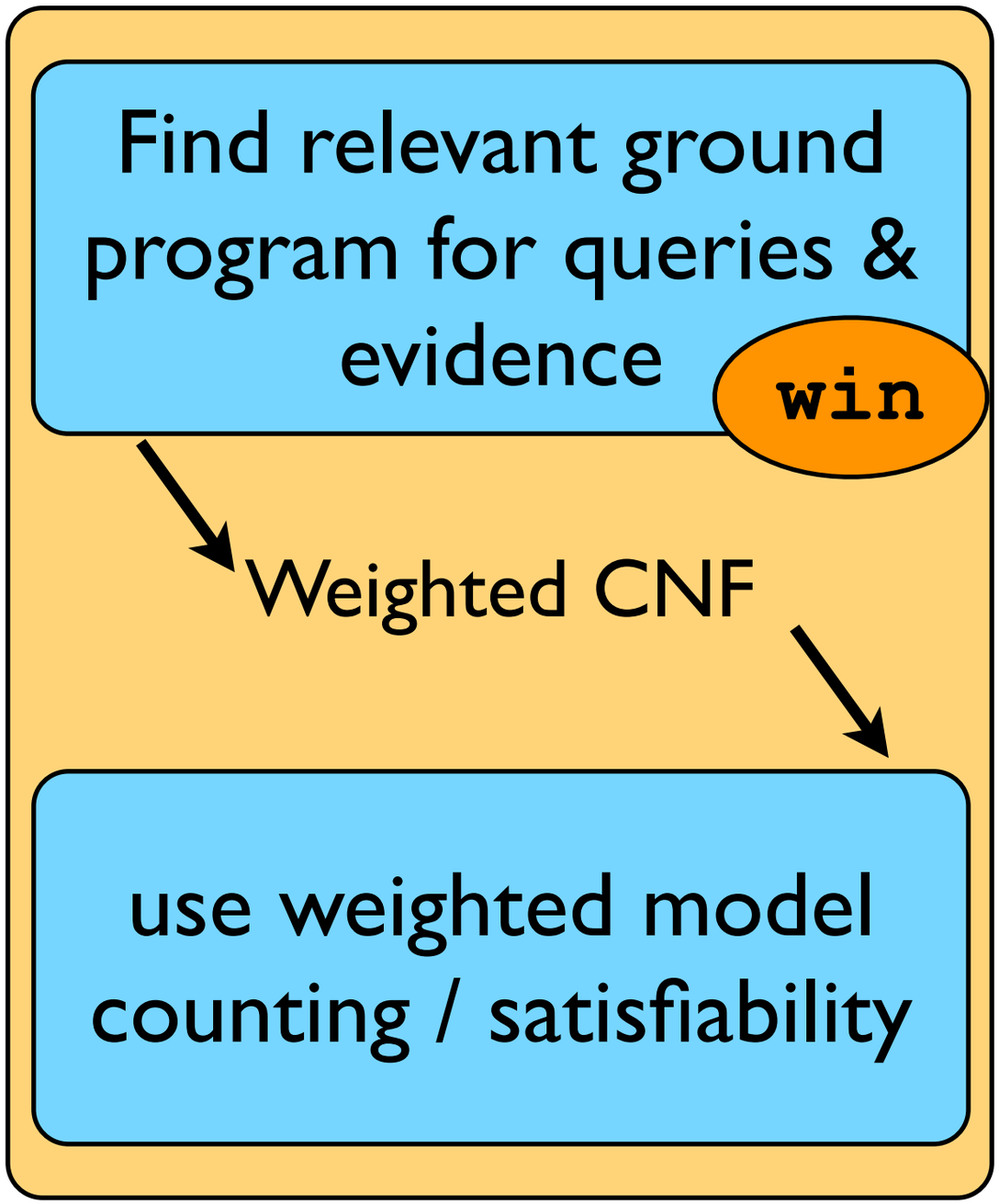


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$$\text{win} \leftrightarrow h(1) \vee (h(2) \wedge h(3))$$

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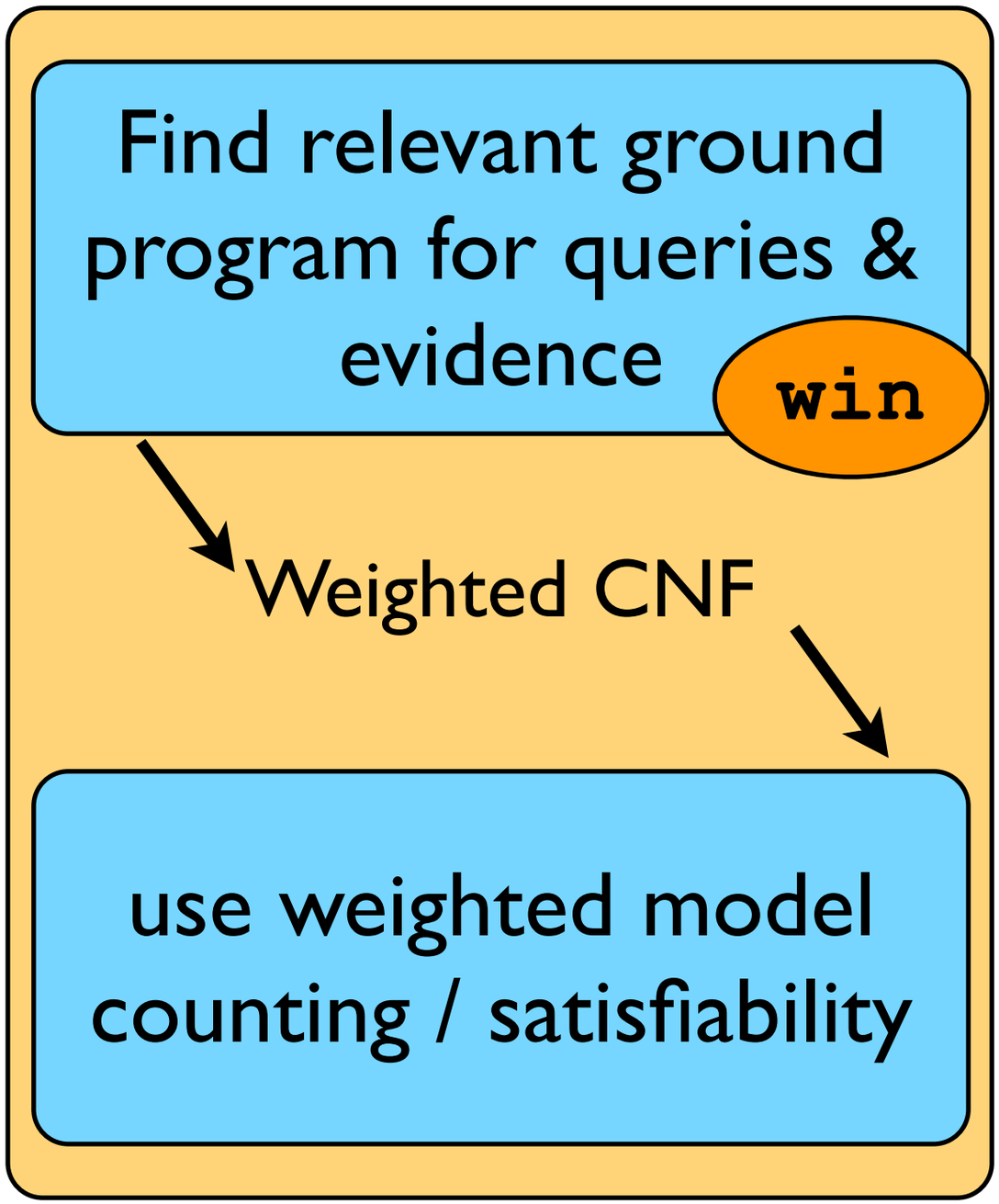


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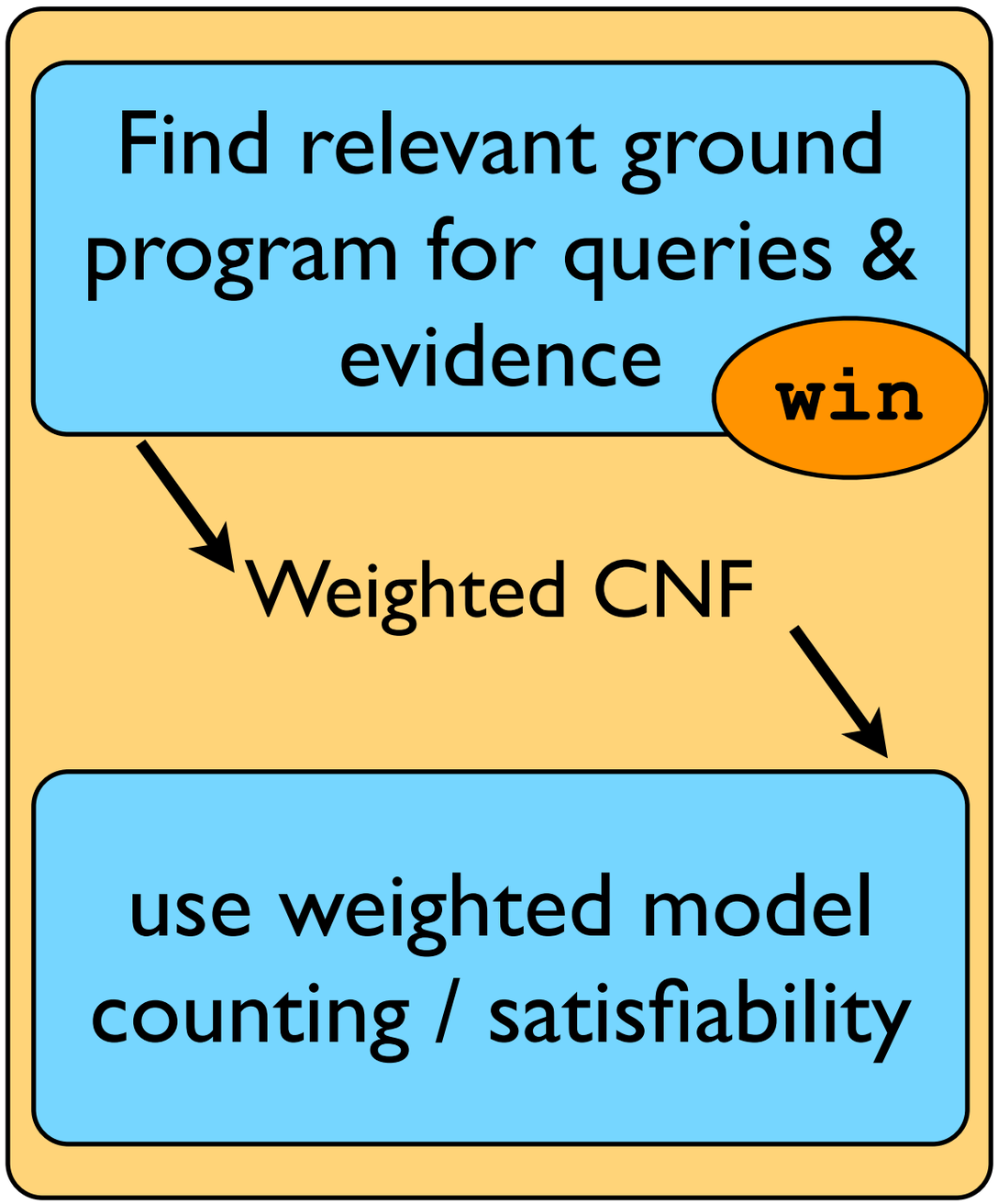


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$h(1) \rightarrow 0.4$	$h(2) \rightarrow 0.7$	$h(3) \rightarrow 0.5$
$\neg h(1) \rightarrow 0.6$	$\neg h(2) \rightarrow 0.3$	$\neg h(3) \rightarrow 0.5$

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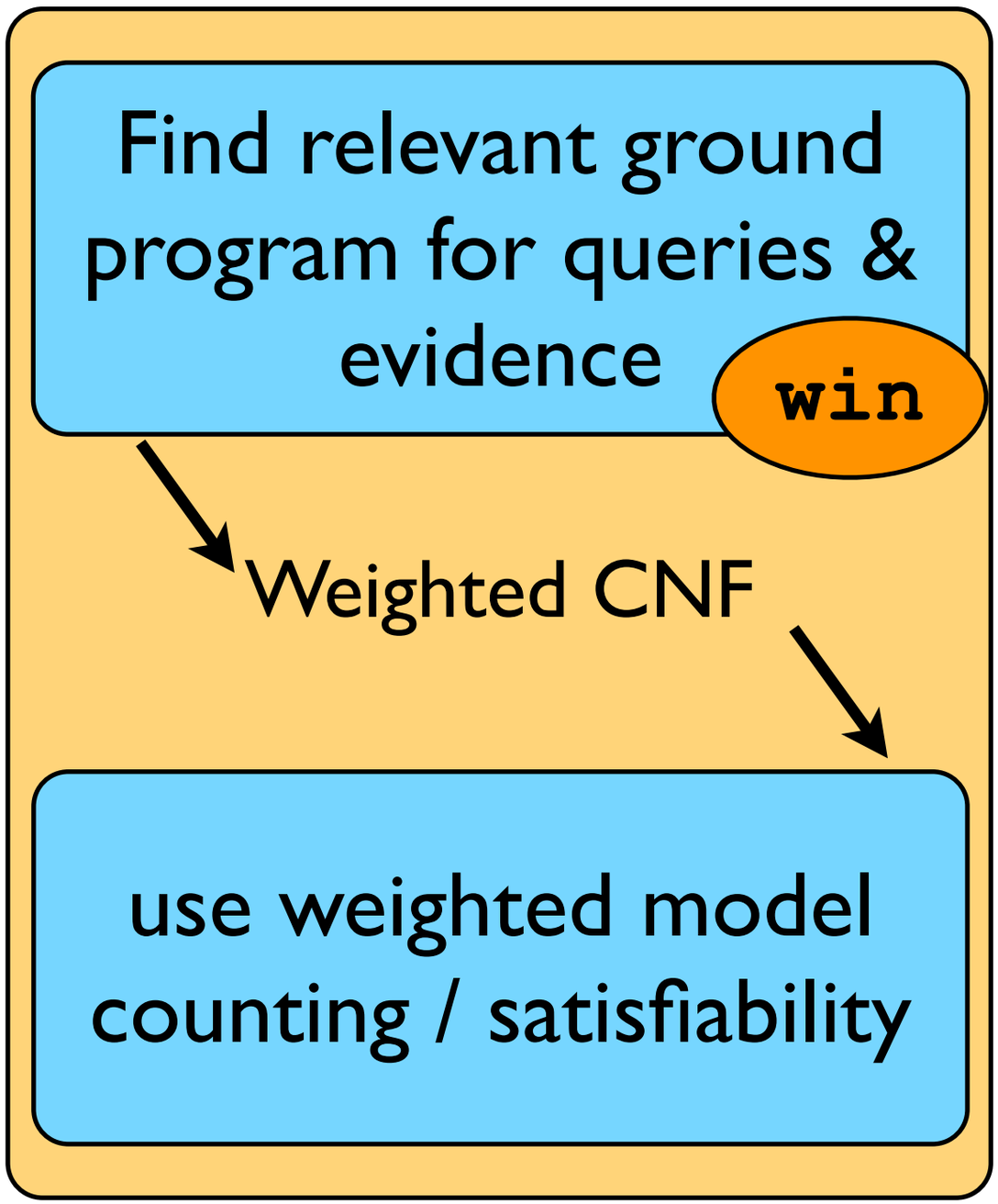


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use standard tool

$h(1) \rightarrow 0.4$ $\quad h(2) \rightarrow 0.7$ $\quad h(3) \rightarrow 0.5$

$\neg h(1) \rightarrow 0.6$ $\quad \neg h(2) \rightarrow 0.3$ $\quad \neg h(3) \rightarrow 0.5$

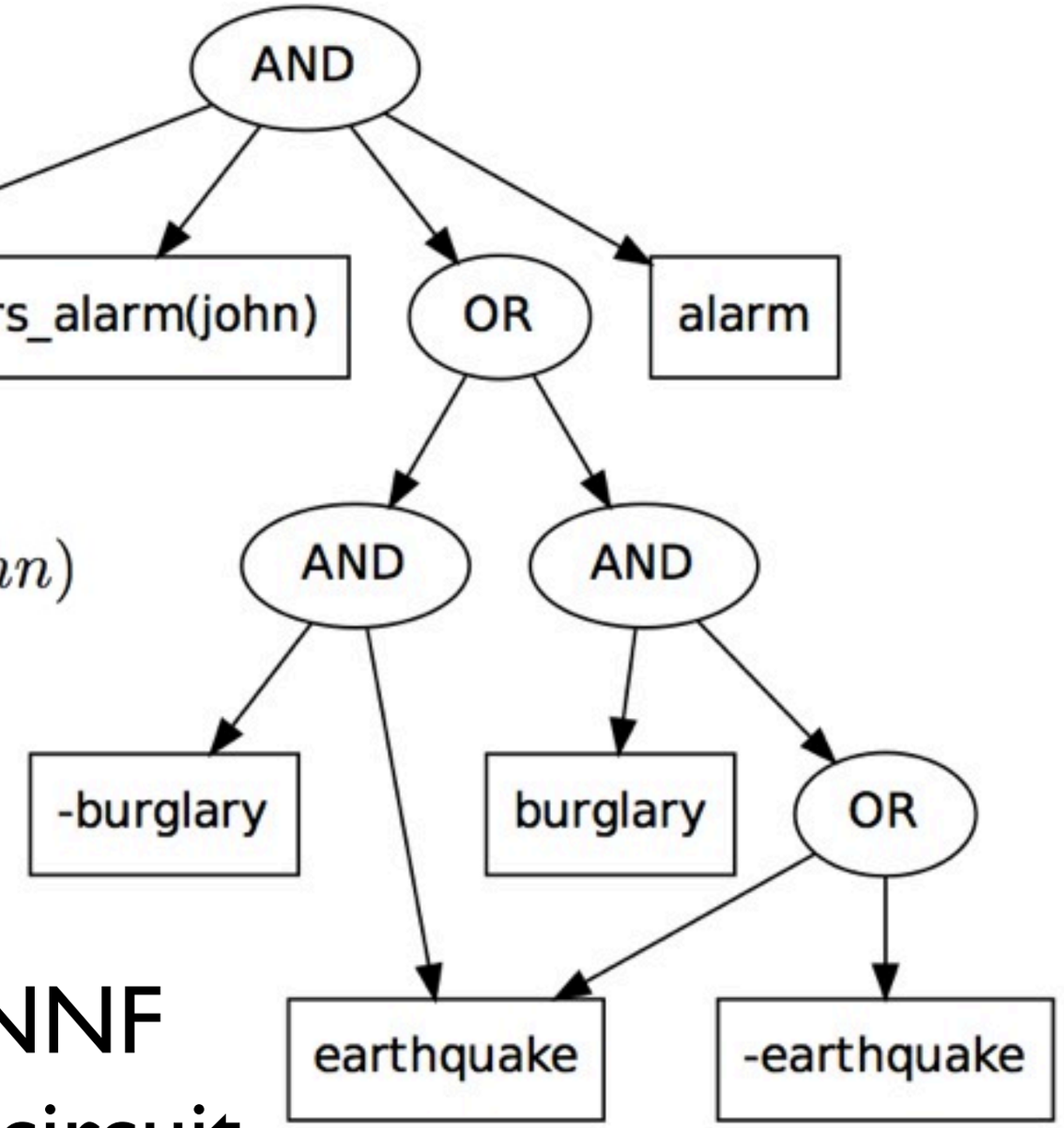
[Fierens et al, TPLP 14]

WMC using d-DNNFs

$$alarm \leftrightarrow burglary \vee earthquake$$

$$calls(john) \leftrightarrow alarm, hears\_alarm(john)$$

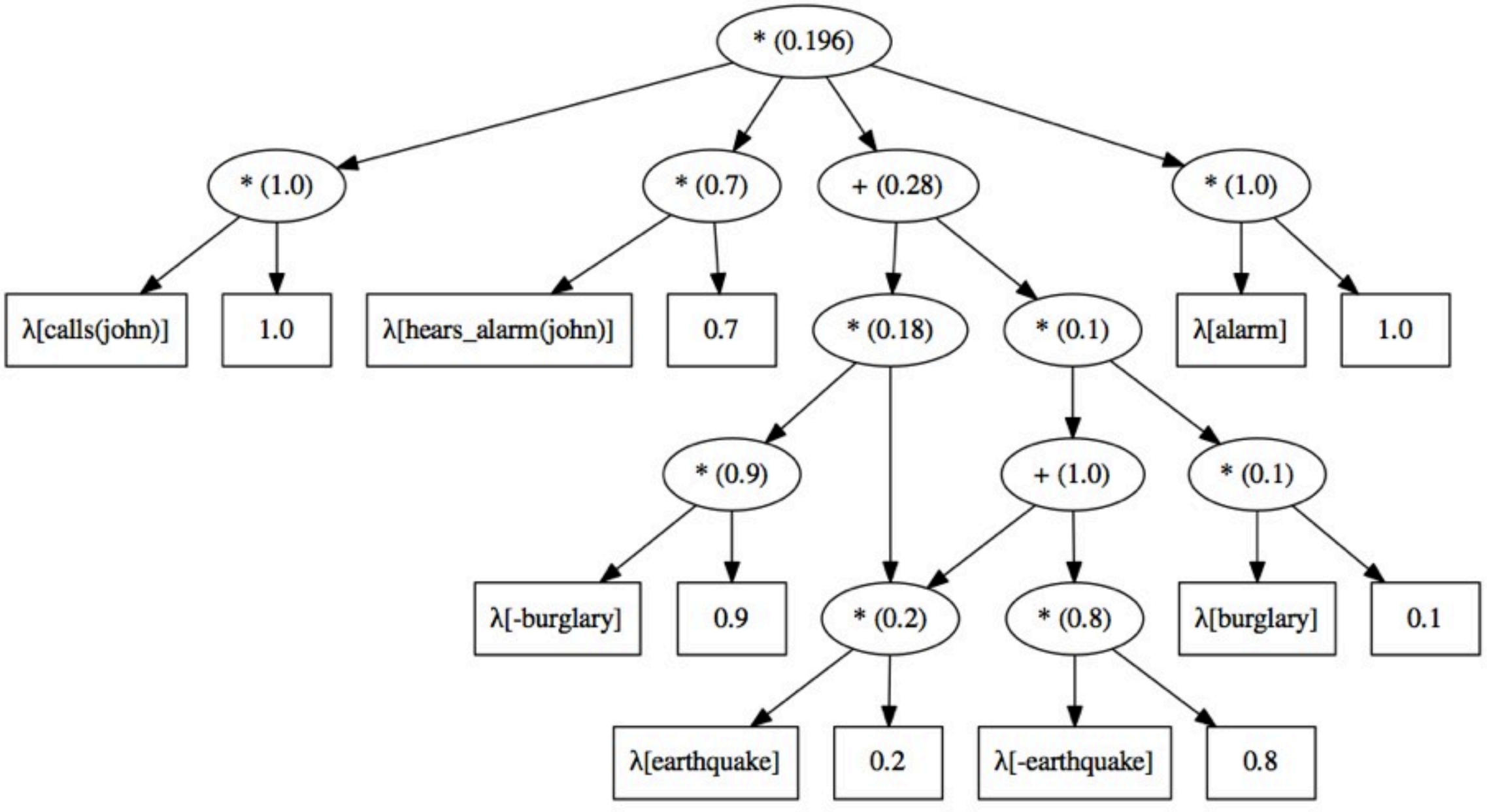
$$calls(john)$$



1. represent formula as d-DNNF
2. transform into arithmetic circuit
3. evaluate bottom-up

[Figure: Fierens et al, TPLP 14]

WMC using d-DNNFs



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[Figure: Fierens et al, TPLP 14]

ProbLog Inference

- reduction to propositional formula
- addresses disjoint-sum-problem
- **but**: not all probabilistic logic programs face this problem! e.g., weather
- more generally: mutually exclusive proofs as assumed in PRISM

PRISM

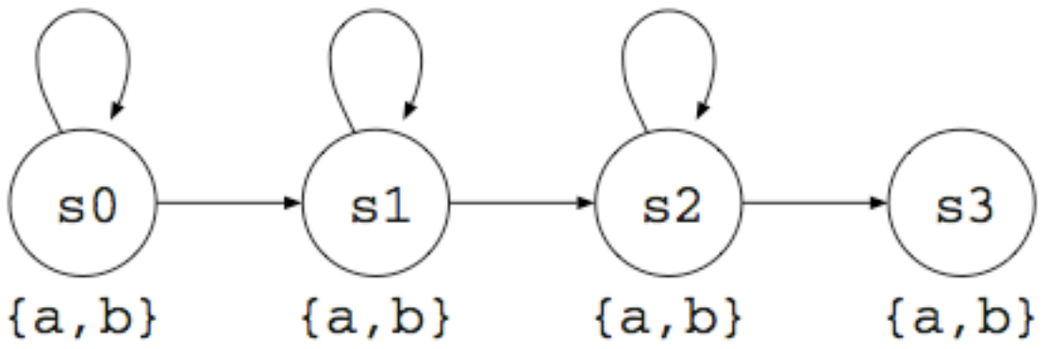


Fig. 1. Example of a left-to-right HMM with four states

```
target(hmm/1).
values(tr(s0),[s0,s1]).
values(tr(s1),[s1,s2]).
values(tr(s2),[s2,s3]).
values(out(_),[a,b]).

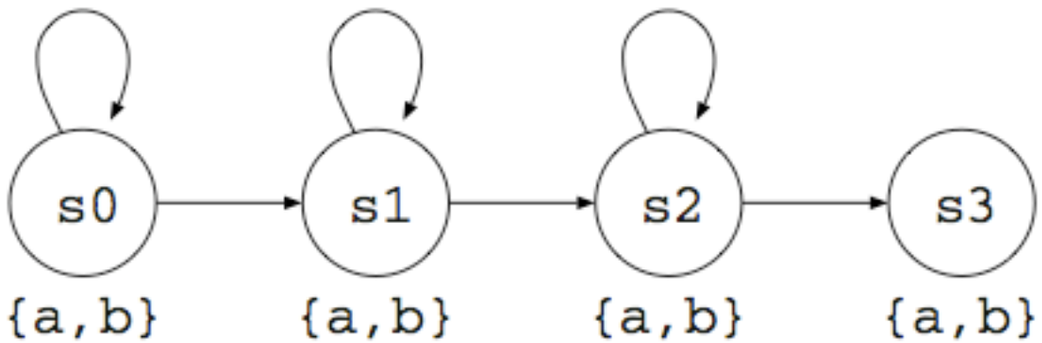
hmm(Cs):- hmm(0,s0,Cs).

hmm(T,s3,[C]):- msw(out(s3),  C). % If at the final state:
                                  %   output a symbol and then terminate.
hmm(T,S,[C|Cs]):- S\==s3,         % If not at the final state:
  msw(out(S),  C),                %   choose a symbol to be output,
  msw(tr(S),  Next),              %   choose the next state,
  T1 is T+1,                      %   Put the clock ahead,
  hmm(T1,Next,Cs).                %   and enter the next loop.
```

Fig. 2. PRISM program for the left-to-right HMM

[Figures: Sato and Kameya 08]

PRISM



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values(tr(s1),[s1,s2]).
values(tr(s2),[s2,s3]).
values(out(_),[a,b]).
```

define switches

```
hmm(Cs):- hmm(0,s0,Cs).

hmm(T,s3,[C]):- msw(out(s3),  C). % If at the final state:
                                  %   output a symbol and then terminate.
hmm(T,S,[C|Cs]):- S\==s3,         % If not at the final state:
  msw(out(S),  C),                %   choose a symbol to be output,
  msw(tr(S),  Next),              %   choose the next state,
  T1 is T+1,                      %   Put the clock ahead,
  hmm(T1,Next,Cs).                %   and enter the next loop.
```

Fig. 2. PRISM program for the left-to-right HMM

[Figures: Sato and Kameya 08]

PRISM

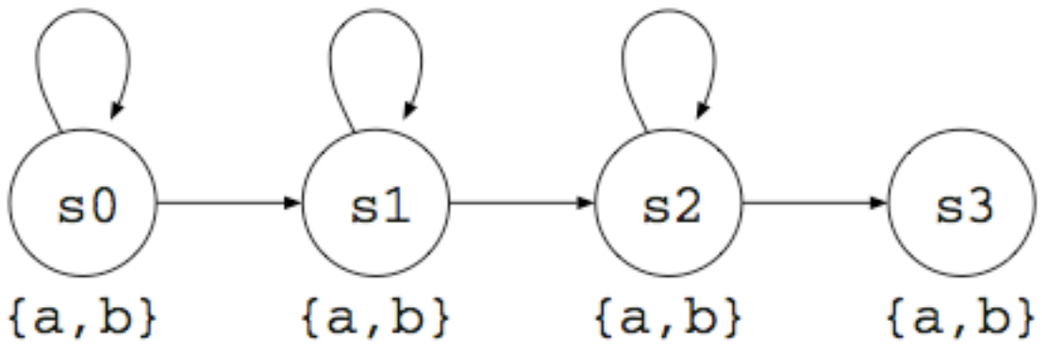


Fig. 1. Example of a left-to-right HMM with four states

```
target(hmm/1).
values(tr(s0),[s0,s1]).
values(tr(s1),[s1,s2]).
values(tr(s2),[s2,s3]).
values(out(_),[a,b]).

hmm(Cs):- hmm(0,s0,Cs).

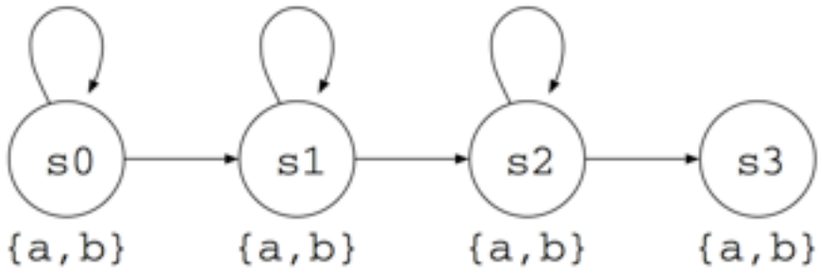
hmm(T,s3,[C]):- msw(out(s3), C). % If at the final state:
                                 %   output a symbol and then terminate.
hmm(T,S,[C|Cs]):- S\==s3,        % If not at the final state:
  msw(out(S), C),                %   choose a symbol to be output,
  msw(tr(S), Next),              %   choose the next state,
  T1 is T+1,                     %   Put the clock ahead,
  hmm(T1,Next,Cs).               %   and enter the next loop.
```

use switches

Fig. 2. PRISM program for the left-to-right HMM

[Figures: Sato and Kameya 08]

PRISM inference




```
hmm([a,b,b,b,b,a])
```

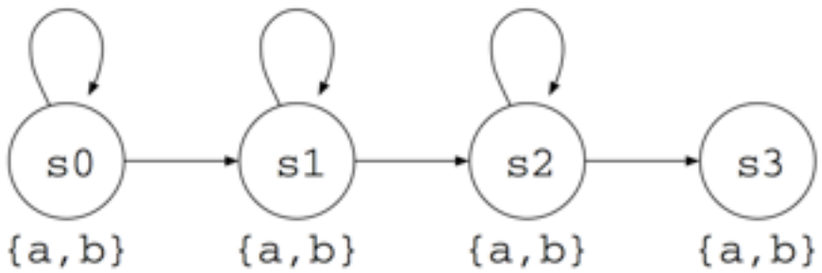
no memoization: two different RVS

$$
\begin{aligned}
E_1 = {} & \mathtt{m(out(s0),a)} \wedge \mathtt{m(tr(s0),s0)} \wedge \mathtt{m(out(s0),b)} \wedge \mathtt{m(tr(s0),s0)} \wedge \mathtt{m(out(s0),b)} \\
& \wedge \mathtt{m(tr(s0),s1)} \wedge \mathtt{m(out(s1),b)} \wedge \mathtt{m(tr(s1),s2)} \wedge \mathtt{m(out(s2),b)} \wedge \mathtt{m(tr(s2),s3)} \\
& \wedge \mathtt{m(out(s3),a)} \\
E_2 = {} & \mathtt{m(out(s0),a)} \wedge \mathtt{m(tr(s0),s0)} \wedge \mathtt{m(out(s0),b)} \wedge \mathtt{m(tr(s0),s1)} \wedge \mathtt{m(out(s1),b)} \\
& \wedge \mathtt{m(tr(s1),s1)} \wedge \mathtt{m(out(s1),b)} \wedge \mathtt{m(tr(s1),s2)} \wedge \mathtt{m(out(s2),b)} \wedge \mathtt{m(tr(s2),s3)} \\
& \wedge \mathtt{m(out(s3),a)} \\
& \vdots \\
E_6 = {} & \mathtt{m(out(s0),a)} \wedge \mathtt{m(tr(s0),s1)} \wedge \mathtt{m(out(s1),b)} \wedge \mathtt{m(tr(s1),s2)} \wedge \mathtt{m(out(s2),b)} \\
& \wedge \mathtt{m(tr(s2),s2)} \wedge \mathtt{m(out(s2),b)} \wedge \mathtt{m(tr(s2),s2)} \wedge \mathtt{m(out(s2),b)} \wedge \mathtt{m(tr(s2),s3)} \\
& \wedge \mathtt{m(out(s3),a)}
\end{aligned}
$$

Fig. 3. Six explanations for `hmm([a, b, b, b, b, a])`. Due to the space limit, the predicate name `msw` is abbreviated to `m`.

[Figures: Sato and Kameya 08]

PRISM inference

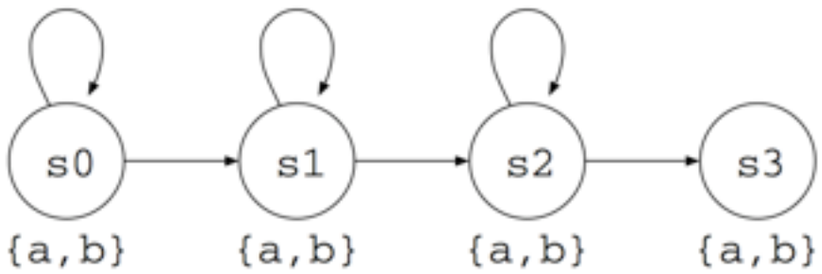


$$
\begin{aligned}
\texttt{hmm}([\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{hmm}(0,\texttt{s0},[\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(0,\texttt{s0},[\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{a}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s0}) \wedge \texttt{hmm}(1,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
&\quad \vee \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{a}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s1}) \wedge \texttt{hmm}(1,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(1,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s0}) \wedge \texttt{hmm}(2,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
&\quad \vee \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s1}) \wedge \texttt{hmm}(2,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\ddagger\\
\texttt{hmm}(2,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s1}) \wedge \texttt{hmm}(3,\texttt{s1},[\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(1,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s1}) \wedge \texttt{hmm}(2,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\ddagger\\
&\quad \vee \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s2}) \wedge \texttt{hmm}(2,\texttt{s2},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(2,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\dagger &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s1}) \wedge \texttt{hmm}(3,\texttt{s1},[\texttt{b},\texttt{b},\texttt{a}])\\
&\quad \vee \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s2}) \wedge \texttt{hmm}(3,\texttt{s2},[\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(3,\texttt{s1},[\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s2}) \wedge \texttt{hmm}(4,\texttt{s2},[\texttt{b},\texttt{a}])\\
\texttt{hmm}(2,\texttt{s2},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s2}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s2}),\texttt{s2}) \wedge \texttt{hmm}(3,\texttt{s2},[\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(3,\texttt{s2},[\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s2}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s2}),\texttt{s2}) \wedge \texttt{hmm}(4,\texttt{s2},[\texttt{b},\texttt{a}])\\
\texttt{hmm}(4,\texttt{s2},[\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s2}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s2}),\texttt{s3}) \wedge \texttt{hmm}(5,\texttt{s3},[\texttt{a}])\\
\texttt{hmm}(5,\texttt{s3},[\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s3}),\texttt{a})
\end{aligned}
$$

Fig. 4. Factorized explanations for $\texttt{hmm}([\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])$

[Figures: Sato and Kameya 08]

PRISM inference



$$
\begin{aligned}
\texttt{hmm}([\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{hmm}(0,\texttt{s0},[\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(0,\texttt{s0},[\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{a}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s0}) \wedge \texttt{hmm}(1,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
&\quad \vee \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{a}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s1}) \wedge \texttt{hmm}(1,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(1,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s0}) \wedge \texttt{hmm}(2,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
&\quad \vee \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s1}) \wedge \texttt{hmm}(2,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\ddagger\\
\texttt{hmm}(2,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s1}) \wedge \texttt{hmm}(3,\texttt{s1},[\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(1,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s1}) \wedge \texttt{hmm}(2,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\ddagger\\
&\quad \vee \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s2}) \wedge \texttt{hmm}(2,\texttt{s2},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(2,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\dagger &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s1}) \wedge \texttt{hmm}(3,\texttt{s1},[\texttt{b},\texttt{b},\texttt{a}])\\
&\quad \vee \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s2}) \wedge \texttt{hmm}(3,\texttt{s2},[\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(3,\texttt{s1},[\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s1}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s1}),\texttt{s2}) \wedge \texttt{hmm}(4,\texttt{s2},[\texttt{b},\texttt{a}])\\
\texttt{hmm}(2,\texttt{s2},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s2}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s2}),\texttt{s2}) \wedge \texttt{hmm}(3,\texttt{s2},[\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(3,\texttt{s2},[\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s2}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s2}),\texttt{s2}) \wedge \texttt{hmm}(4,\texttt{s2},[\texttt{b},\texttt{a}])\\
\texttt{hmm}(4,\texttt{s2},[\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s2}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s2}),\texttt{s3}) \wedge \texttt{hmm}(5,\texttt{s3},[\texttt{a}])\\
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\end{aligned}
$$

Fig. 4. Factorized explanations for $\texttt{hmm}([\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])$

[Figures: Sato and Kameya 08]

PRISM: compute probability by dynamic programming

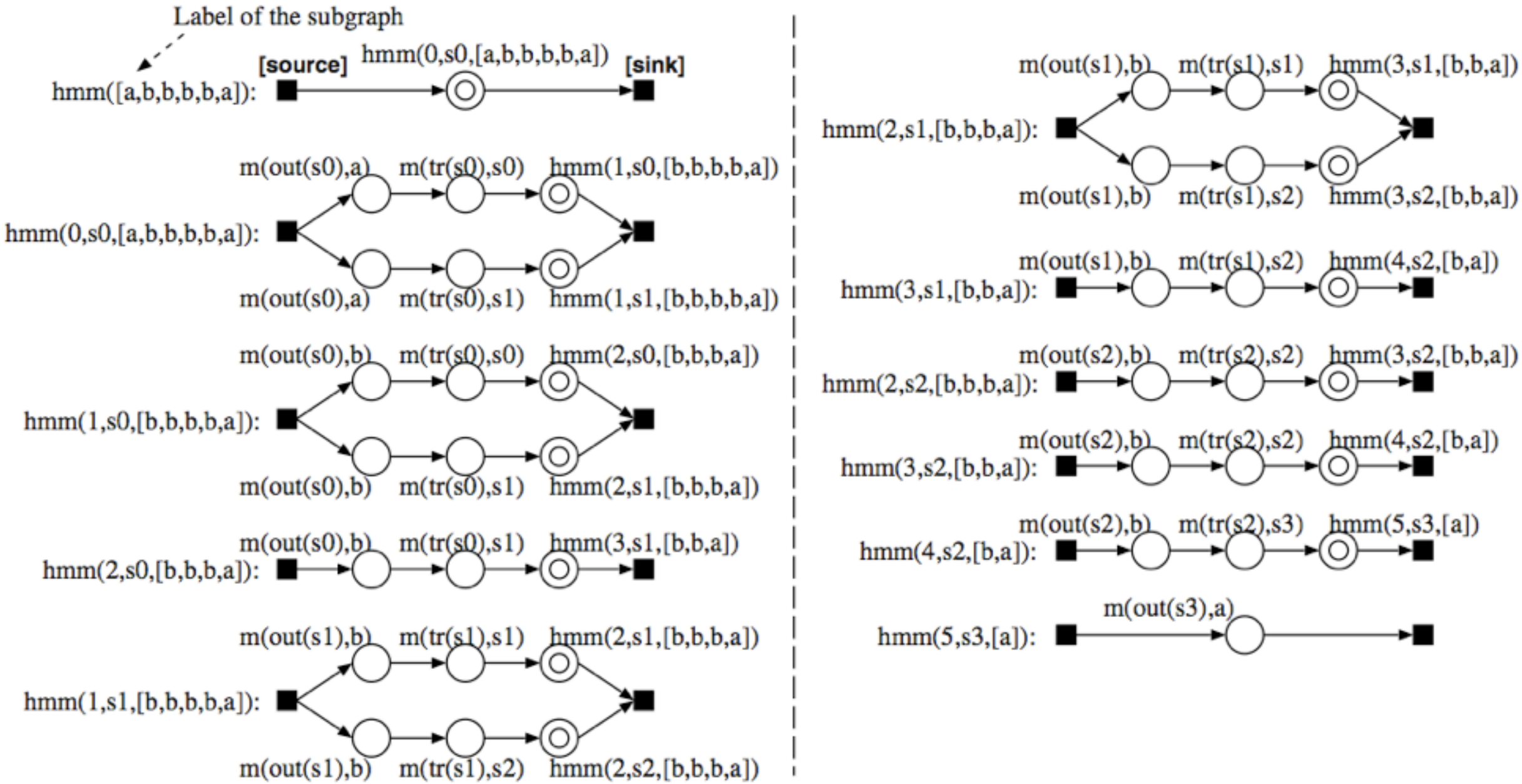


Fig. 5. Explanation graph

[Figures: Sato and Kameya 08]

PRISM: compute probability by dynamic programming

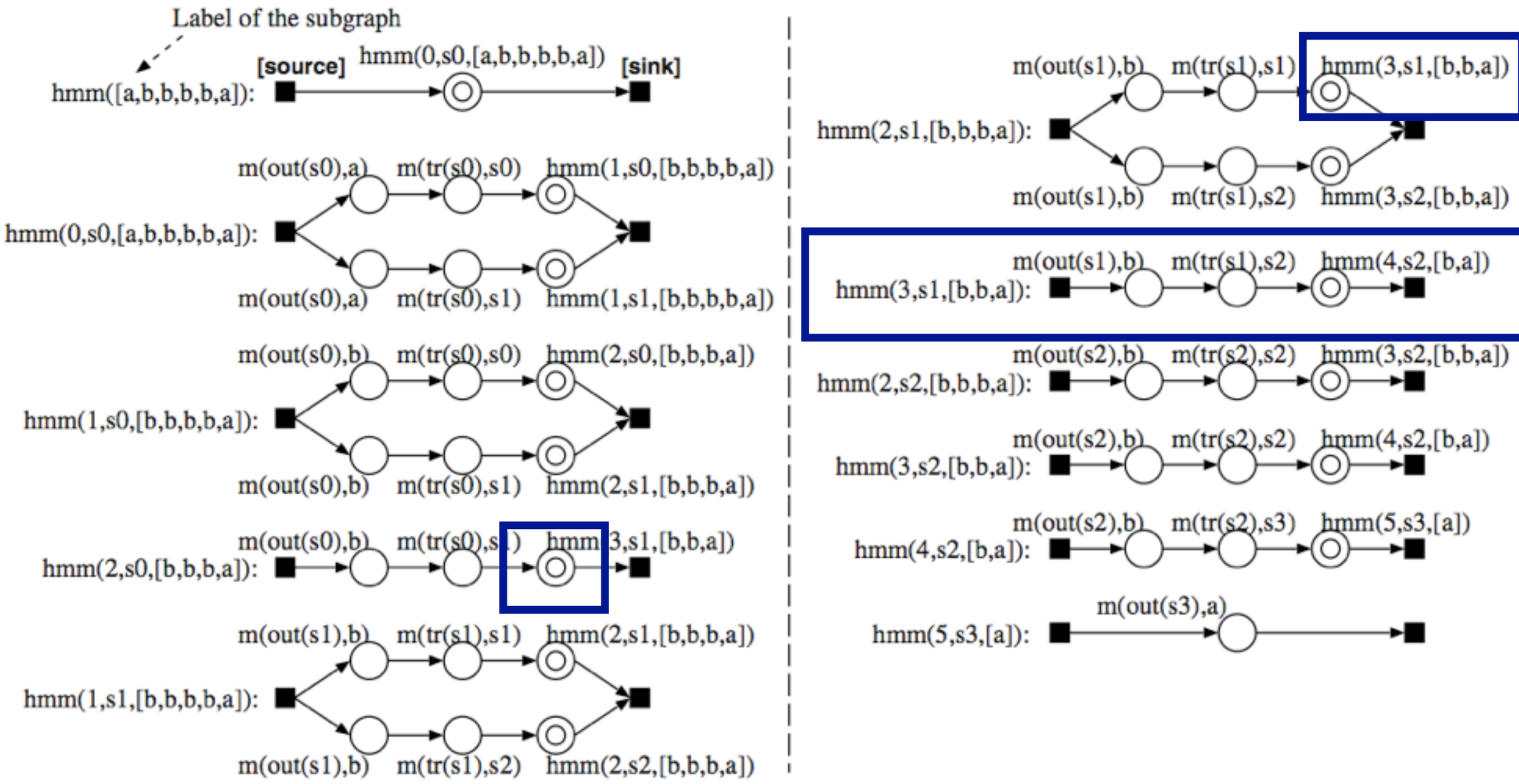


Fig. 5. Explanation graph

[Figures: Sato and Kameya 08]

Query Evaluation in PDB

Query Evaluation in PDB

- **Extensional** evaluation
  - guided by query expression only
  - exploit DB technology
  - for queries known to have polytime evaluation

Query Evaluation in PDB

- **Extensional** evaluation
  - guided by query expression only
  - exploit DB technology
  - for queries known to have polytime evaluation
- **Intensional** evaluation
  - construct **lineage** (= propositional formula)
  - compute probability of lineage
  - all queries

same idea as for ProbLog

Approximate Inference

- Lower and upper bounds

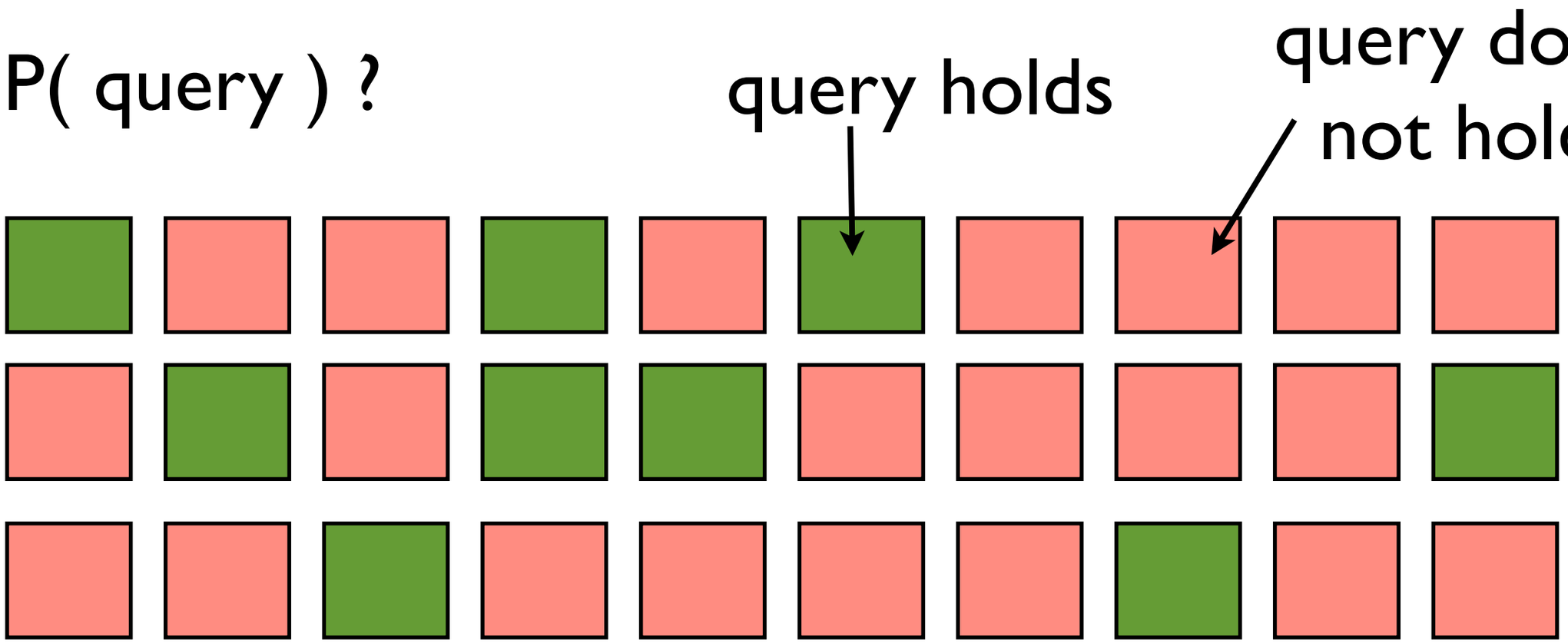
$$\phi_L \models \phi \models \phi_U$$

$$P(\phi_L) \leq P(\phi) \leq P(\phi_U)$$

- Sampling

Sampling

- P(query) ?



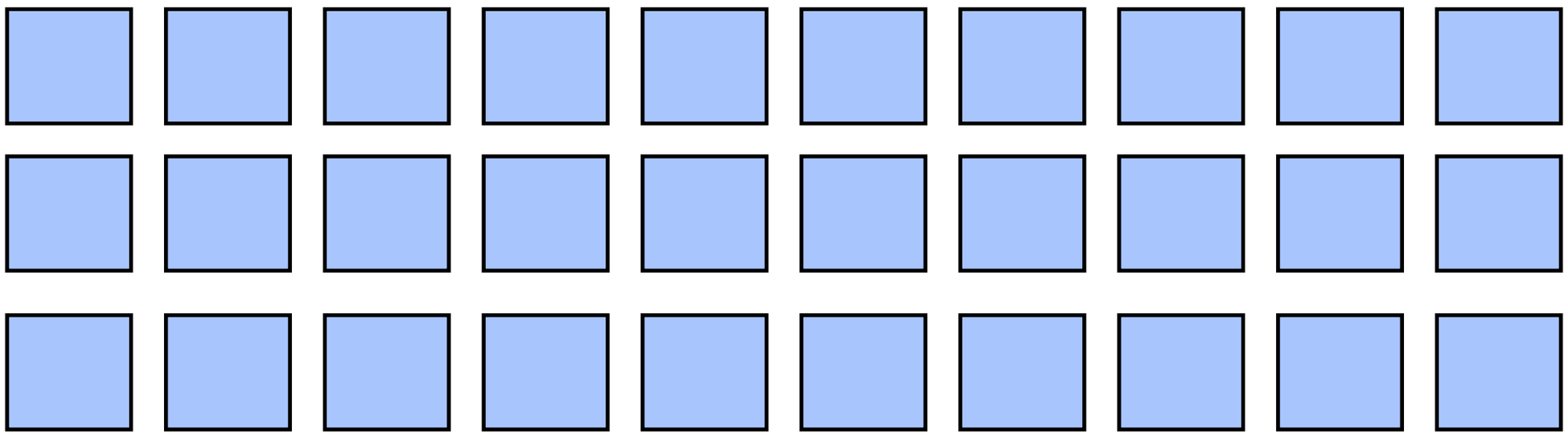
$$P(\text{query}) \approx \frac{\#\text{ query holds}}{\#\text{ worlds sampled}}$$

Rejection Sampling

- P(query | evidence) ?

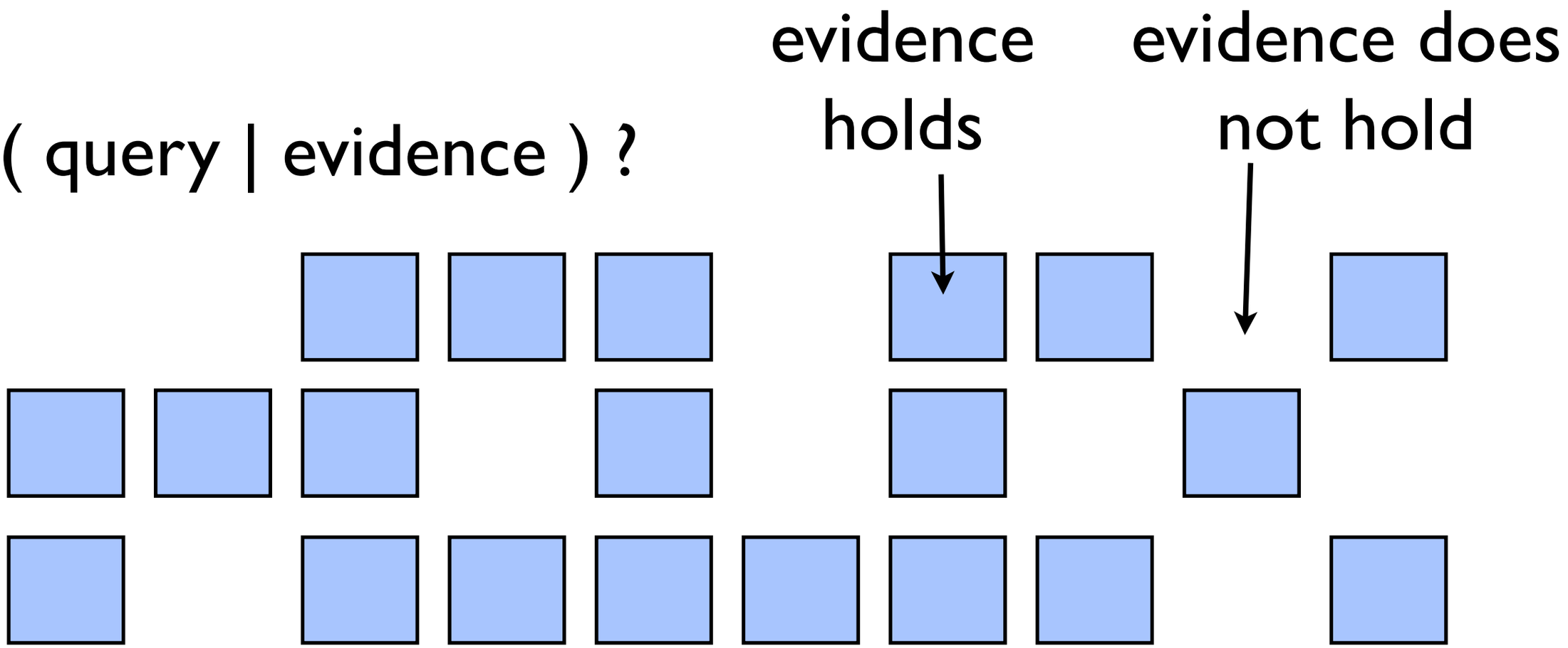
Rejection Sampling

- P(query | evidence) ?



Rejection Sampling

- P(query | evidence) ?



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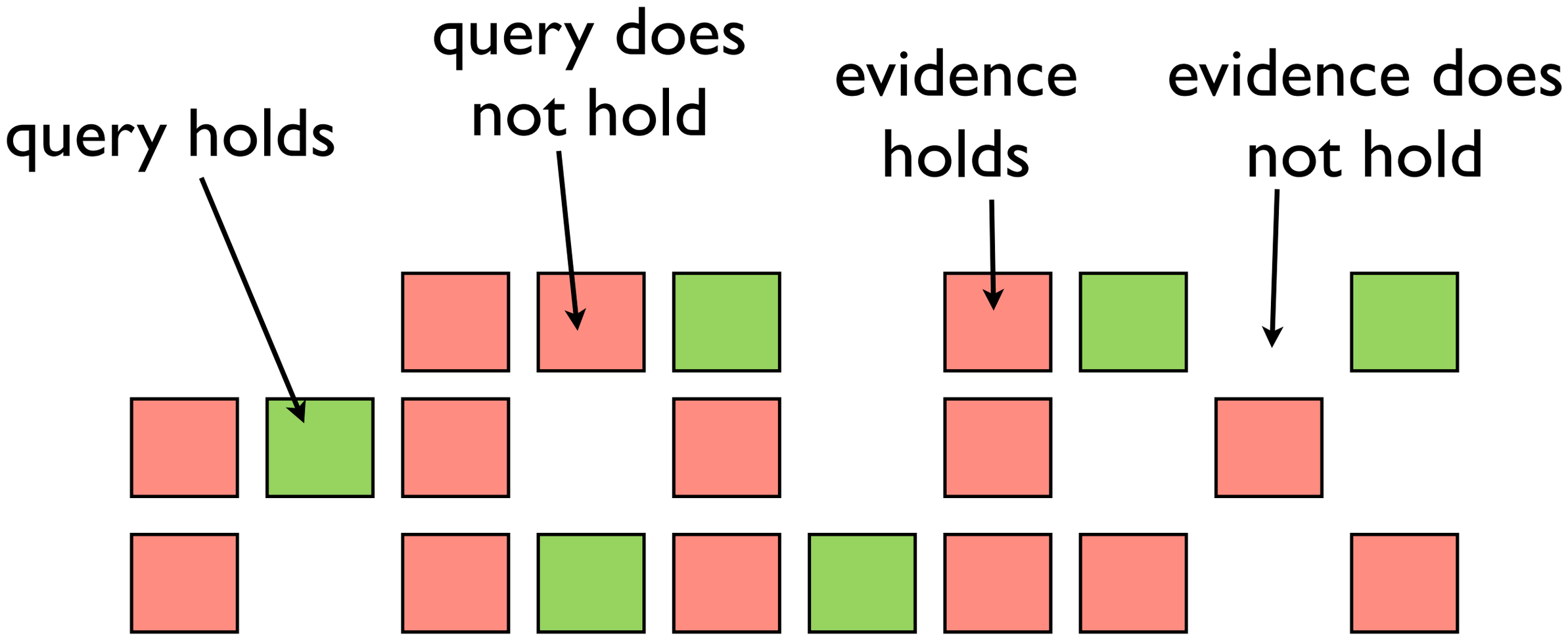


$$P(\text{query} \mid \text{evidence}) \approx \frac{\#\ \text{query \& evidence holds}}{\#\ \text{evidence holds}}$$

Markov Chain Monte Carlo (MCMC)

- Generate next sample by modifying current one
- Most common inference approach for PP languages such as Church, BLOG, ...
- Also considered for PRISM and ProbLog

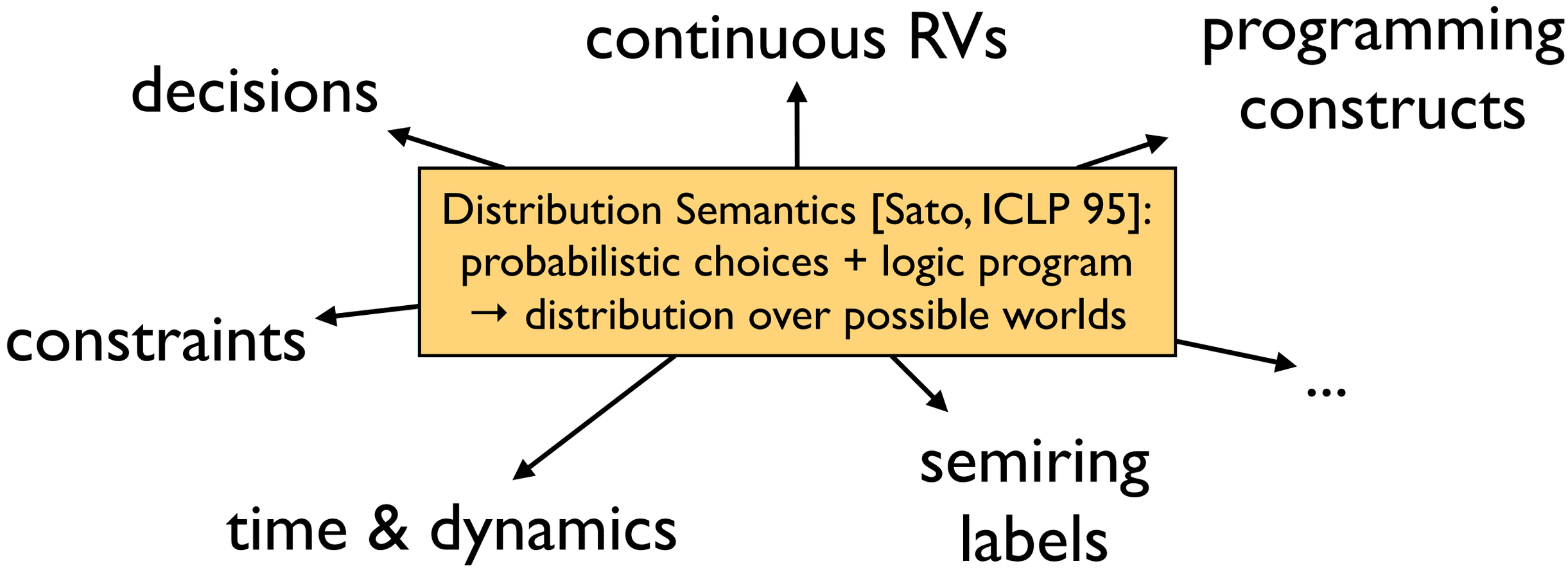
Key challenges:
- how to propose next sample
- how to handle evidence

Roadmap

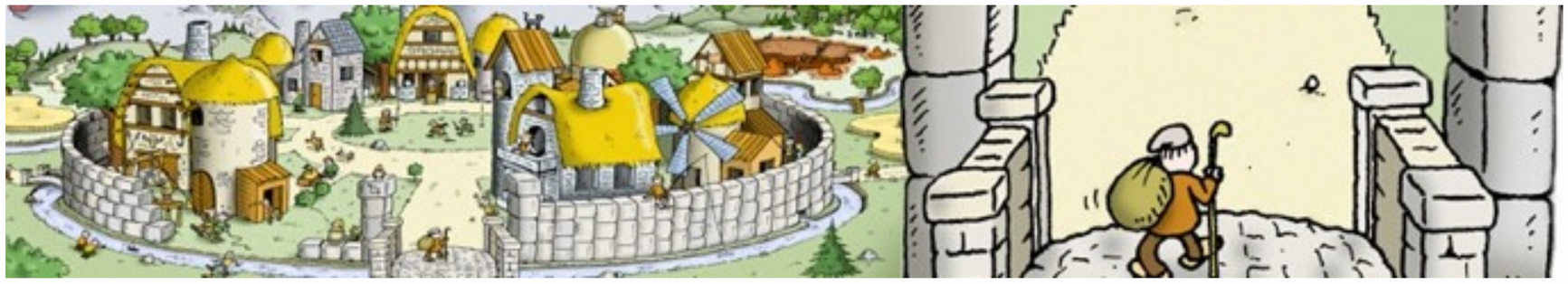
- Modeling
- Reasoning
- Language extensions
- Advanced topics

... with some detours on the way

Extensions of basic PLP

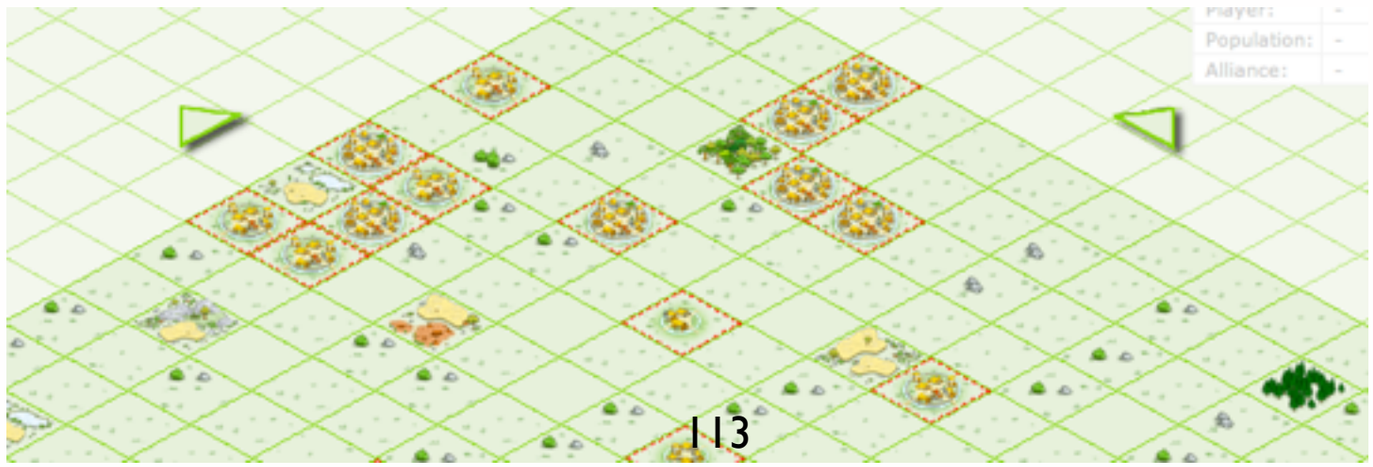


Dynamics: Evolving Networks



- *Travian*: A massively multiplayer real-time strategy game
- Commercial game run by TravianGames GmbH
- ~3.000.000 players spread over different "worlds"
- ~25.000 players in one world

[Thon et al. ECML 08]



World Dynamics

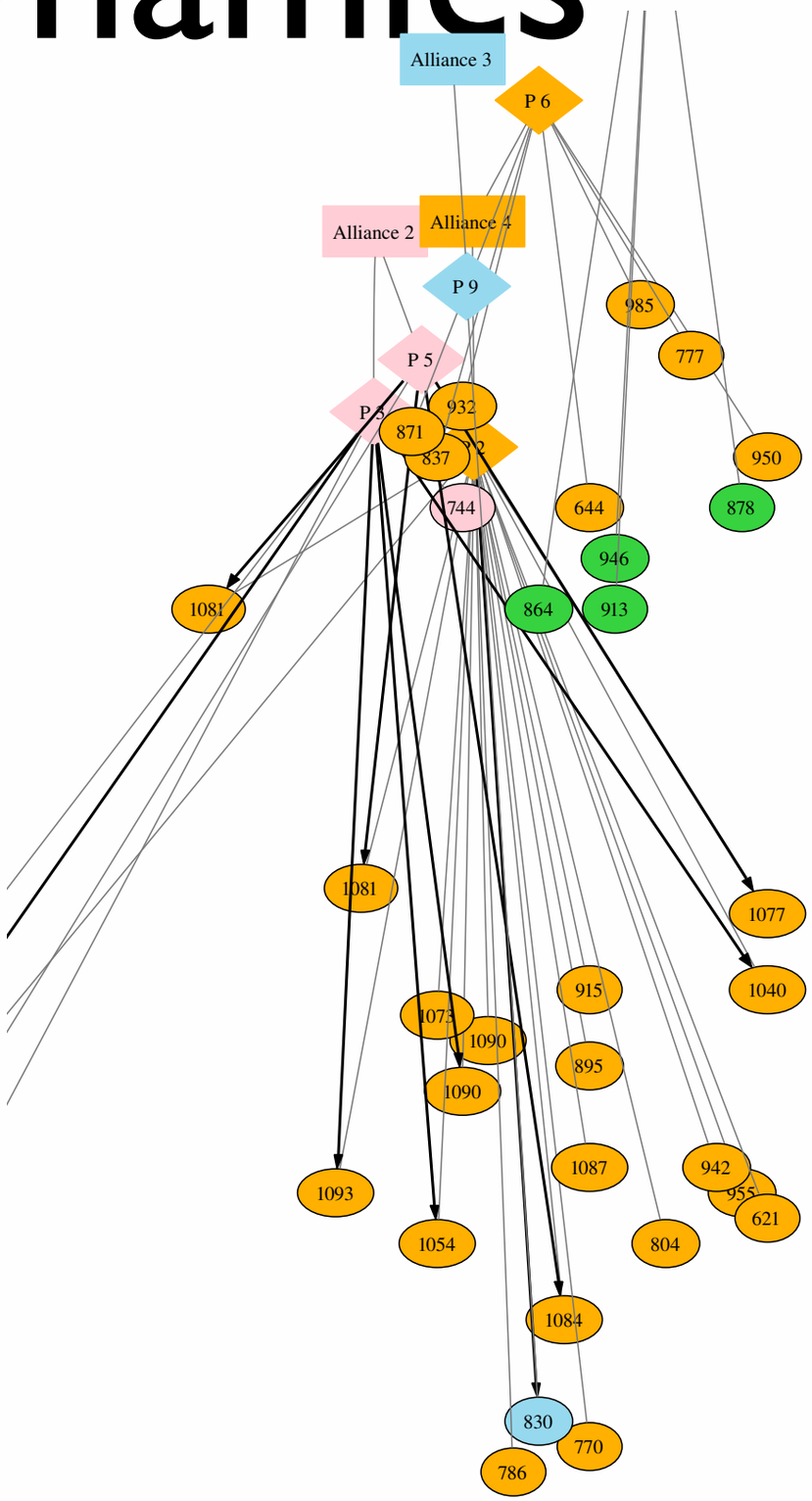
Fragment of world with

~10 alliances
~200 players
~600 cities

alliances color-coded

Can we build a model
of this world ?
Can we use it for playing
better ?

[Thon, Landwehr, De Raedt, ECML08]



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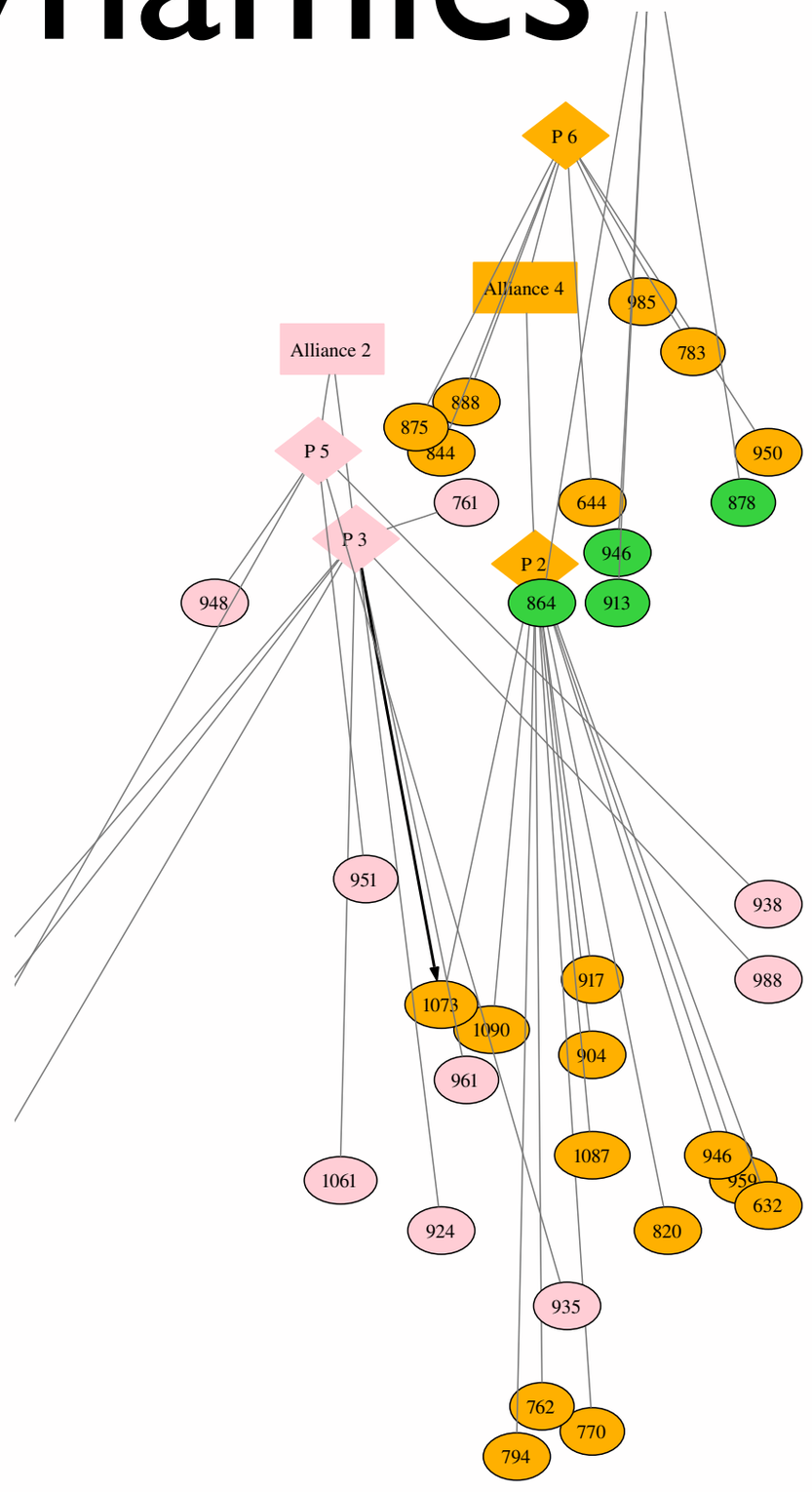

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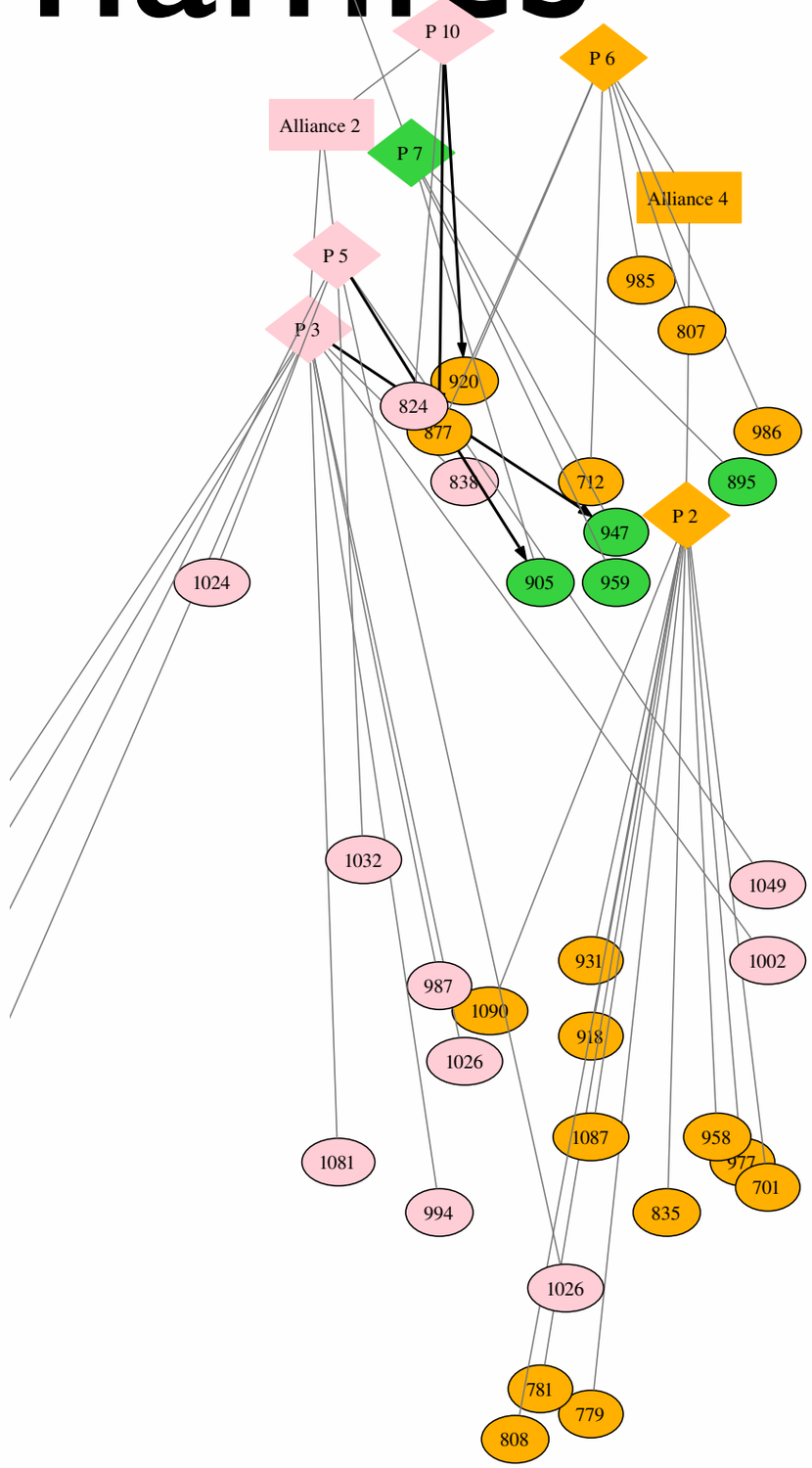

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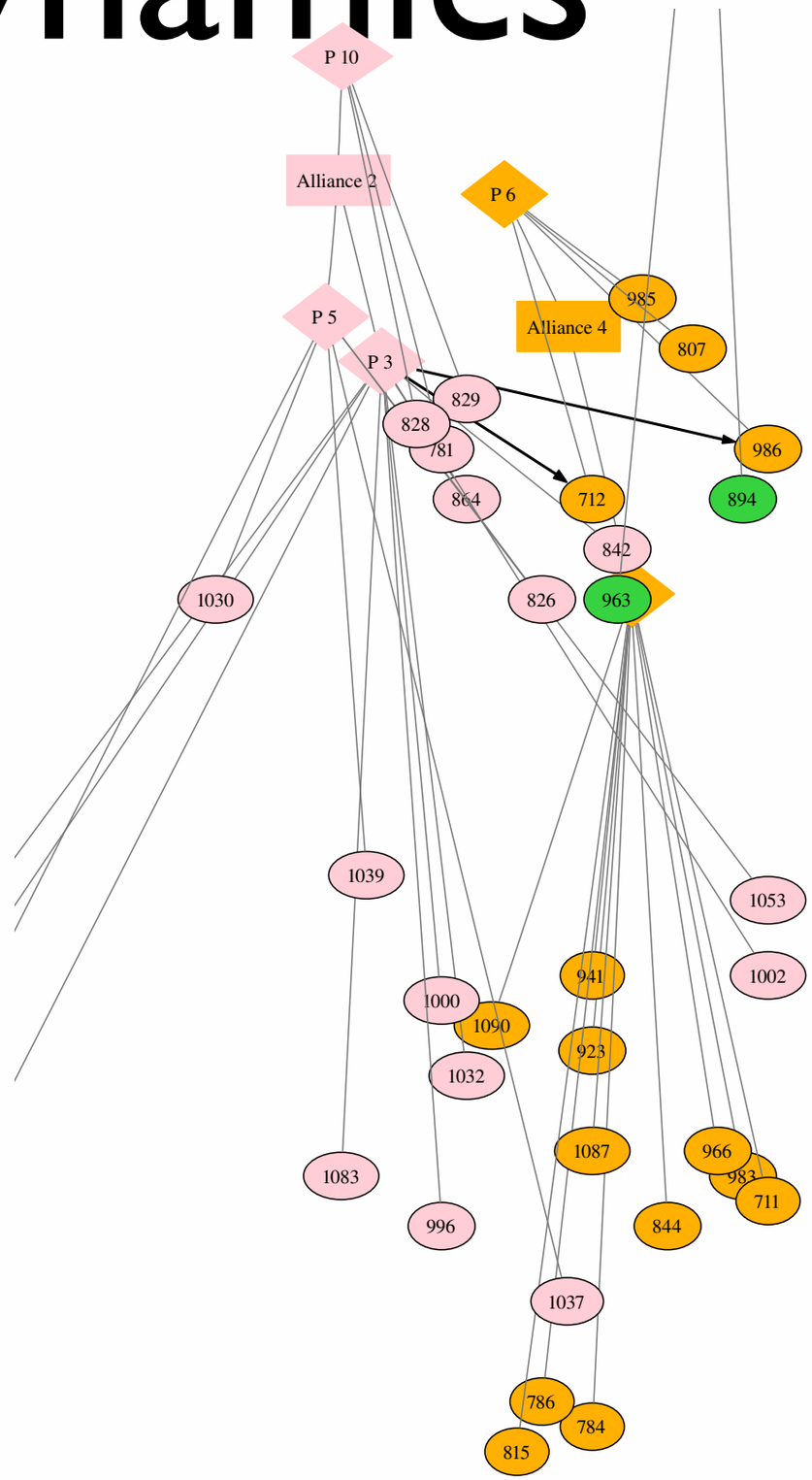

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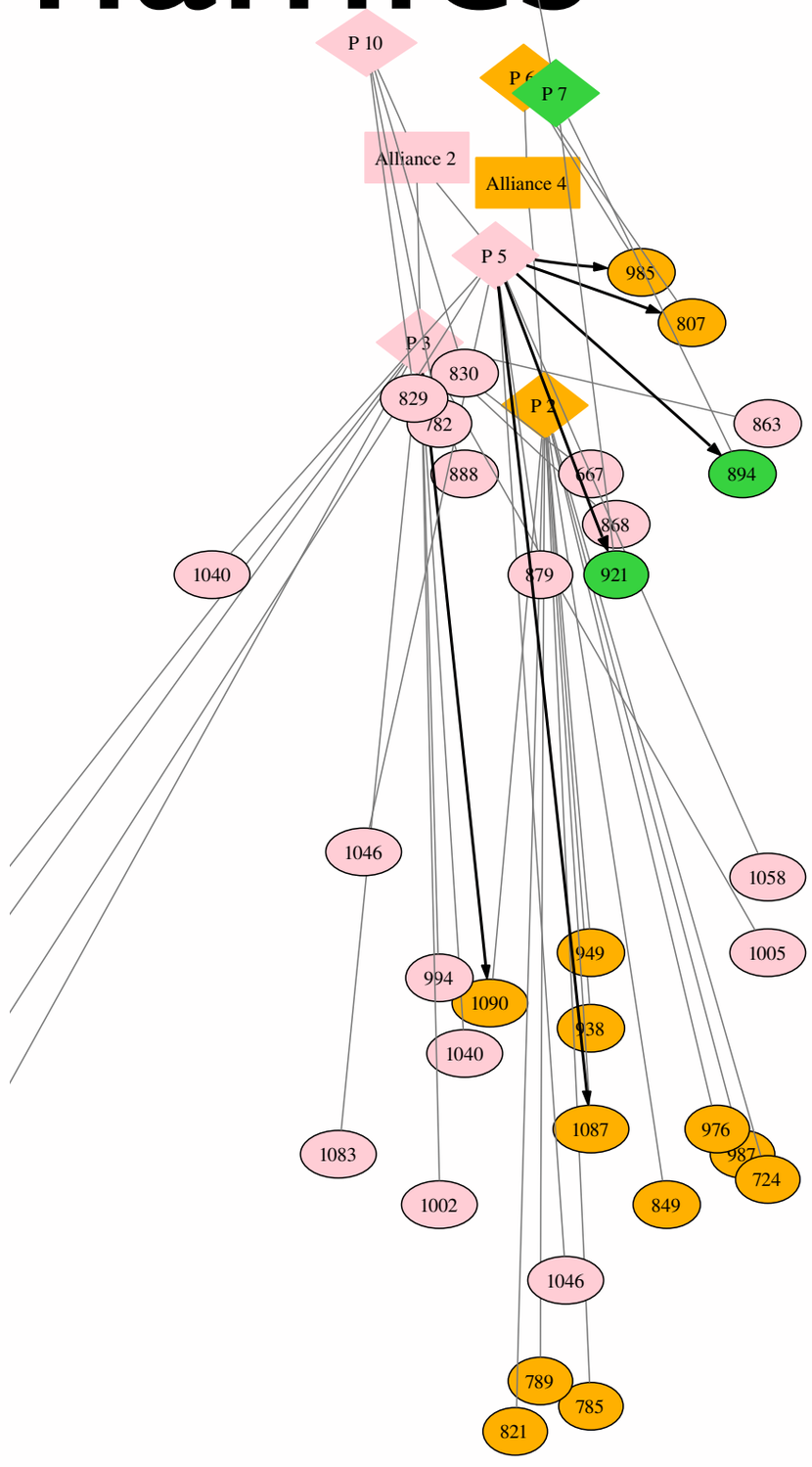

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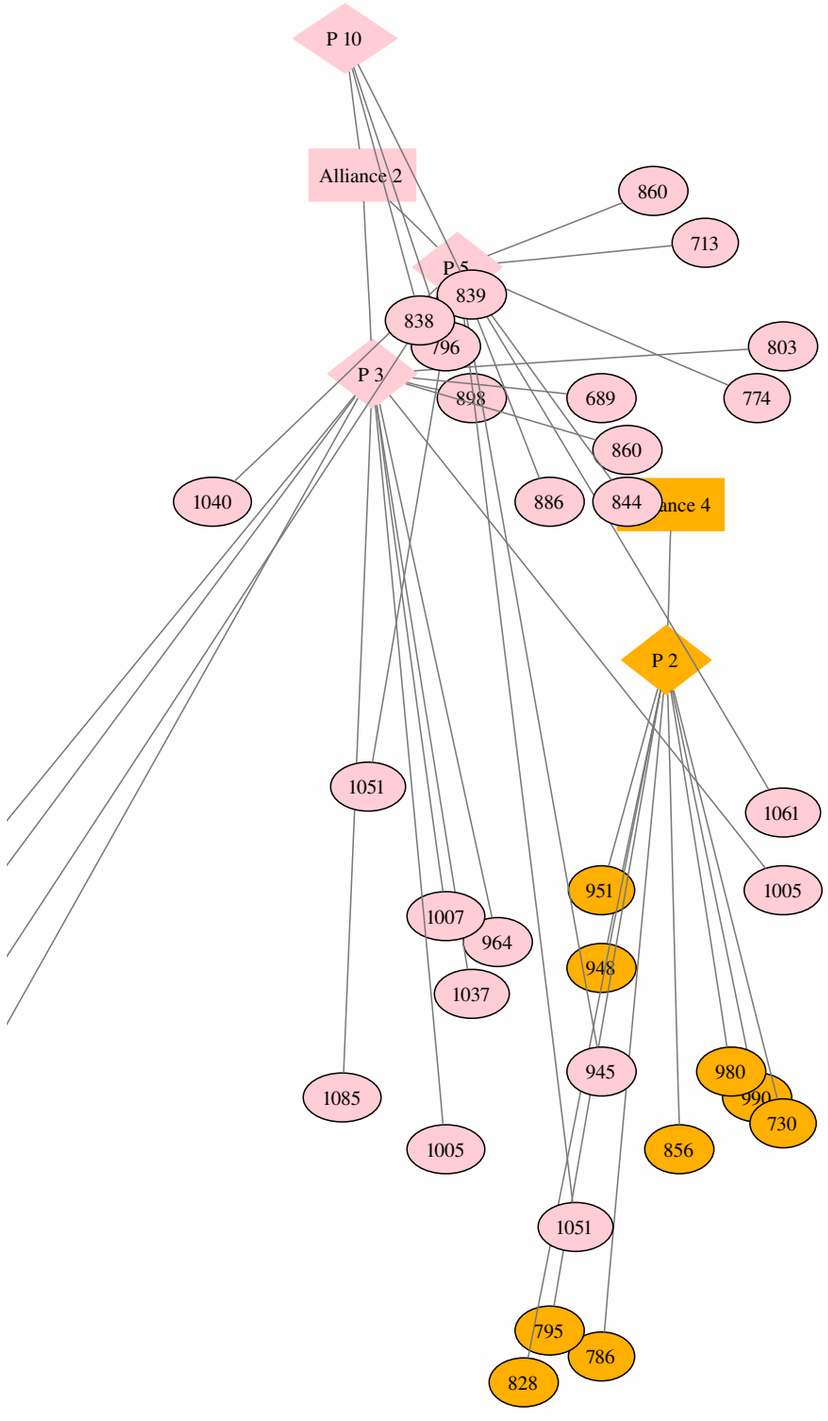
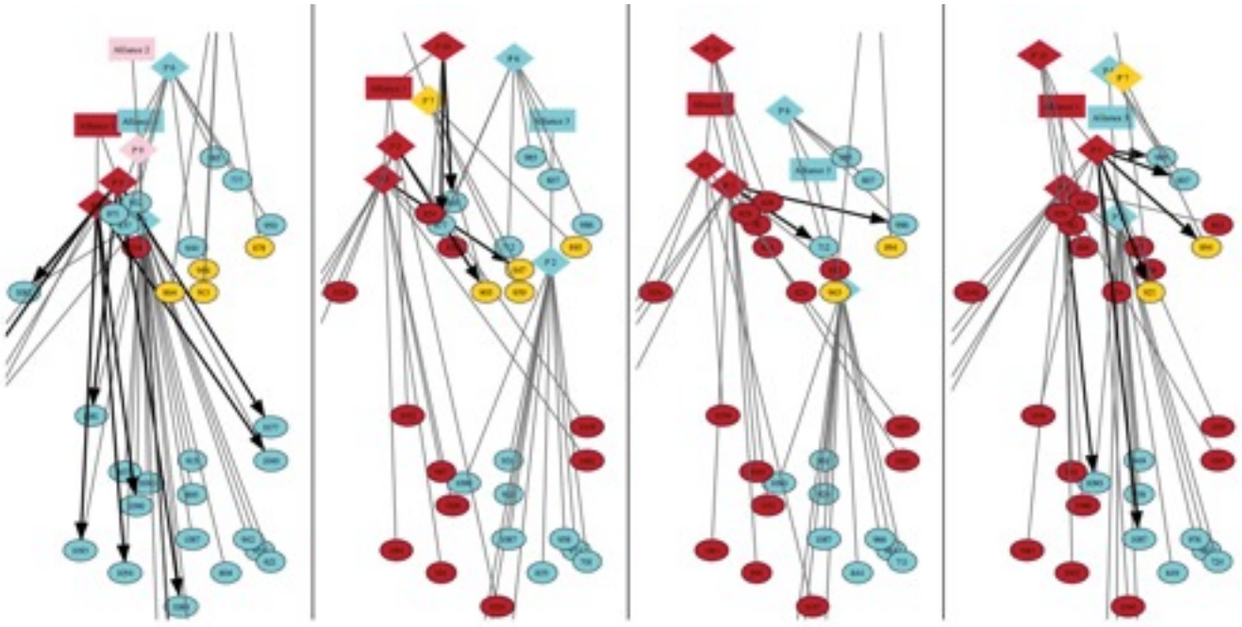


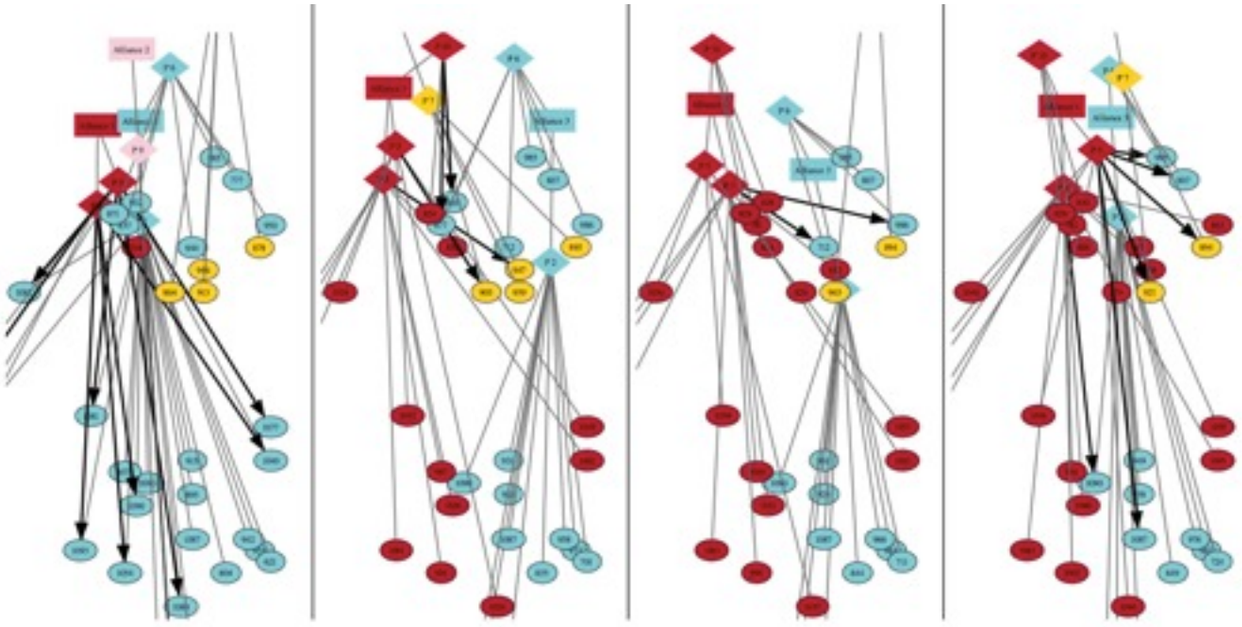
Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

[Thon et al, MLJ 11]

Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

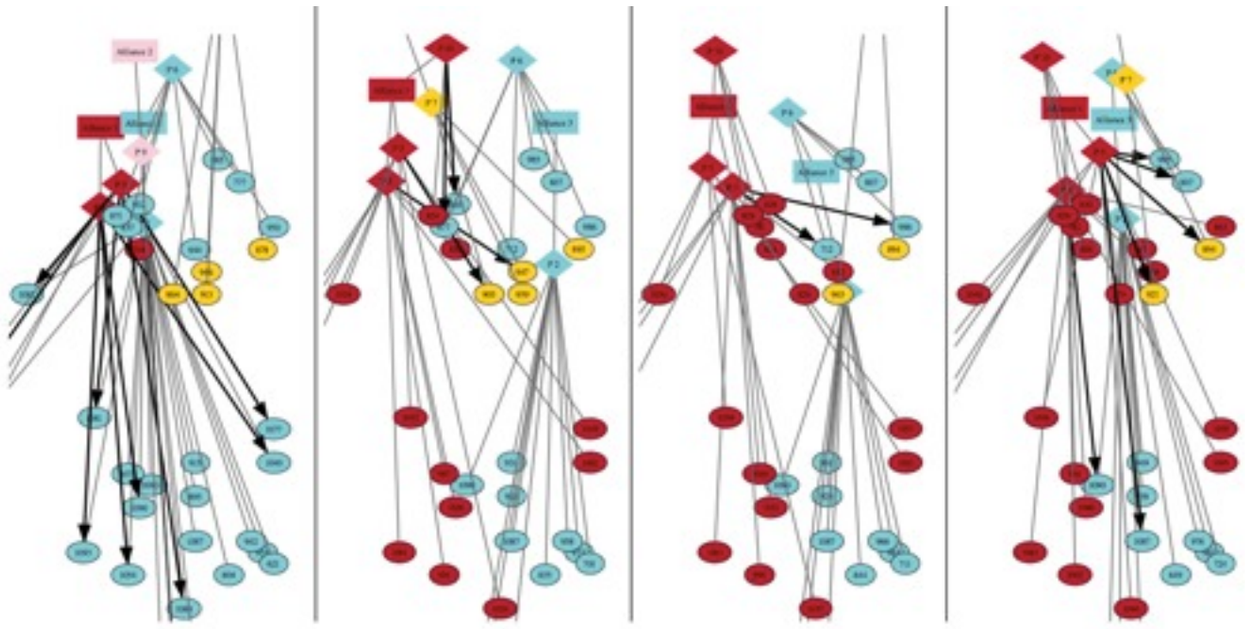
```
0.4::conquest(Attacker,C); 0.6::nil <-
```

```
city(C,Owner),city(C2,Attacker),close(C,C2).
```

if **cause** holds at time T

[Thon et al, MLJ 11]

Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

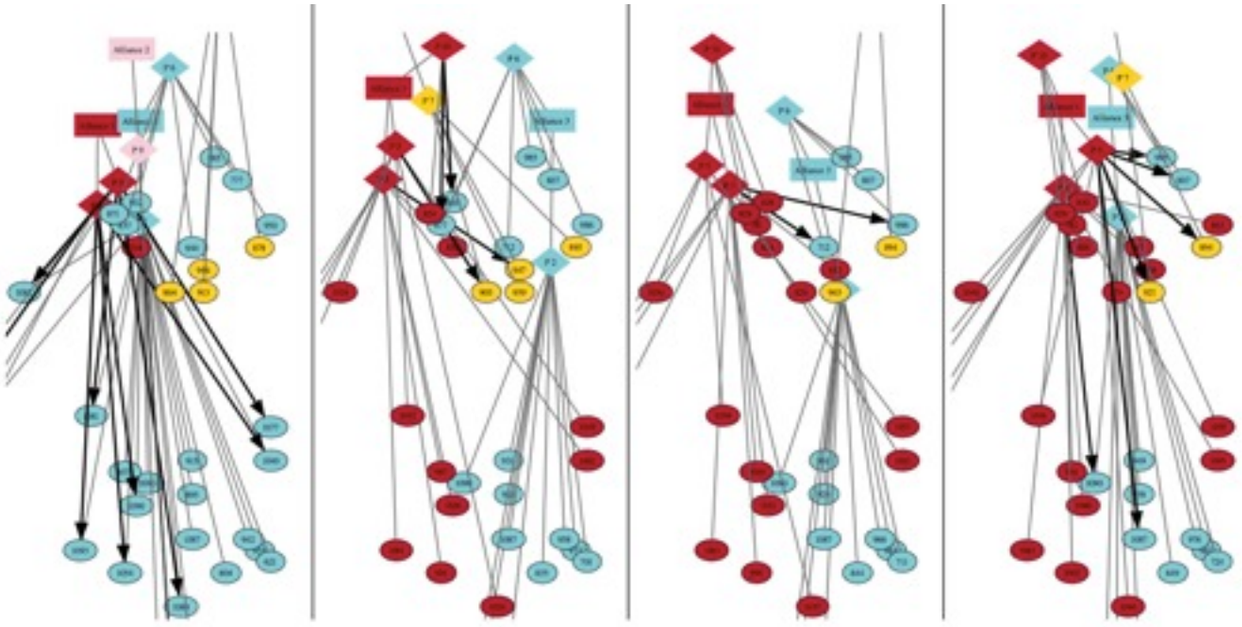
one of the **effects** holds at time T+1

```
0.4::conquest(Attacker,C); 0.6::nil <-
        city(C,Owner),city(C2,Attacker),close(C,C2).
```

if **cause** holds at time T

[Thon et al, MLJ 11]

Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

one of the **effects** holds at time T+1

```
0.4::conquest(Attacker,C); 0.6::nil <-

                city(C,Owner),city(C2,Attacker),close(C,C2).
```

if **cause** holds at time T

[Thon et al, MLJ 11]

continuous RVs

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables
- Inference: particle filter

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables
- Inference: particle filter

random variable with Gaussian distribution

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
```

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables
- Inference: particle filter

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
stackable(OBot,OTop) :-
    ≃length(OBot) ≥ ≃length(OTop),
    ≃width(OBot) ≥ ≃width(OTop).
```

comparing values of random variables

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables
- Inference: particle filter

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
stackable(OBot,OTop) :-
      ≃length(OBot) ≥ ≃length(OTop),
      ≃width(OBot) ≥ ≃width(OTop).
ontype(Obj,plate) ~ finite([0 : glass, 0.0024 : cup,
                            0 : pitcher, 0.8676 : plate,
                            0.0284 : bowl, 0 : serving,
                            0.1016 : none])
                        :- obj(Obj), on(Obj,O2), type(O2,plate).
```

random variable with discrete distribution

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

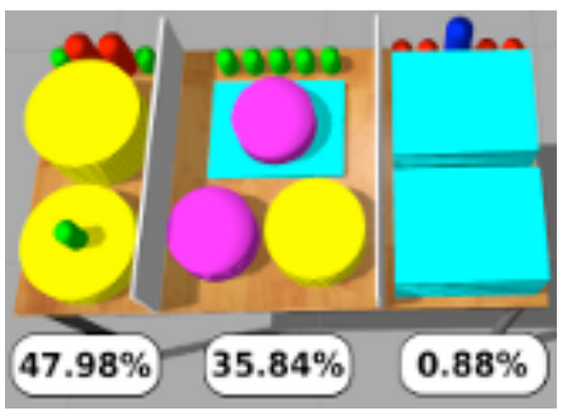
- Discrete- and continuous-valued random variables
- Inference: particle filter

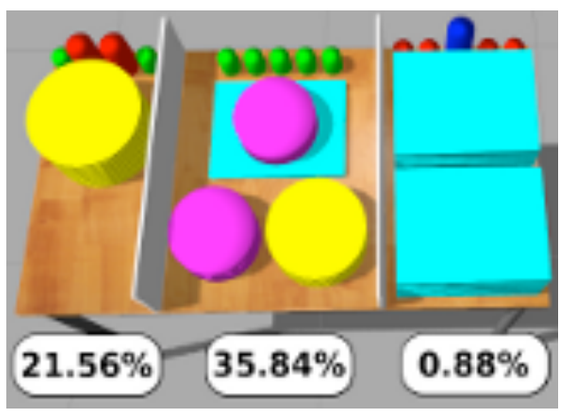
```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
stackable(OBot,OTop) :-
      ≃length(OBot) ≥ ≃length(OTop),
      ≃width(OBot) ≥ ≃width(OTop).
ontype(Obj,plate) ~ finite([0 : glass, 0.0024 : cup,
                            0 : pitcher, 0.8676 : plate,
                            0.0284 : bowl, 0 : serving,
                            0.1016 : none])
                    :- obj(Obj), on(Obj,O2), type(O2,plate).
```

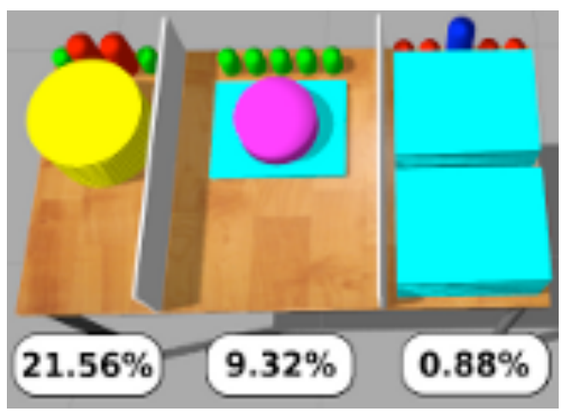
[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

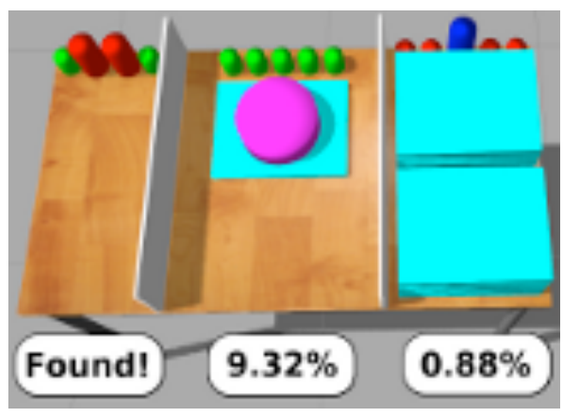
Occluded Object Search

- DC model of objects and their spatial arrangement
- different types of objects suitable for different tasks
- shelves with objects of different shape and size
- given a task, find an object to perform that task







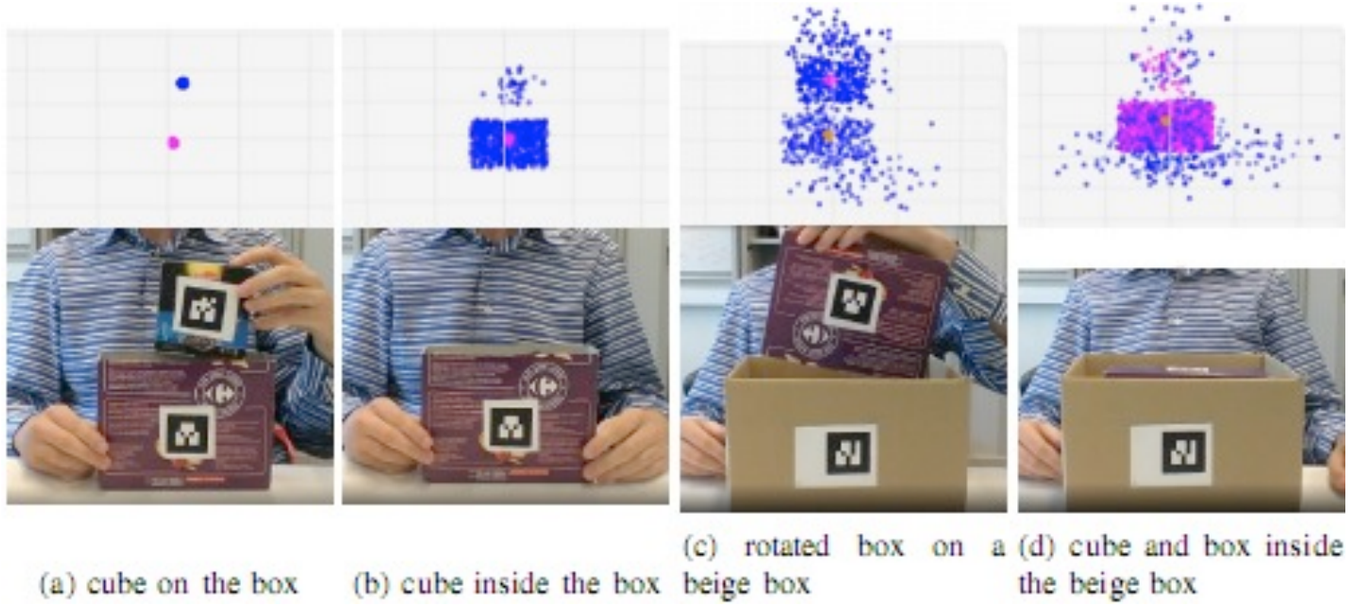


[Moldovan et al, ICRA 12, 14]

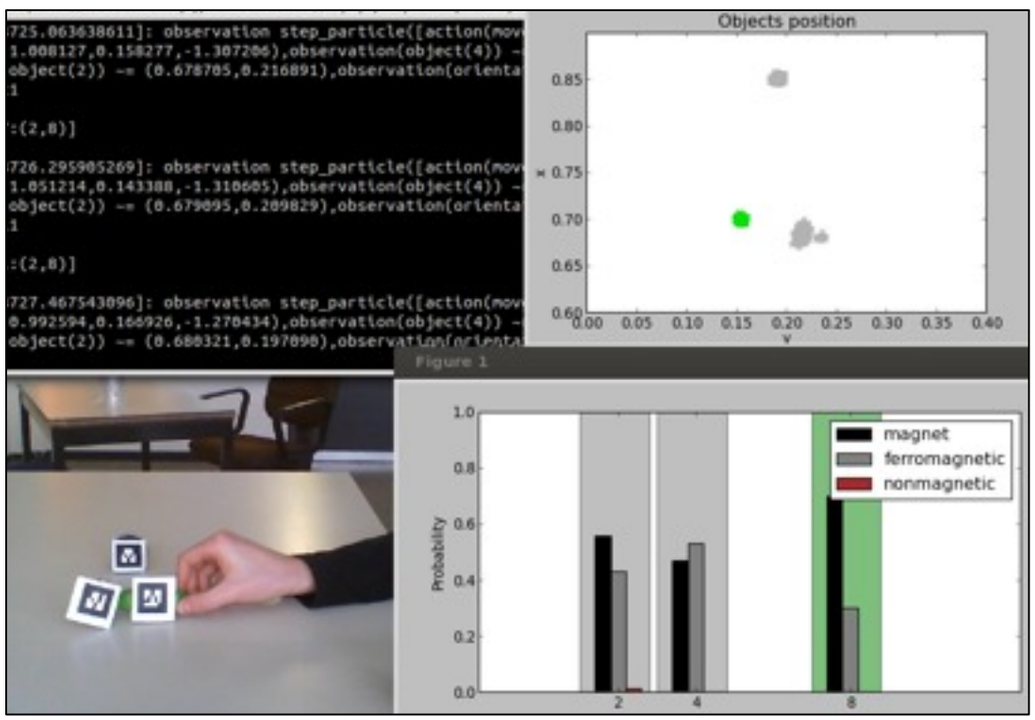
Relational State Estimation over Time

Magnetism scenario

- object tracking
- category estimation from interactions



(a) cube on the box (b) cube inside the box (c) rotated box on a beige box (d) cube and box inside the beige box



Box scenario

- object tracking even when invisible
- estimate spatial relations

[Nitti et al, IROS 13]

Dynamic Distributional Clauses

3 object types: magnetic, ferromagnetic, nonmagnetic

$\text{type}(X)_t \sim \text{finite}([1/3:\text{magnet},1/3:\text{ferromagnetic},1/3:\text{nonmagnetic}]) \leftarrow \text{object}(X).$

2 magnets attract or repulse

$\text{interaction}(A,B)_t \sim \text{finite}([0.5:\text{attraction},0.5:\text{repulsion}])$

$\leftarrow A \neq B, \text{type}(A)_t = \text{magnet}, \text{type}(B)_t = \text{magnet}.$

Next position after attraction

$\text{pos}(A)_{t+1} \sim \text{gaussian}(\text{middlepoint}(A,B)_t, Cov) \leftarrow$

$\text{near}(A,B)_t, \text{not}(\text{held}(A)), \text{not}(\text{held}(B)), \text{interaction}(A,B)_t = \text{attr}, c/\text{dist}(A,B)^2 > \text{friction}(A).$

$\text{pos}(A)_{t+1} \sim \text{gaussian}(\text{pos}(A)_t, Cov) \leftarrow \text{not}(\text{attraction}(A,B)).$

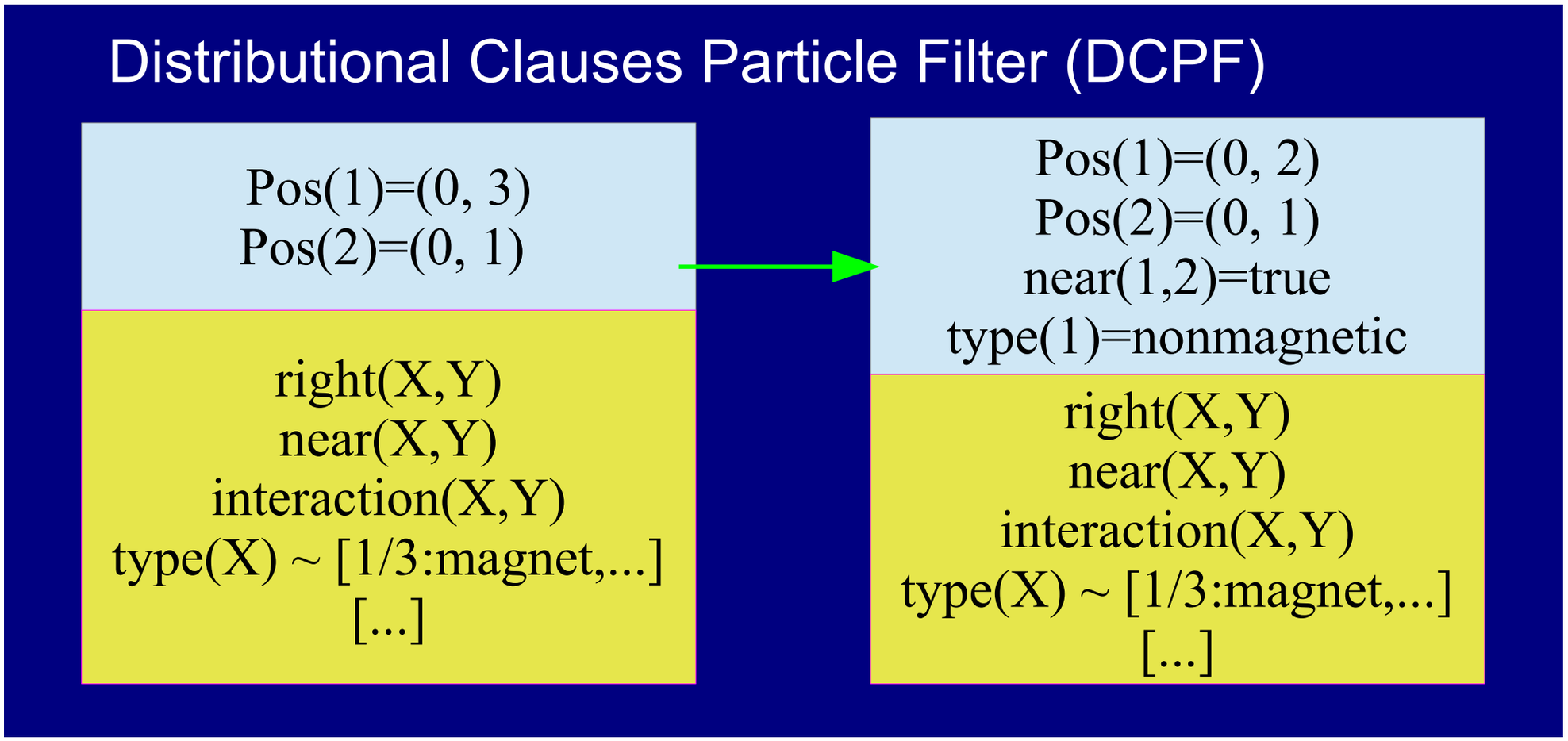
Nitti et al, IROS 13, ICRA14

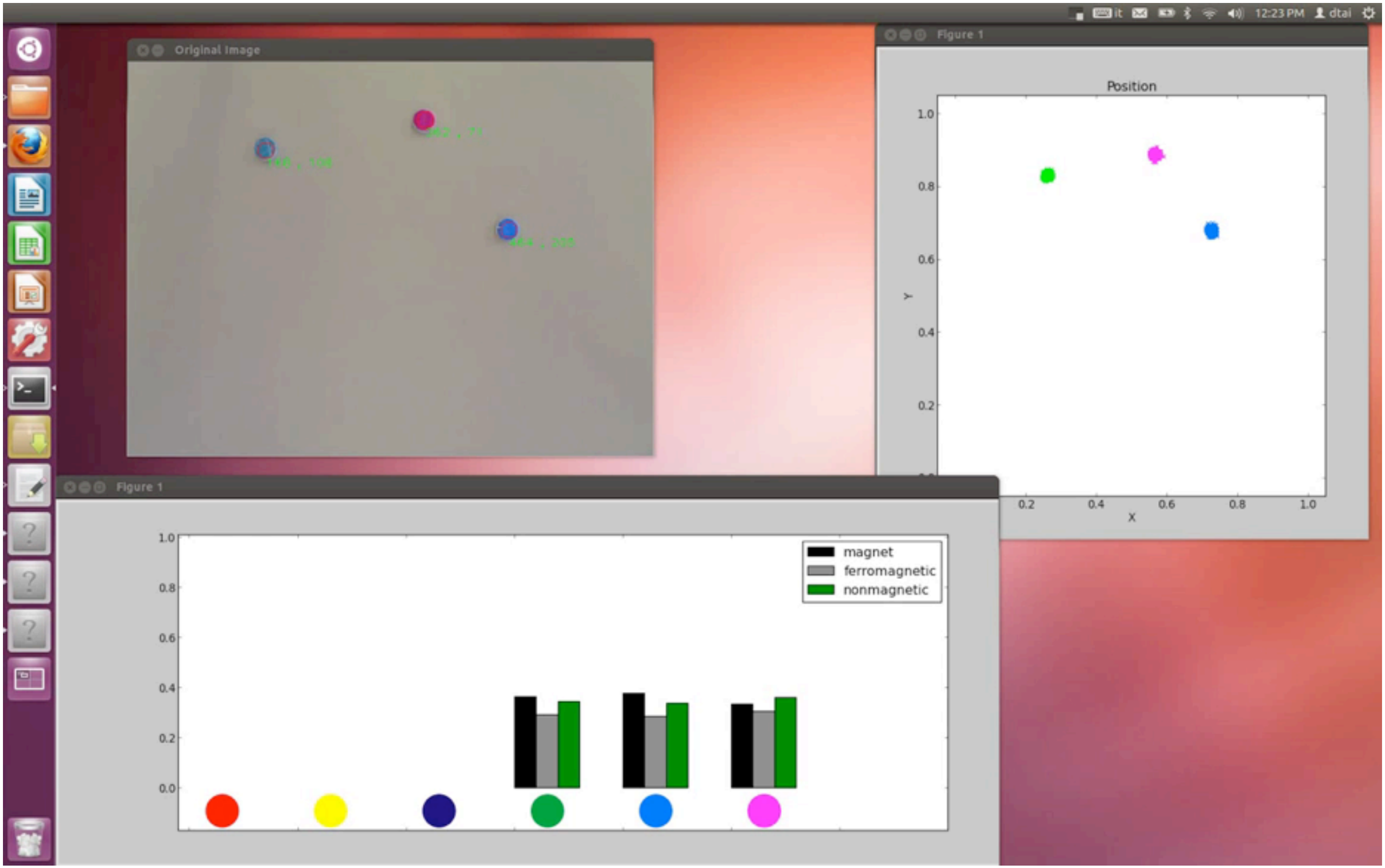
Magnet scenario

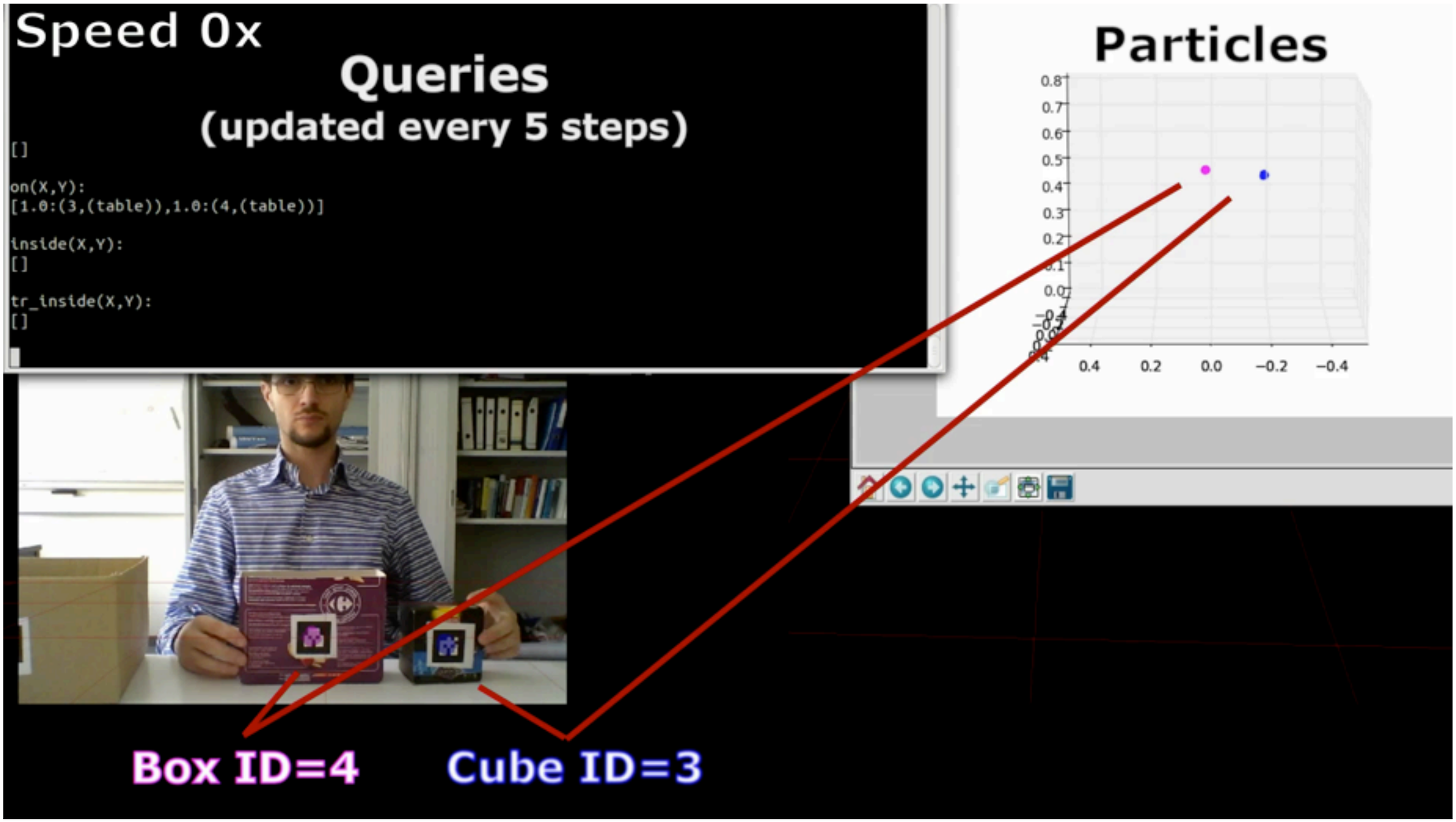
- 3 object types: magnetic, ferromagnetic, nonmagnetic
    - nonmagnetic objects do not interact
    - a magnet and a ferromagnetic object attract each other with a force that depends on the distance
    - if an object is held magnetic force is compensated.

Two complications : 1) observations uncertain
2) no discrete states

Inference and Learning




Original Image
Figure 1
Position
1.0
0.8
0.6
0.4
0.2
Y
0.2
0.4
0.6
0.8
1.0
X
Figure 1
1.0
0.8
0.6
0.4
0.2
0.0
magnet
ferromagnetic
nonmagnetic


Speed 0x
Queries
(updated every 5 steps)
[]
on(X,Y):
[1.0:(3,(table)),1.0:(4,(table))]
inside(X,Y):
[]
tr_inside(X,Y):
[]
Particles
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.4
0.2
0.0
−0.2
−0.4
Box ID=4
Cube ID=3

IROS 13

cProbLog: constraints on possible worlds

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :-
  weight(Item,Weight),
  P is 1.0/Weight.

excess(Limit) :- ...

not excess(10).
pack(helmet) v pack(boots).
```


[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).
```

distribution over **all** possible worlds

```
P::pack(Item) :-
  weight(Item,Weight),
  P is 1.0/Weight.

excess(Limit) :- ...

not excess(10).
pack(helmet) v pack(boots).
```

sbhg e(10)	sb g e(10)	sbh e(10)	sb
s hg e(10)	s g	s h	s
bhg	b g	bh	b
hg	g	h	

[Fierens et al, PP 12; Shterionov et al]

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weight(skis,6).
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constraints as first-order logic formulas

sbhg e(10)	sb g e(10)	sbh e(10)	sb
s hg e(10)	s g	s h	s
bhg	b g	bh	b
hg	g	h	

[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

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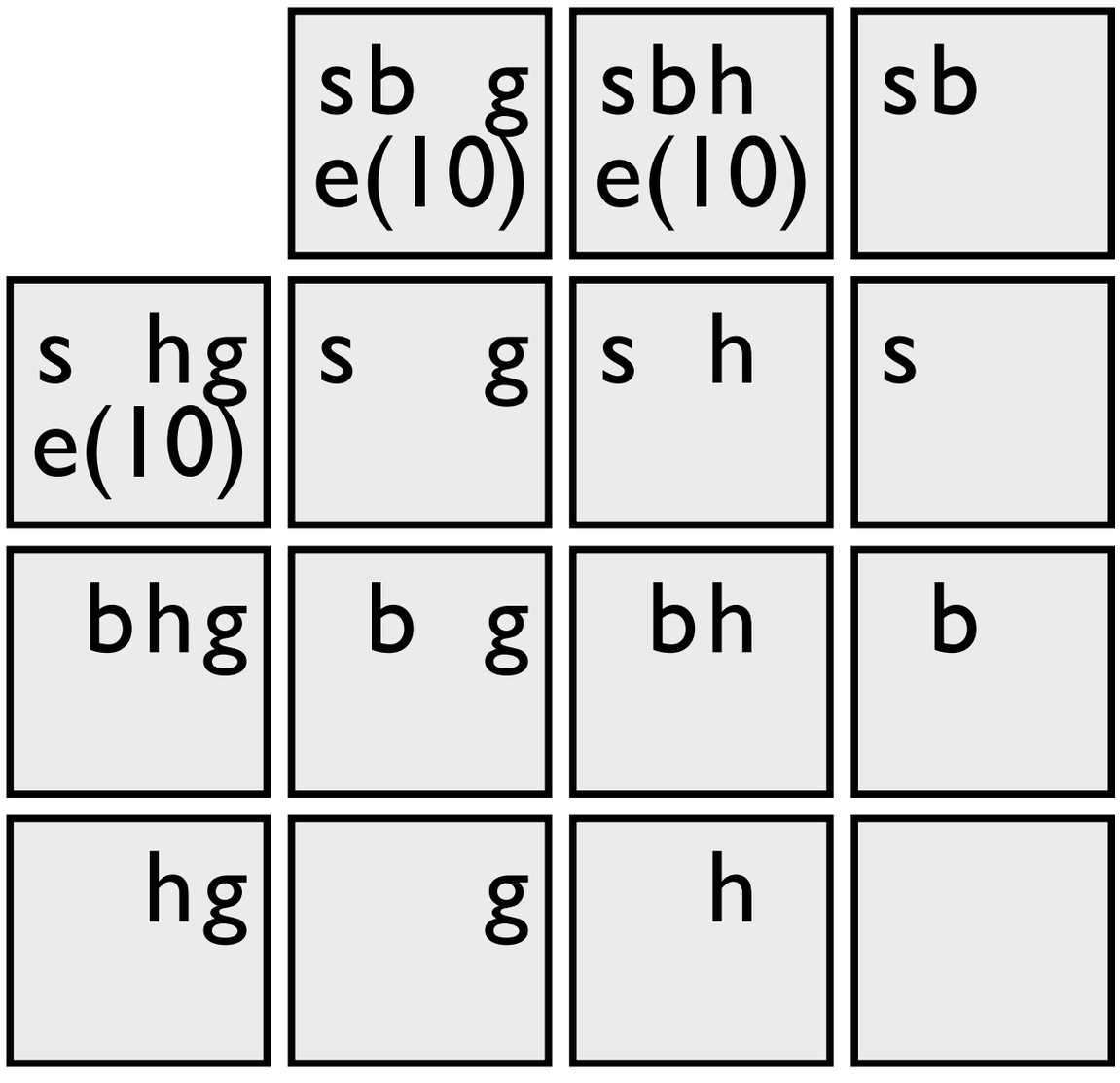
```

constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

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constraints as first-order logic formulas

		sbh e(10)	sb
s hg e(10)	s g	s h	s
bhg	b g	bh	b
hg	g	h	

[Fierens et al, PP 12; Shterionov et al]

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constraints as first-order logic formulas

s hg e(10)	s g	s h	s
bhg	b g	bh	b
hg	g	h	

[Fierens et al, PP 12; Shterionov et al]

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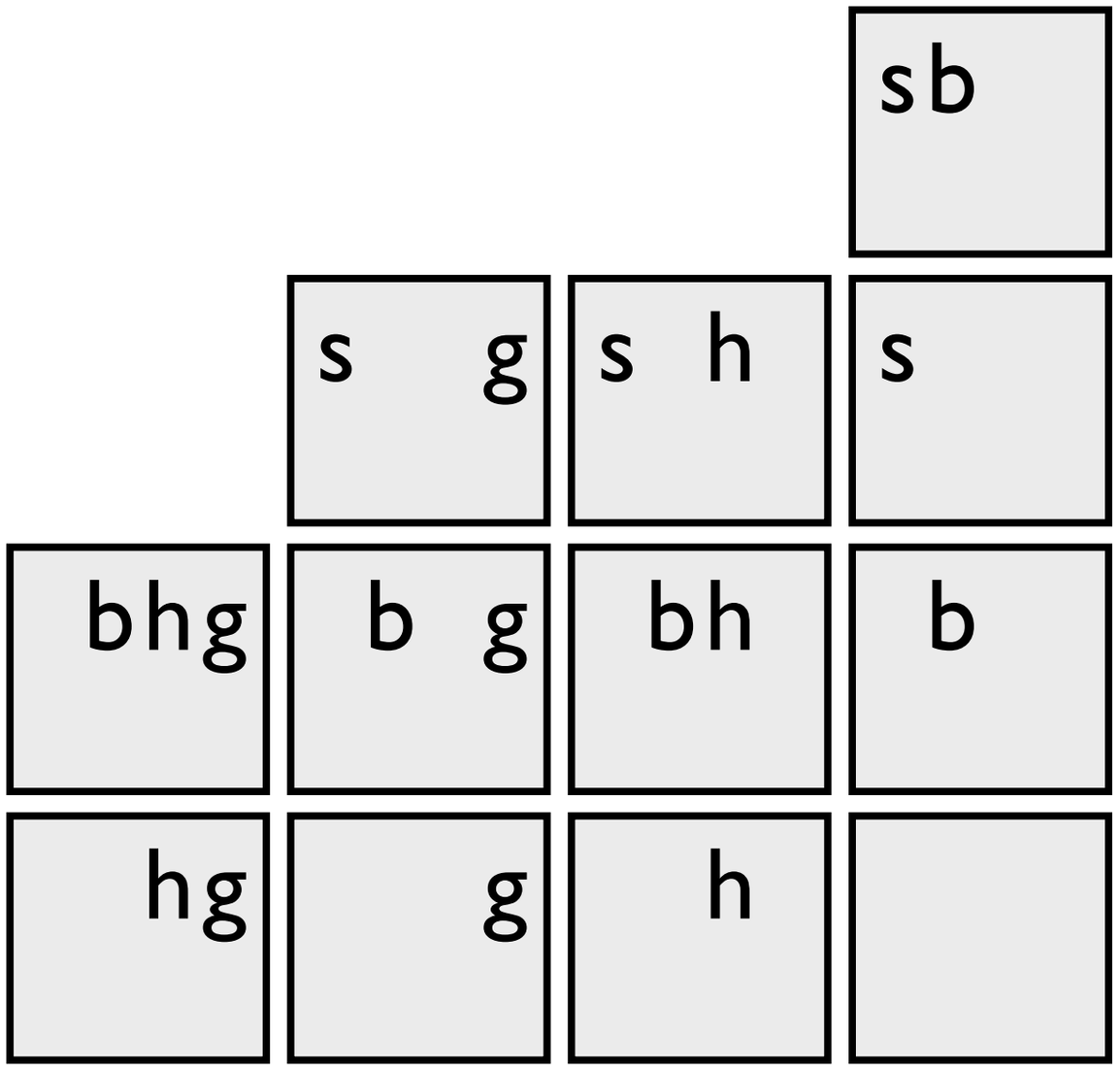
constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

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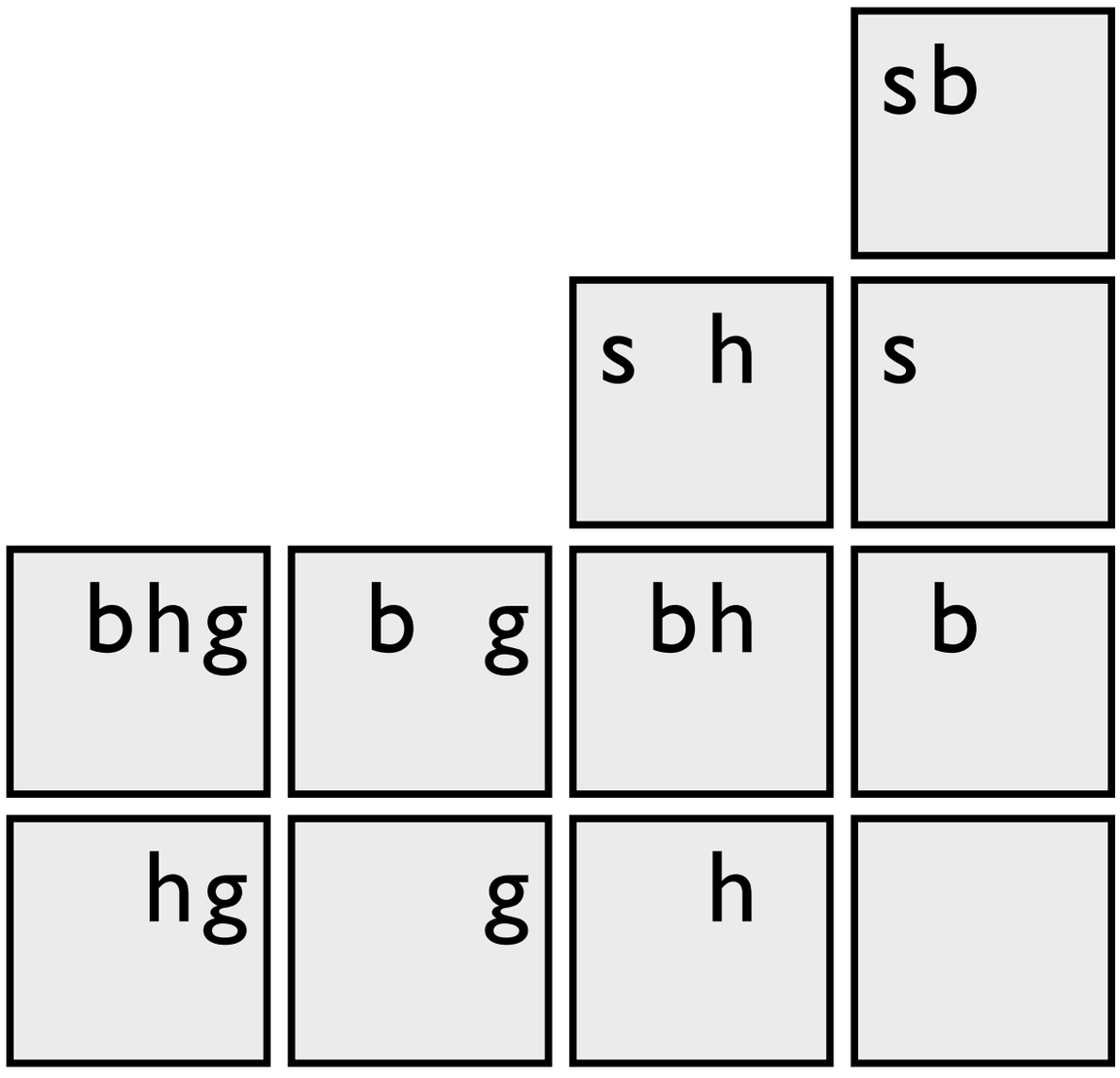
constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

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  weight(Item,Weight),
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excess(Limit) :- ...
```

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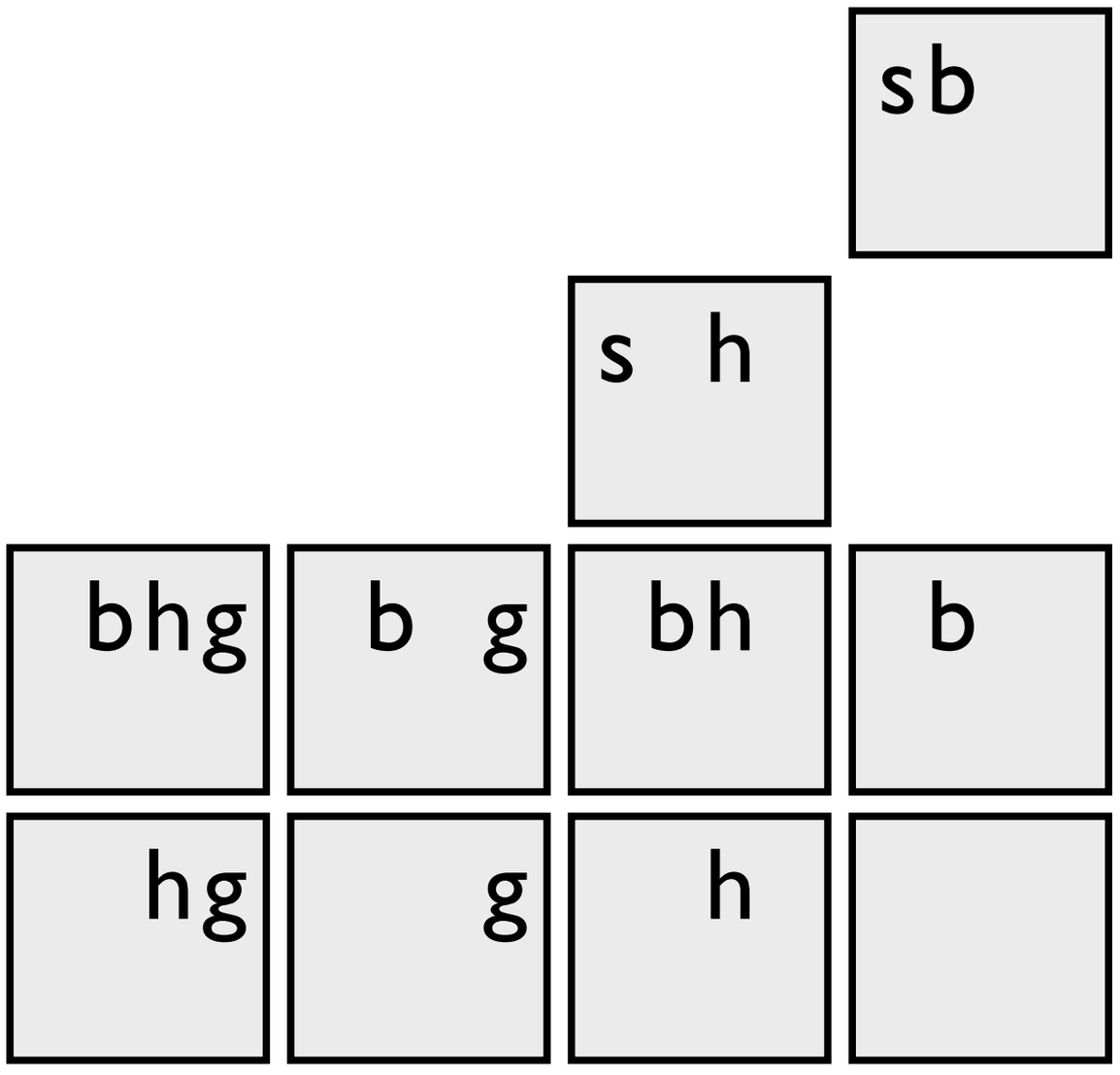
not excess(10).
pack(helmet) v pack(boots).
```

constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

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excess(Limit) :- ...
```

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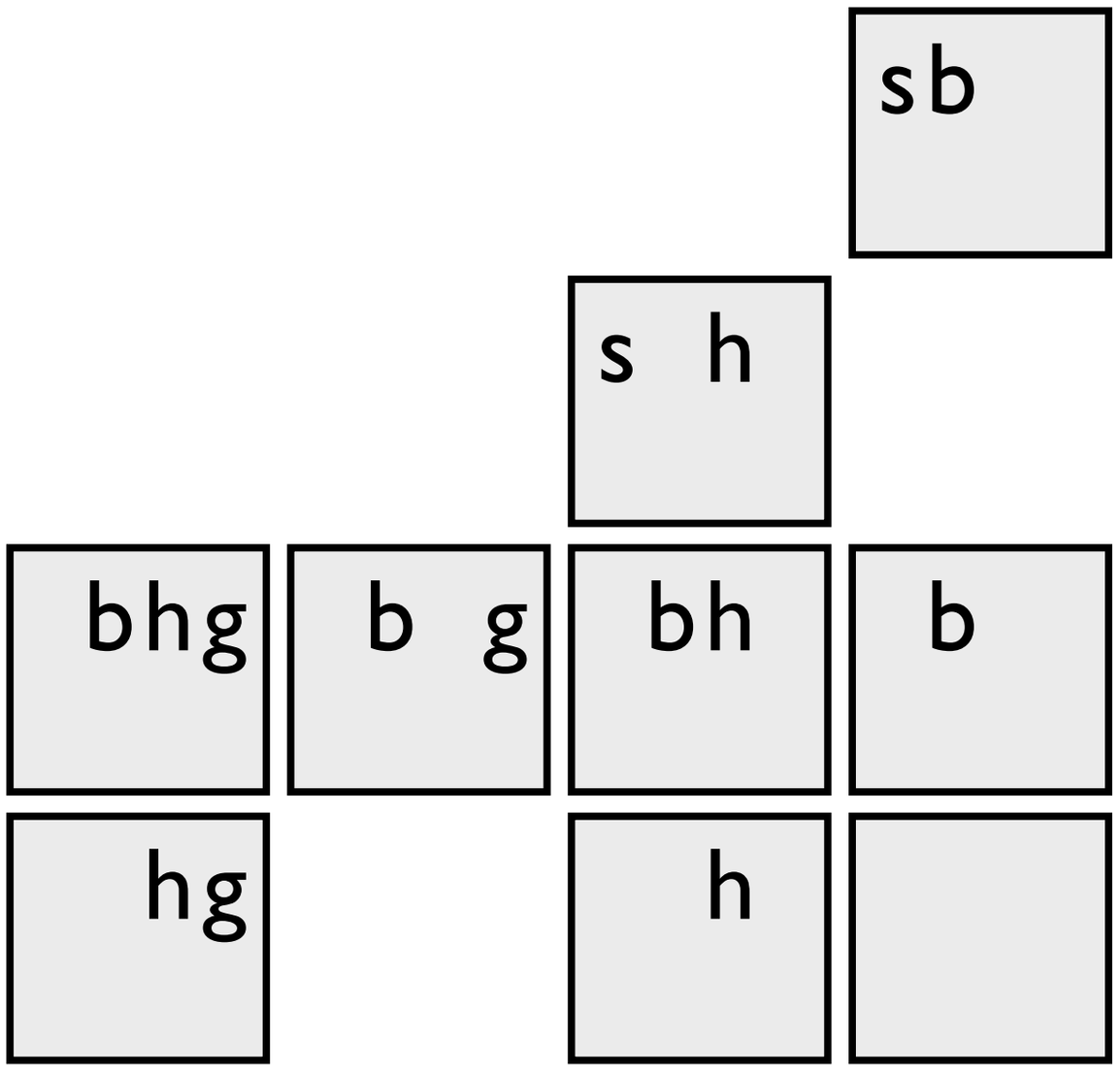
pack(helmet) v pack(boots).
```

constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

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excess(Limit) :- ...
```

```
not excess(10).
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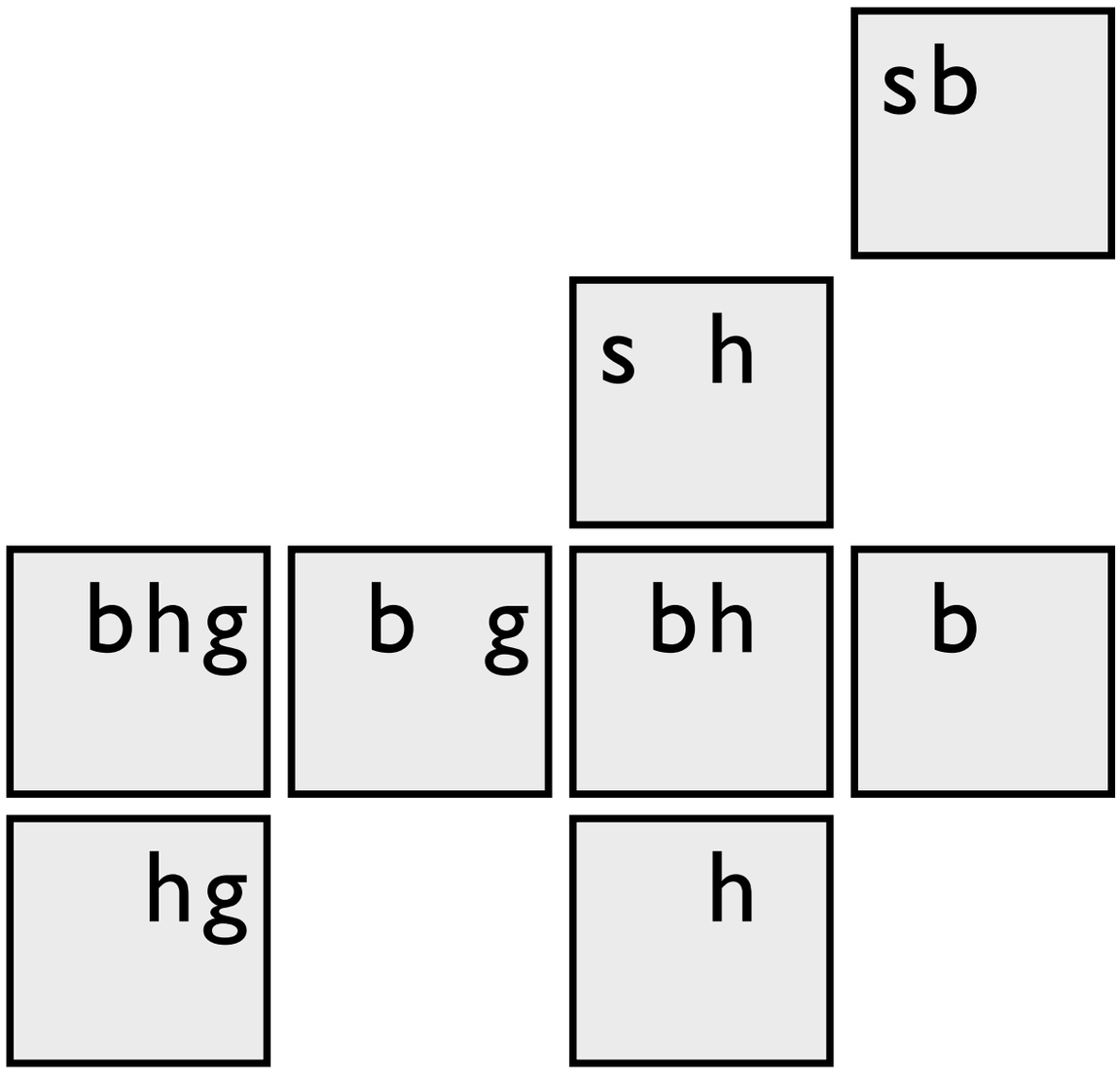
```

constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

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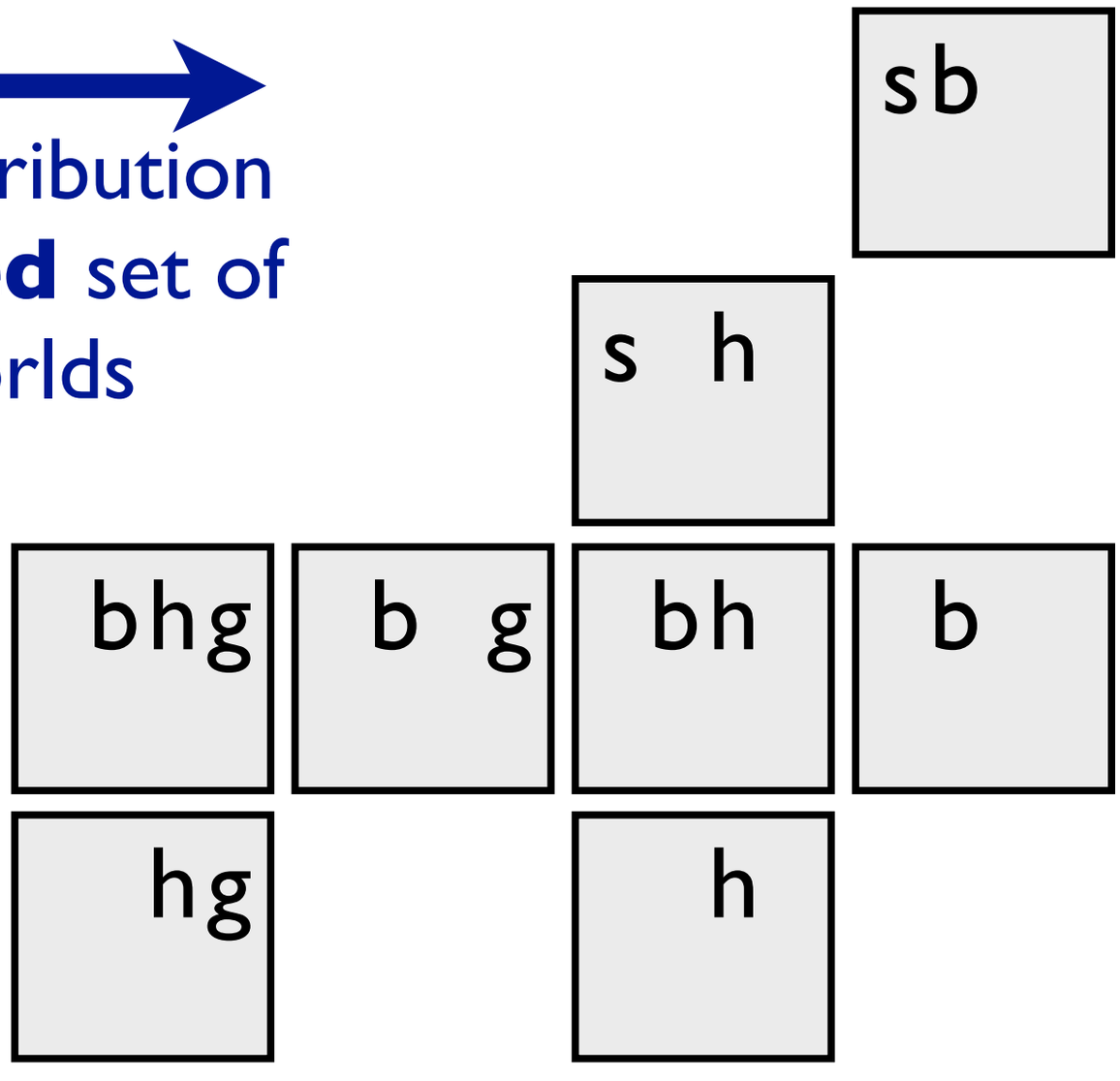
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :-
  weight(Item,Weight),
  P is 1.0/Weight.

excess(Limit) :- ...

not excess(10).
pack(helmet) v pack(boots).
```

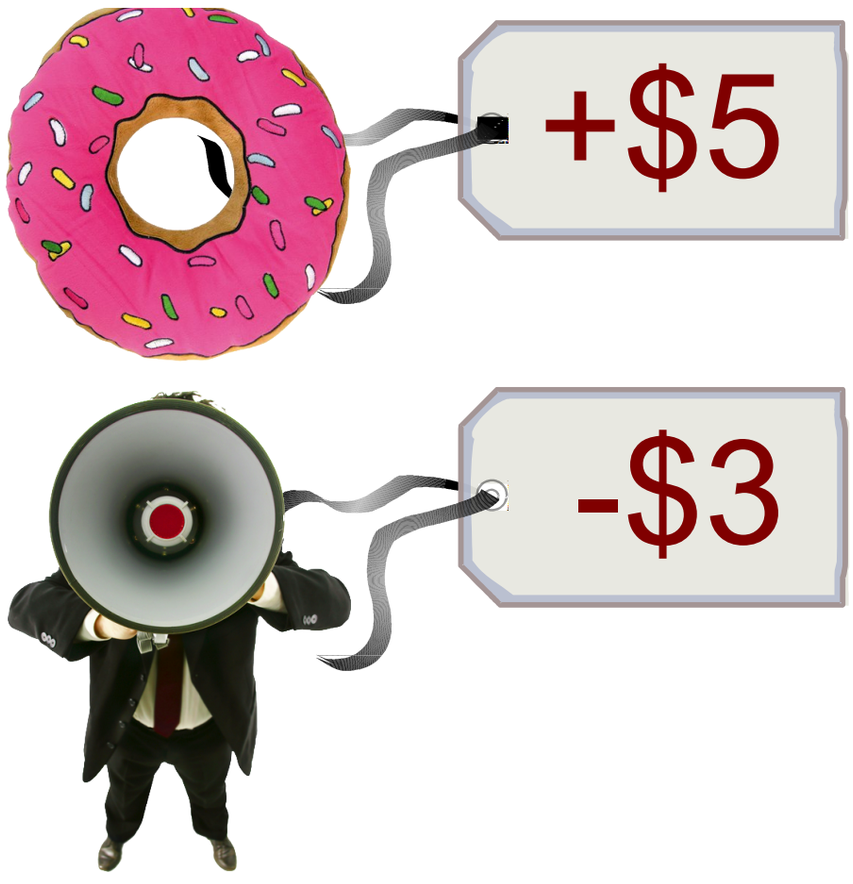
normalized distribution over **restricted** set of possible worlds



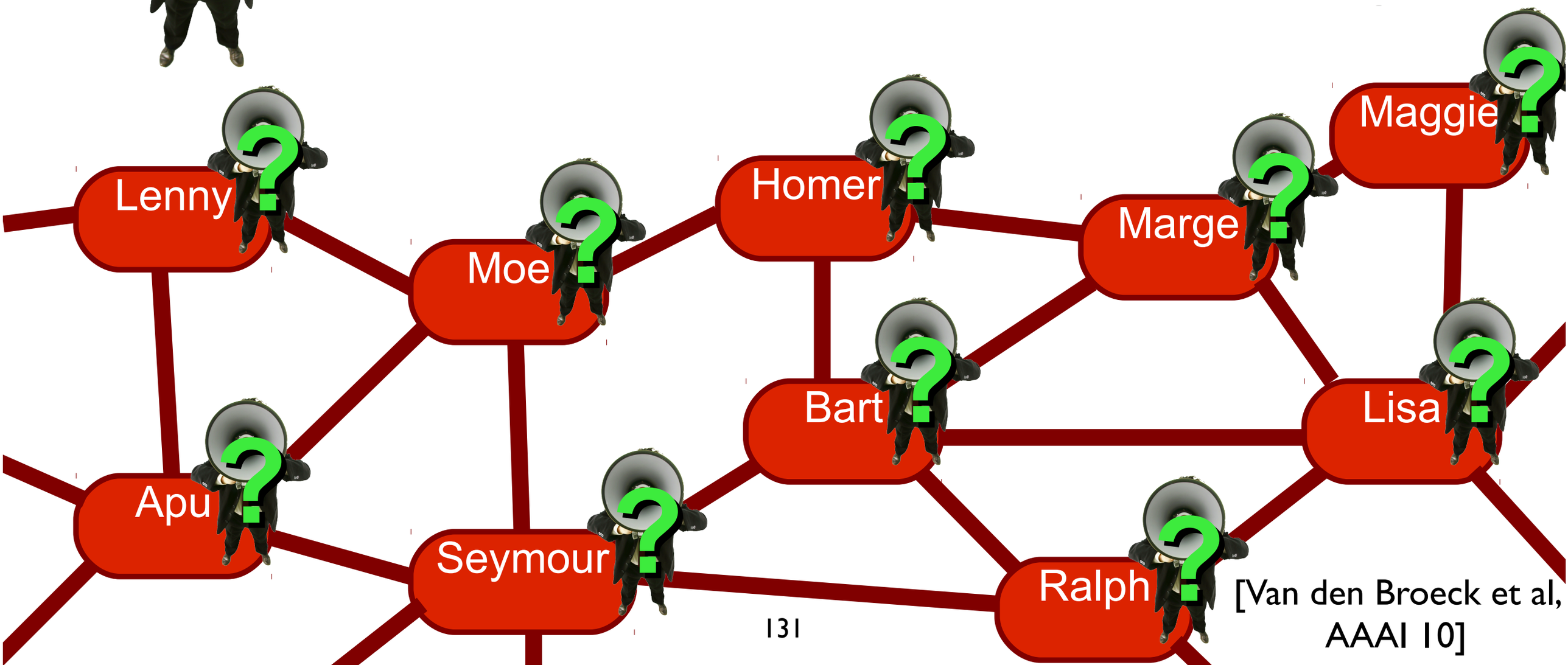
constraints as first-order logic formulas

[Fierens et al, PP 12; Shterionov et al]

Viral Marketing

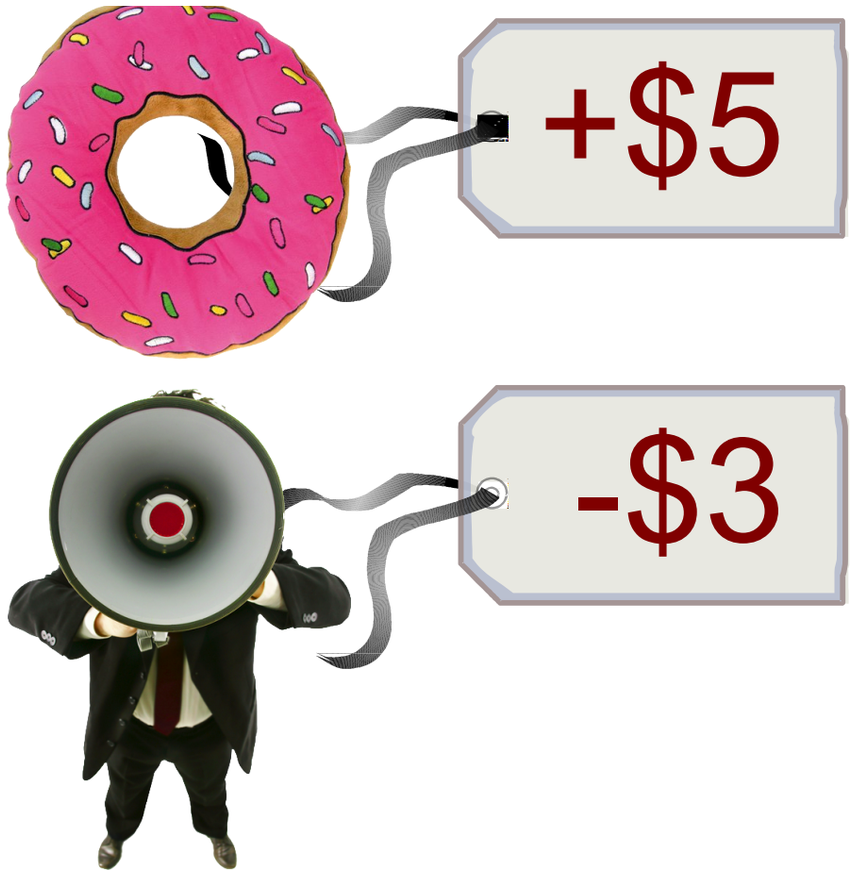


Which advertising strategy maximizes expected profit?

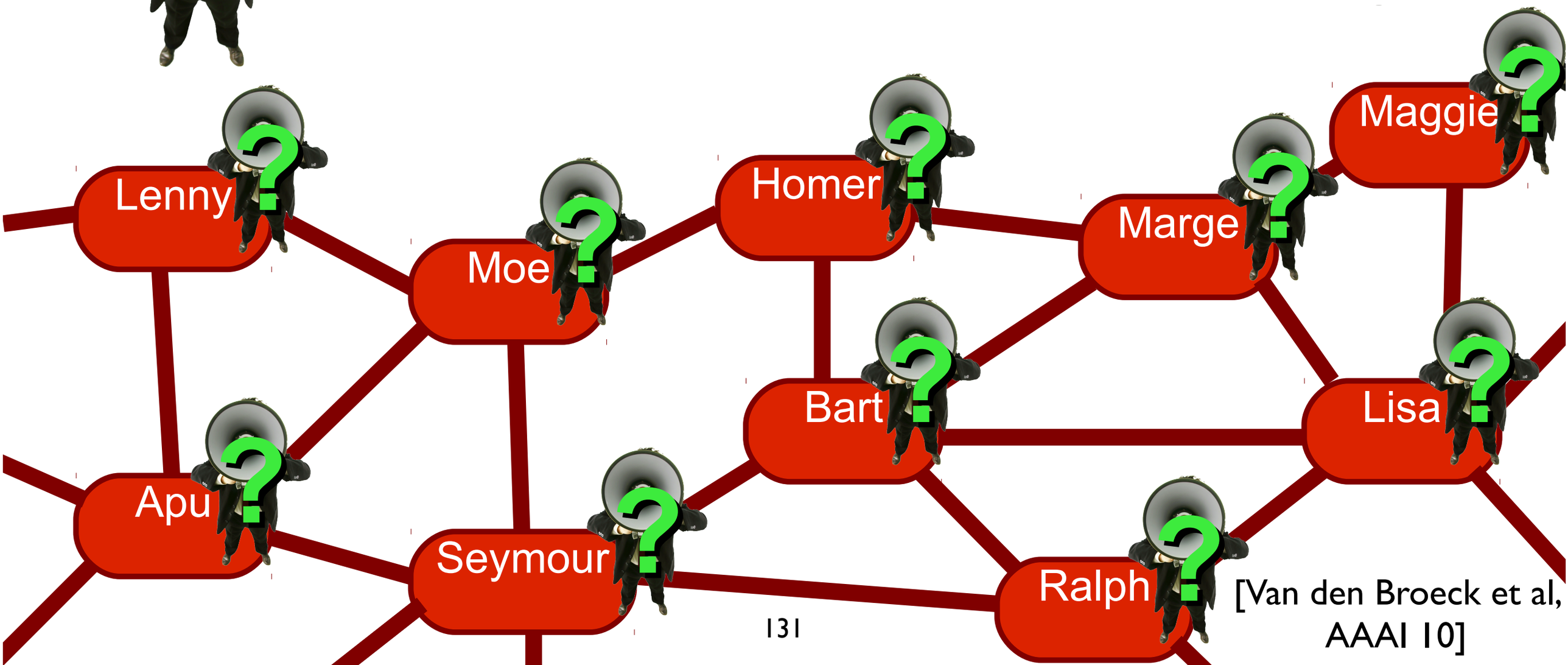


[Van den Broeck et al, AAAI 10]

Viral Marketing

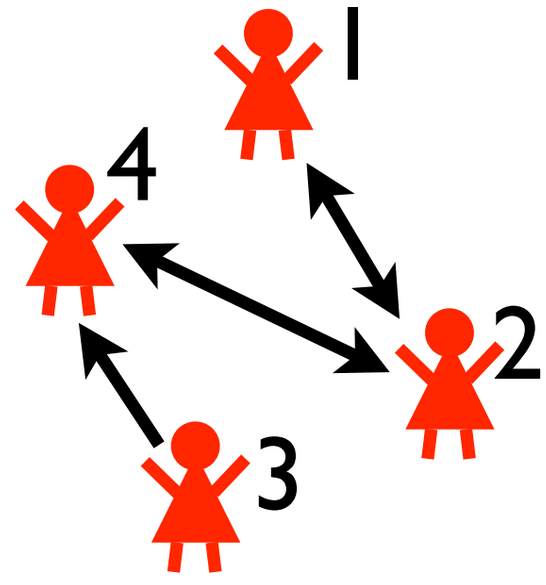


decide truth values of some atoms



[Van den Broeck et al, AAAI 10]

DTProbLog



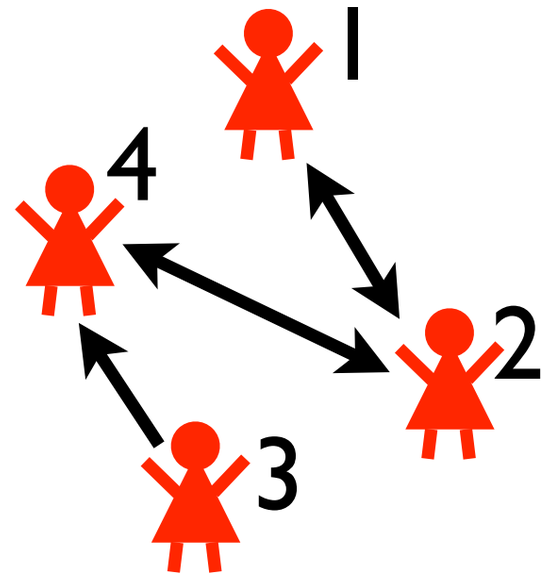

```
person(1).
person(2).
person(3).
person(4).
```

```
friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

DTProbLog

```
? :: marketed(P) :- person(P).
```

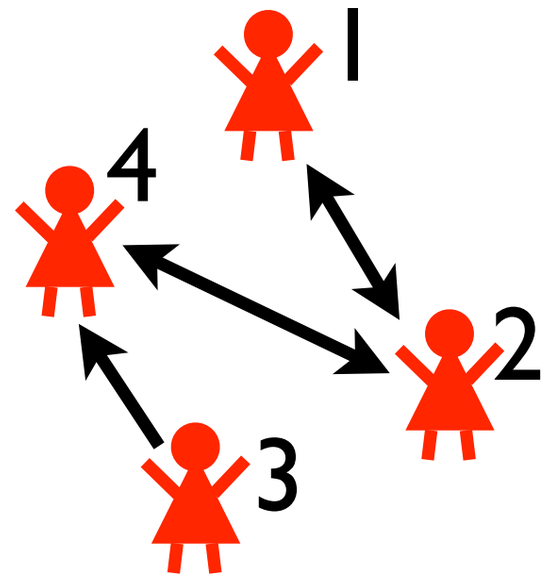
decision fact: true or false?




```
person(1).
person(2).
person(3).
person(4).
```

```
friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

DTProbLog




```
? :: marketed(P) :- person(P).
```

```
0.3 :: buy_trust(X,Y) :- friend(X,Y).
0.2 :: buy_marketing(P) :- person(P).

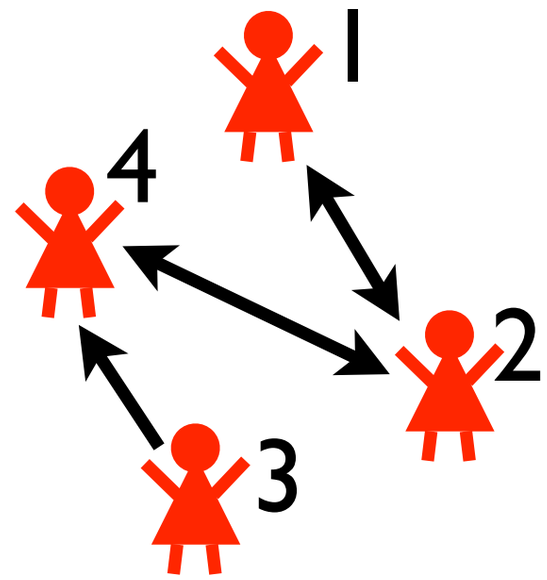
buys(X) :- friend(X,Y), buys(Y), buy_trust(X,Y).
buys(X) :- marketed(X), buy_marketing(X).
```

probabilistic facts
+ logical rules

```
person(1).
person(2).
person(3).
person(4).
```

```
friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

DTProbLog




```
? :: marketed(P) :- person(P).

0.3 :: buy_trust(X,Y) :- friend(X,Y).
0.2 :: buy_marketing(P) :- person(P).

buys(X) :- friend(X,Y), buys(Y), buy_trust(X,Y).
buys(X) :- marketed(X), buy_marketing(X).

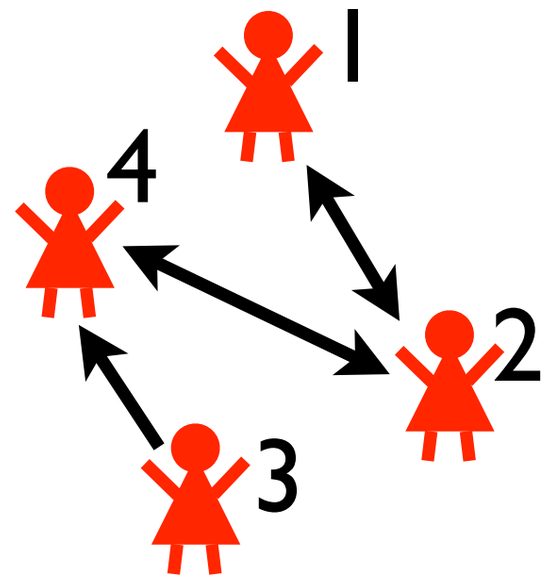
buys(P) => 5 :- person(P).
marketed(P) => -3 :- person(P).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

utility facts: cost/reward if true

DTProbLog

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0.2 :: buy_marketing(P) :- person(P).

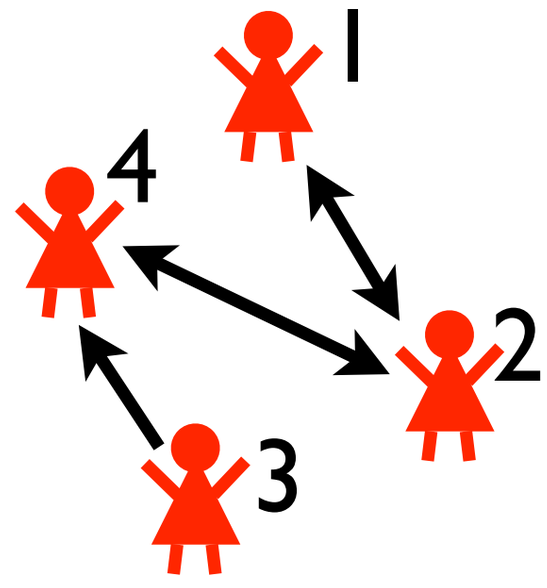
buys(X) :- friend(X,Y), buys(Y), buy_trust(X,Y).
buys(X) :- marketed(X), buy_marketing(X).

buys(P) => 5 :- person(P).
marketed(P) => -3 :- person(P).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

DTProbLog

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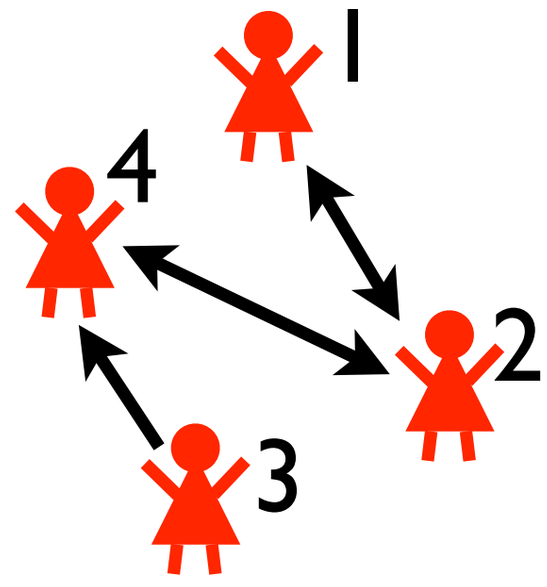


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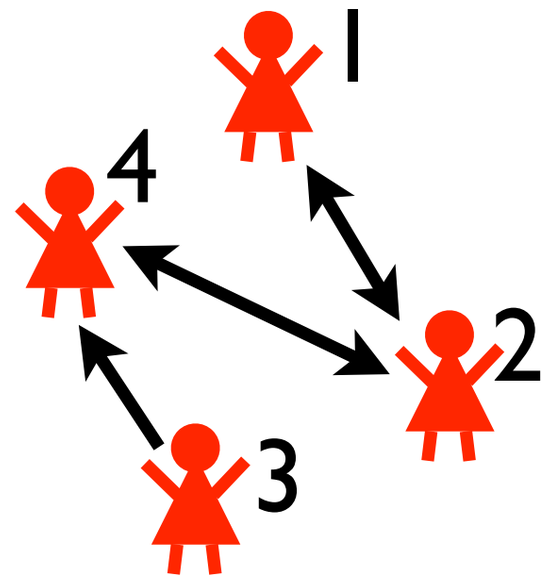
friend(4,2).
```

```
marketed(1)        marketed(3)
```

DTProbLog

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person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

```
marketed(1)        marketed(3)

    bt(2,1)   bt(2,4)          bm(1)
```

DTProbLog

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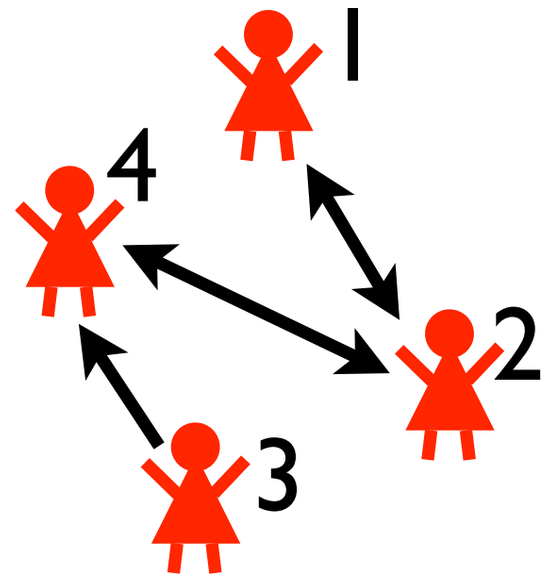


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friend(2,4).
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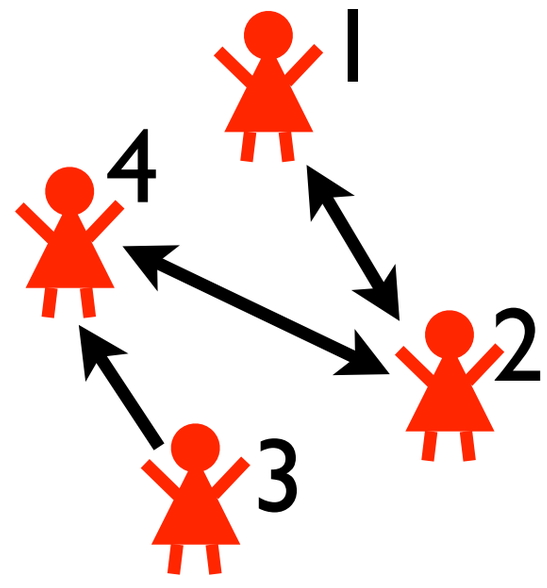


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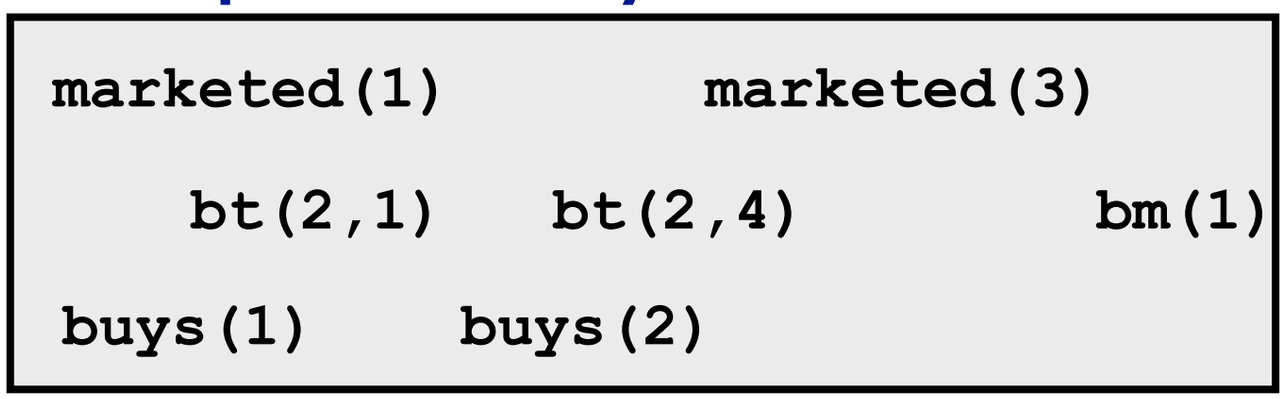
```

utility = −3 + −3 + 5 + 5 = 4

probability = 0.0032



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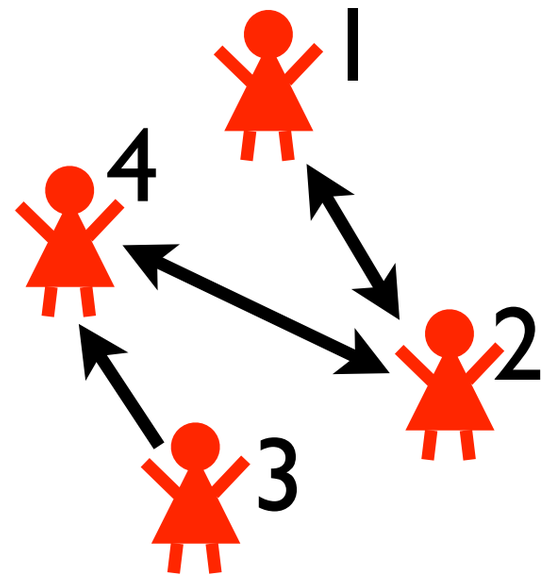


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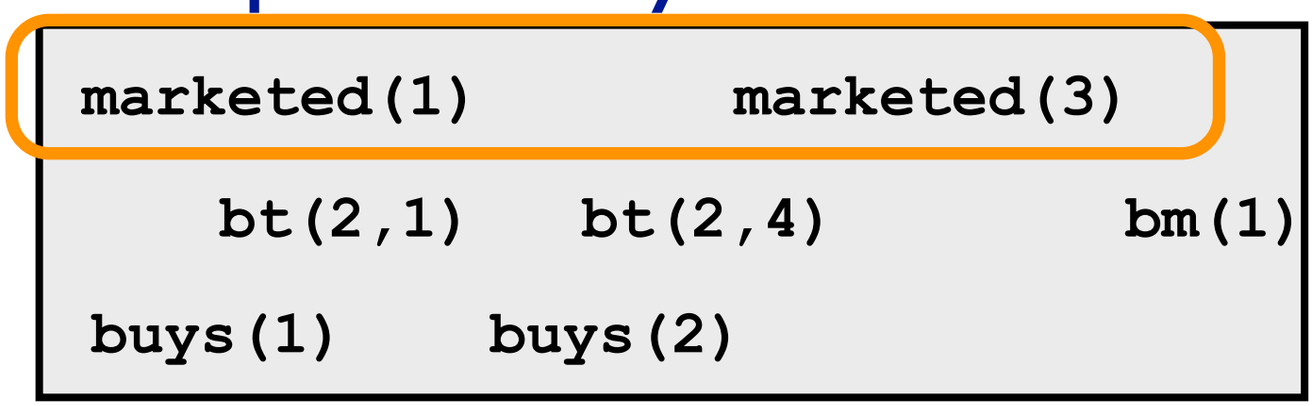
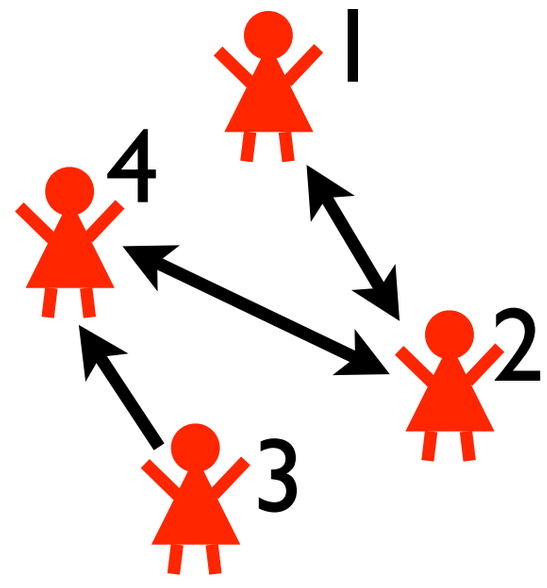


world contributes 0.0032×4 to expected utility of strategy

DTProbLog

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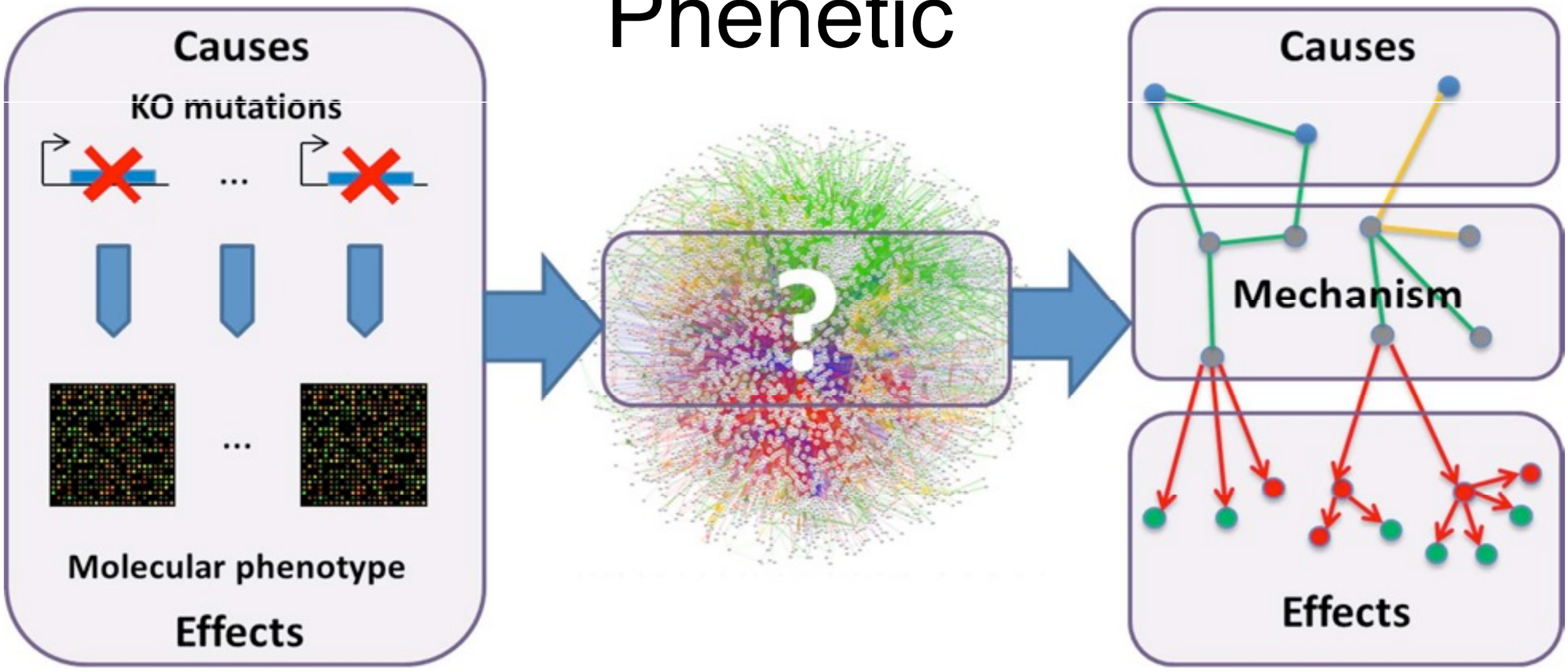
friend(4,2).
```

task: find strategy that maximizes expected utility
solution: using ProbLog technology

Phenetic



- Causes: Mutations
  - All related to similar phenotype
- Effects: Differentially expressed genes
- 27 000 cause effect pairs

- Interaction network:
  - 3063 nodes
    - Genes
    - Proteins
  - 16794 edges
    - Molecular interactions
    - Uncertain

- Goal: connect causes to effects through common subnetwork
  - = Find mechanism
- Techniques:
  - DTProbLog
  - Approximate inference

[De Maeyer et al., Molecular Biosystems 13]

Dyna

[Eisner et al 05]

- Weighted Logic Programs (but not Prolog)
- Inspired by NLP
- Arbitrary semiring weights
- Forward reasoning

CYK parsing in Dyna

```
word(john,0,1)=1
word(loves,1,2)=1
word(mary,2,3)=1
end(3)=1
```

input sentence

```
rewrite(np,mary)=0.003
rewrite(vp,verb,np)=0.5
...
```

probabilistic grammar rules

rules X→W with W spanning I,J

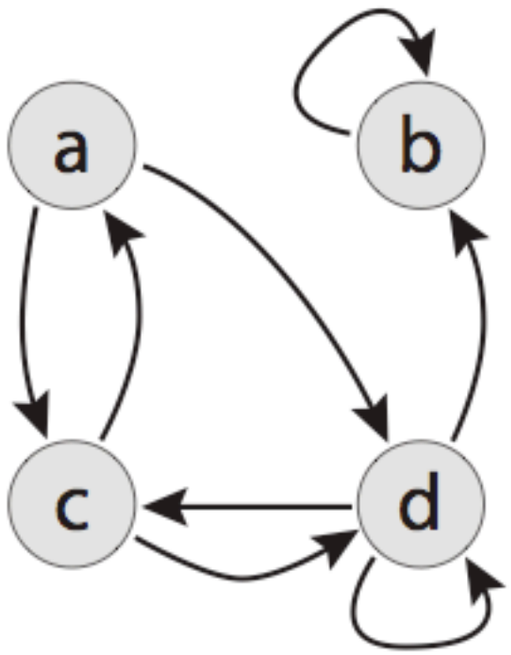
```
constit(X,I,J) += rewrite(X,W) * word(W,I,J).
constit(X,I,K) += rewrite(X,Y,Z) * constit(Y,I,J) * constit(Z,J,K).
goal += constit(s,0,N) * end(N).
```

rules X→YZ, split points J

sum over all parses

```
reachable(Q) :- initial(Q).
reachable(Q) :- reachable(P), edge(P, Q).
```

Fig. 1. A simple bottom-up logic program for graph reachability



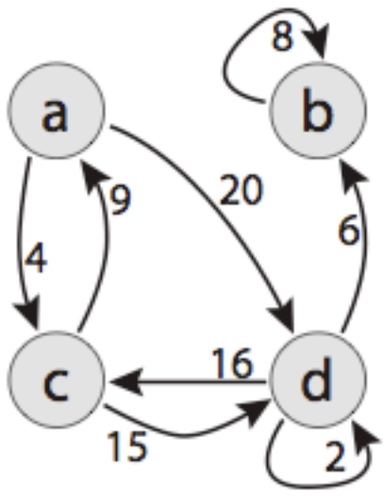
$$
\begin{aligned}
\texttt{initial(a)} &= \mathrm{T} & \texttt{edge(c, d)} &= \mathrm{T} \\
\texttt{edge(a, c)} &= \mathrm{T} & \texttt{edge(d, b)} &= \mathrm{T} \\
\texttt{edge(a, d)} &= \mathrm{T} & \texttt{edge(d, c)} &= \mathrm{T} \\
\texttt{edge(b, b)} &= \mathrm{T} & \texttt{edge(d, d)} &= \mathrm{T} \\
\texttt{edge(c, a)} &= \mathrm{T}
\end{aligned}
$$

Fig. 2. A directed graph and the corresponding initial database

[Figures: Cohen et al, ICLP 08]

$$\texttt{reachable(Q)} \oplus\!= \texttt{initial(Q)}.$$

$$\texttt{reachable(Q)} \oplus\!= \texttt{reachable(P)} \otimes \texttt{edge(P, Q)}.$$



$$\begin{aligned}
\texttt{initial(a)} &= 0 & \texttt{edge(c, d)} &= 15\\
\texttt{edge(a, c)} &= 4 & \texttt{edge(d, b)} &= 6\\
\texttt{edge(a, d)} &= 20 & \texttt{edge(d, c)} &= 16\\
\texttt{edge(b, b)} &= 8 & \texttt{edge(d, d)} &= 2\\
\texttt{edge(c, a)} &= 9
\end{aligned}$$

Fig. 3. A cost graph and the corresponding initial database

[Figures: Cohen et al, ICLP 08]

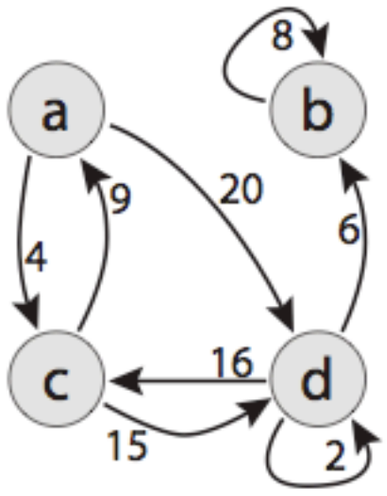
$$\texttt{reachable}(\texttt{Q}) \oplus\!= \texttt{initial}(\texttt{Q}).$$

$$\texttt{reachable}(\texttt{Q}) \oplus\!= \texttt{reachable}(\texttt{P}) \otimes \texttt{edge}(\texttt{P}, \texttt{Q}).$$

$\oplus \quad \rightarrow \quad \min$

$\otimes \quad \rightarrow \quad +$

shortest path



$$\texttt{initial}(\texttt{a}) = 0 \qquad \texttt{edge}(\texttt{c}, \texttt{d}) = 15$$
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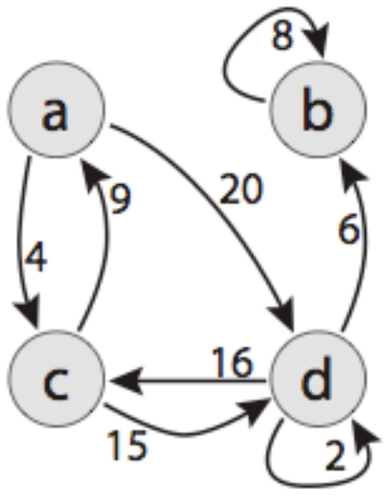

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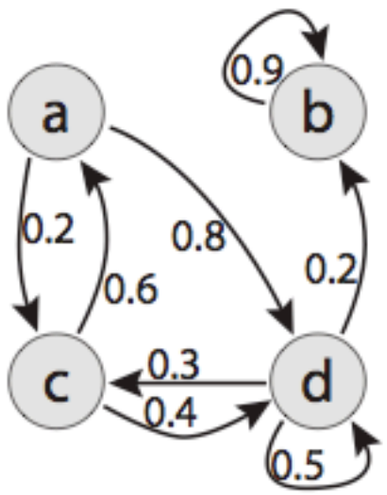

$$\texttt{initial}(\texttt{a}) = 1 \qquad \texttt{edge}(\texttt{c}, \texttt{d}) = 0.4$$
$$\texttt{edge}(\texttt{a}, \texttt{c}) = 0.2 \qquad \texttt{edge}(\texttt{d}, \texttt{b}) = 0.2$$
$$\texttt{edge}(\texttt{a}, \texttt{d}) = 0.8 \qquad \texttt{edge}(\texttt{d}, \texttt{c}) = 0.3$$
$$\texttt{edge}(\texttt{b}, \texttt{b}) = 0.9 \qquad \texttt{edge}(\texttt{d}, \texttt{d}) = 0.5$$
$$\texttt{edge}(\texttt{c}, \texttt{a}) = 0.6$$

Fig. 4. A probabilistic graph and the corresponding initial database. With stopping probabilities made explicit, this would encode a Markov model.

[Figures: Cohen et al, ICLP 08]

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shortest path

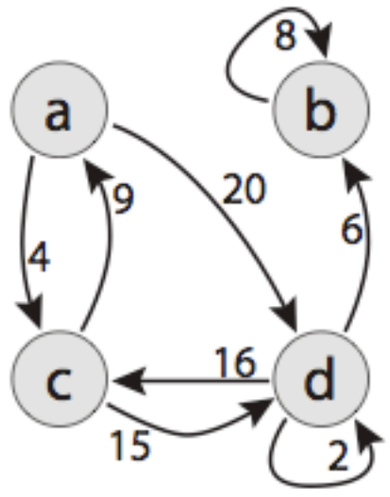

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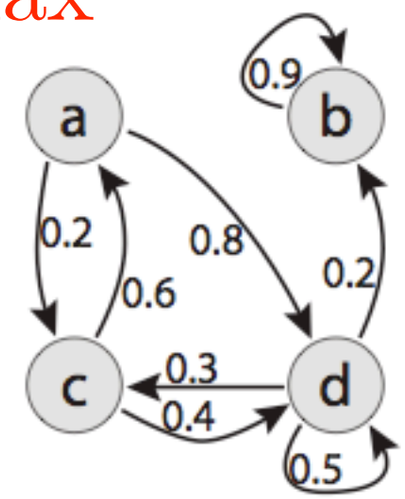
Fig. 3. A cost graph and the corresponding initial database

$\oplus \rightarrow \max$

$\otimes \rightarrow \cdot$

most likely path



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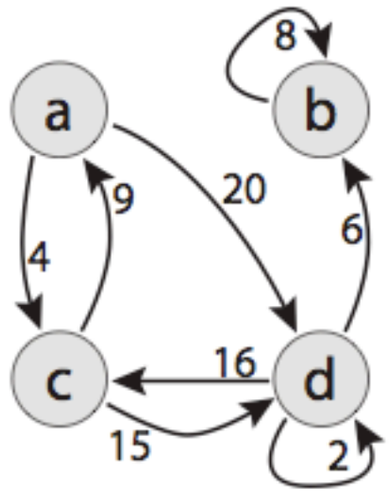
$\oplus \rightarrow \min$

$\otimes \rightarrow +$

shortest path



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$\otimes \rightarrow \cdot$

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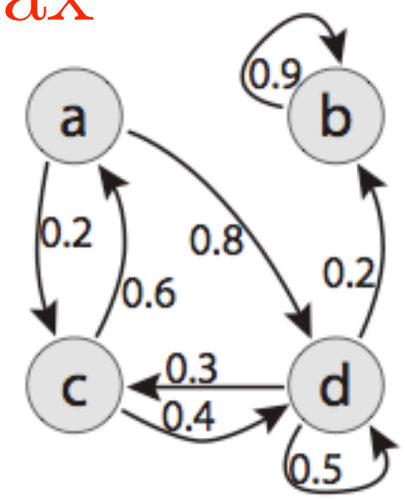

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$\oplus \rightarrow +$

$\otimes \rightarrow \cdot$

PRISM

Fig. 4. A probabilistic graph and the corresponding initial database. With stopping probabilities made explicit, this would encode a Markov model.

[Figures: Cohen et al, ICLP 08]

Semantics

- Semiring *
- valuation function maps provable ground terms to values
- extended to expressions, e.g. $[\![x * y]\!] \stackrel{\text{def}}{=} [\![x]\!] * [\![y]\!]$
- weighted rules $r \oplus_r = E$ constrain evaluation

$$[\![r]\!] = [\![E_1]\!] \oplus_r [\![E_2]\!] \oplus_r \ldots$$

* An algebraic semiring consists of five elements $\langle \mathbb{K}, \oplus, \otimes, \mathbf{0}, \mathbf{1} \rangle$, where $\mathbb{K}$ is a domain closed under $\oplus$ and $\otimes$, $\oplus$ is a binary, associative, commutative operator, $\otimes$ is a binary, associative operator that distributes over $\oplus$, $\mathbf{0} \in \mathbb{K}$ is the $\oplus$-identity, and $\mathbf{1} \in \mathbb{K}$ is the $\otimes$-identity.

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grounding of a rule body

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Roadmap

- Modeling
- Reasoning
- Language extensions
- Advanced topics

... with some detours on the way

Advanced Topics

- parameter estimation
- complexity of querying
- lifted graphical models and KBMC

Parameter Learning

Parameter Learning

e.g., webpage classification model

for each ***CLASS1***, ***CLASS2*** and each ***WORD***

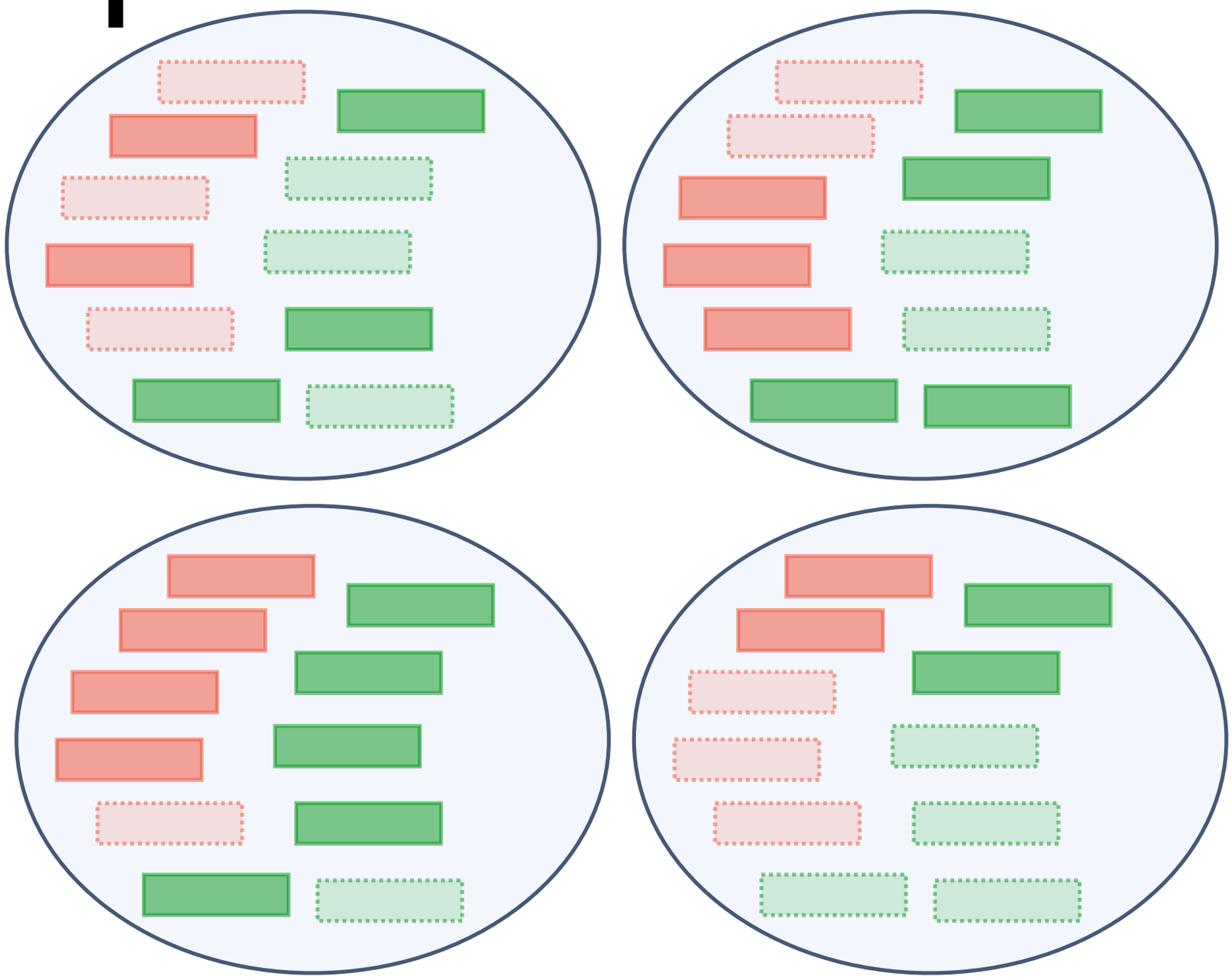
?? :: link_class(Source,Target,***CLASS1***,***CLASS2***).
?? :: word_class(***WORD***,***CLASS***).

```
class(Page,C)    :-  has_word(Page,W), word_class(W,C).

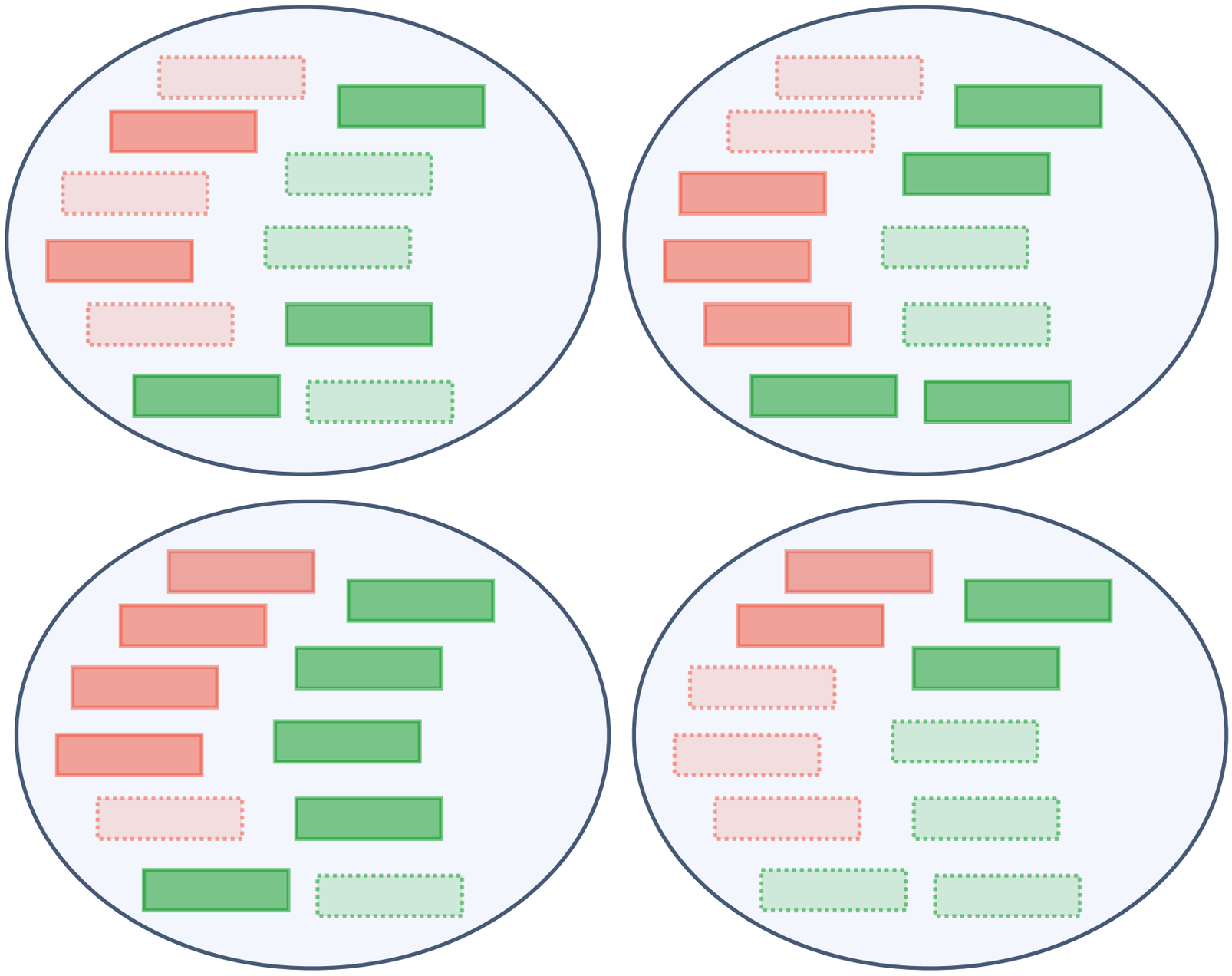
class(Page,C)    :-  links_to(OtherPage,Page),
                     class(OtherPage,OtherClass),
                     link_class(OtherPage,Page,OtherClass,C).
```

Sampling Interpretations

Sampling Interpretations



Parameter Estimation



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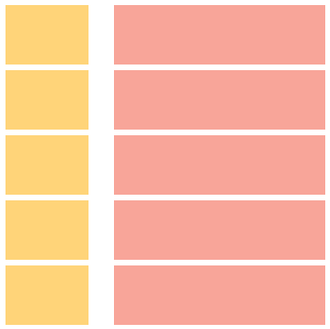
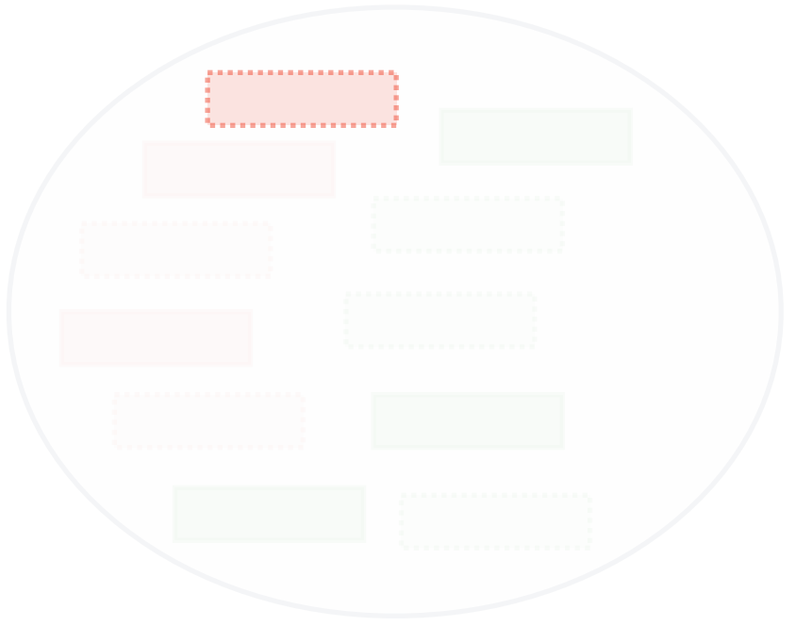
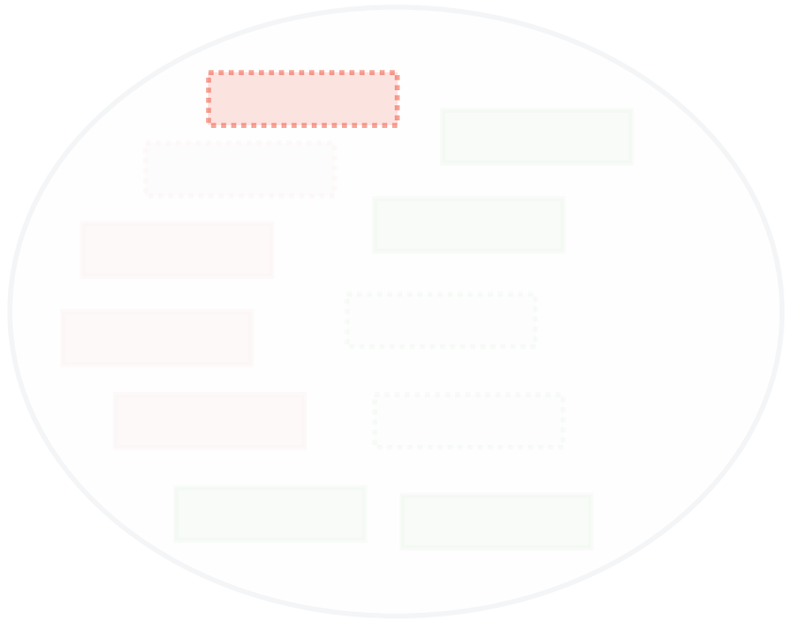
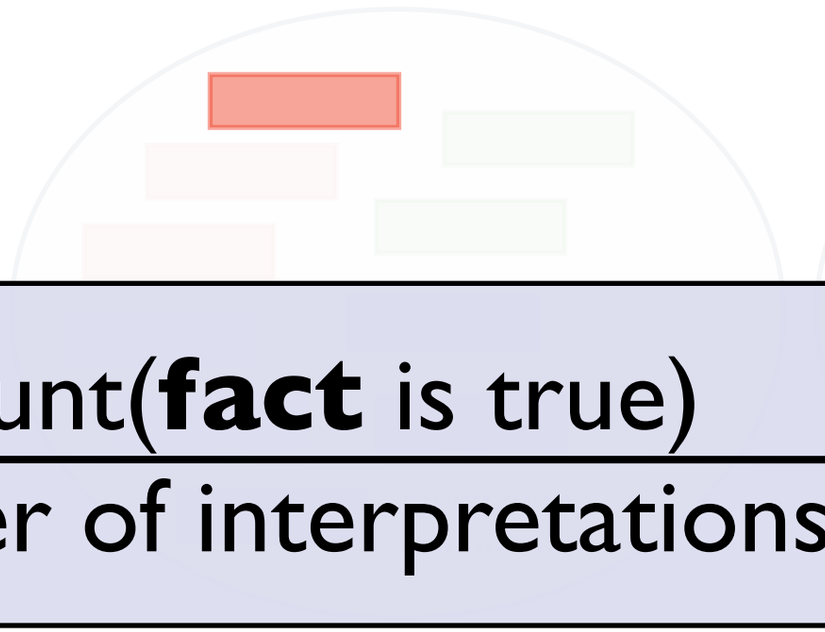


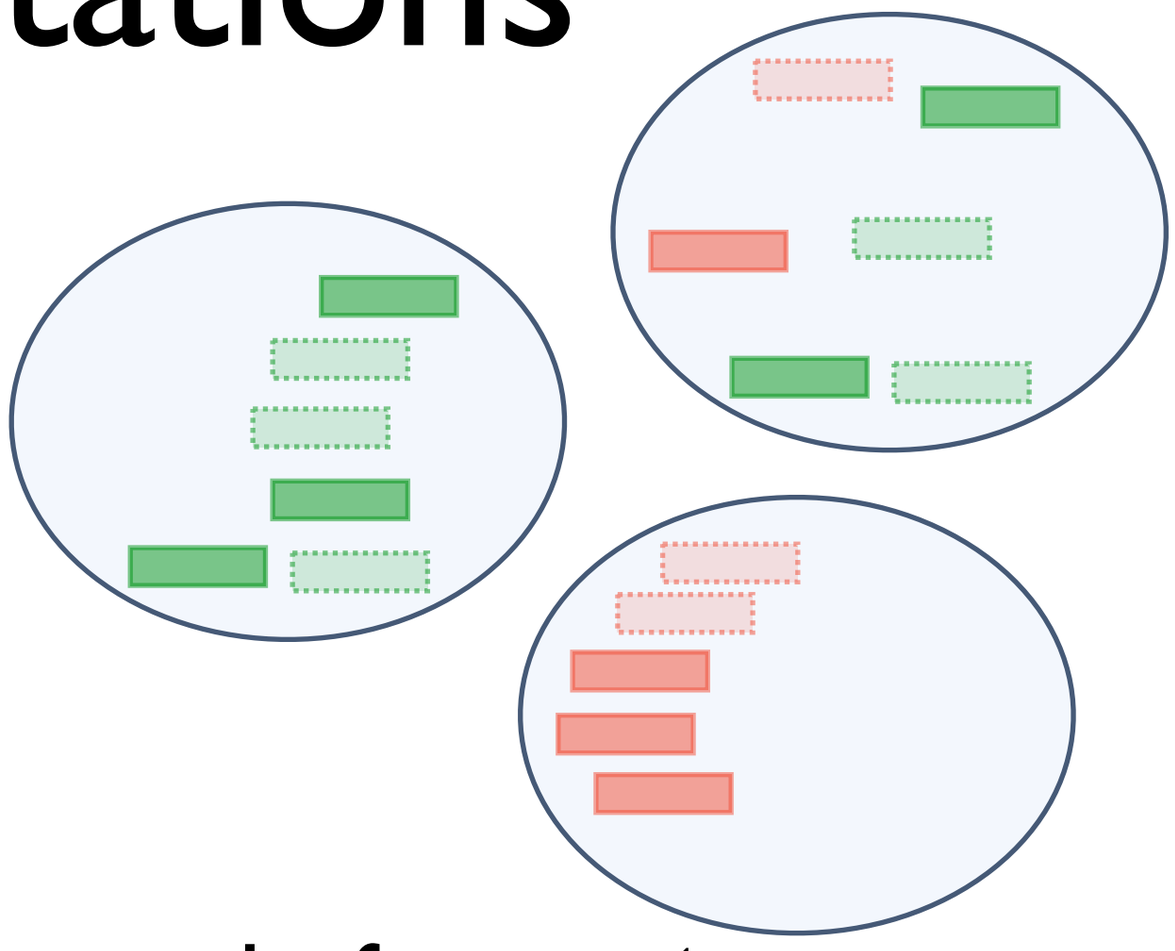






$$p(\textbf{fact}) = \frac{\text{count}(\textbf{fact} \text{ is true})}{\text{Number of interpretations}}$$

Learning from partial interpretations



- Not all facts observed
- Soft-EM
- use **expected count** instead of **count**
- **P(Q |E) -- conditional queries !**

 [Gutmann et al, ECML 11; Fierens et al, TPLP 14]

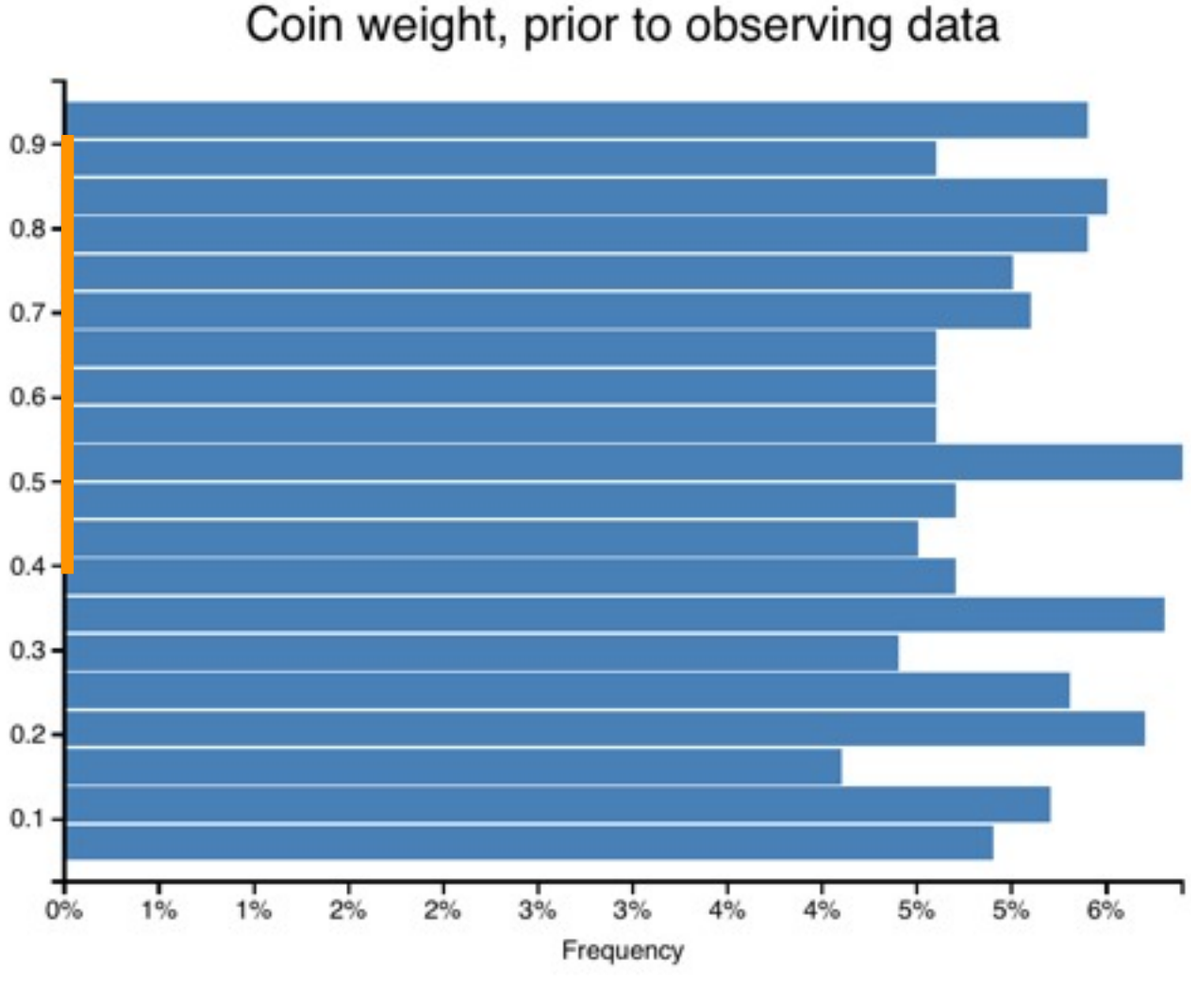
Bayesian Parameter Learning

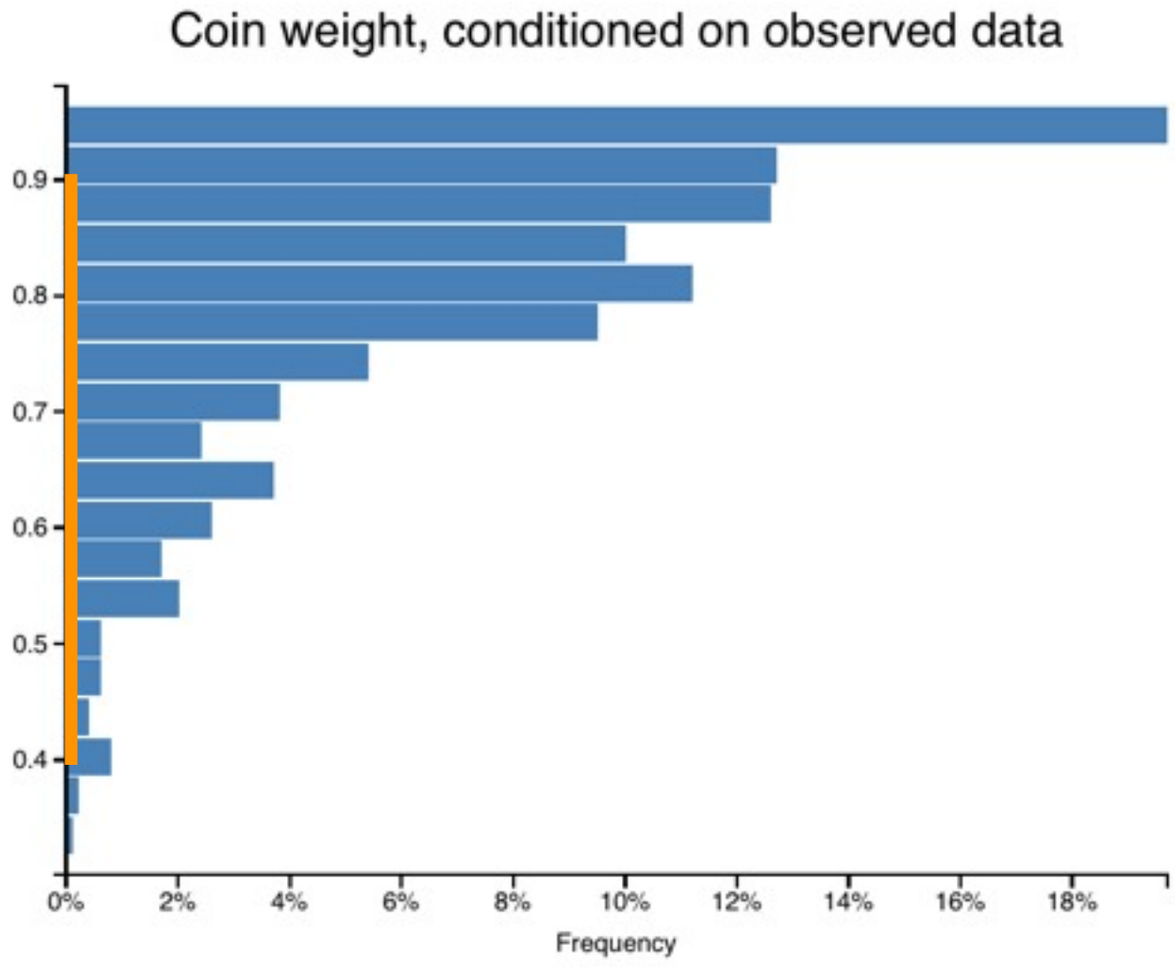
- Learning as inference (e.g., Church)
- Prior distributions for parameters
- Given data, find most likely parameter values

Example

[from probmods.org]

- Flipping a coin with unknown weight
- Prior: uniform distribution on [0,1]
- Observation: 5x heads in a row
- Sampling from Church model:





ProbLog Example

prior

```
0.05::weight(C,0.1); 0.2::weight(C,0.3); 0.5::weight(C,0.5);
                  0.2::weight(C,0.7); 0.05::weight(C,0.9) <- coin(C).
```

```
Param::toss(_,Param,_).
heads(C,R) :- weight(C,Param),toss(C,Param,R).
tails(C,R) :- weight(C,Param),\+toss(C,Param,R).

data(C,[]).
data(C,[h|R]) :- heads(C,R), data(C,R).
data(C,[t|R]) :- tails(C,R), data(C,R).
```

```
coin(c1).
coin(c2).
param(0.1).
param(0.3).
param(0.5).
param(0.7).
param(0.9).
```

```
query(weight(C,X)) :- coin(C),param(X).
```

ask for posterior

```
evidence(data(c1,[h,h,h,h,h,h,h,h,h,h,h,h,h,h]),true).
evidence(data(c2,[h,t,h,h,h,h,h,t,t,h,t,t,h]),true).
```

data

ProbLog Example

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0.05::weight(C,0.1); 0.2::weight(C,0.3); 0.5::weight(C,0.5);
                     0.2::weight(C,0.7); 0.05::weight(C,0.9) <- coin(C).
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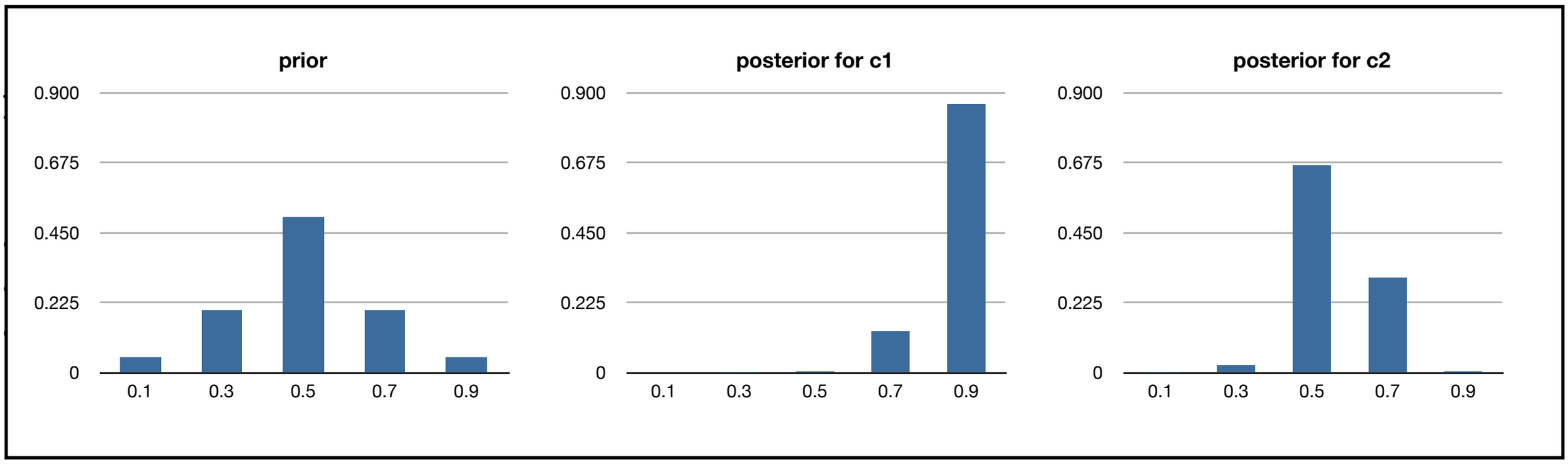


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query(weight(C,X)) :- coin(C),param(X).

evidence(data(c1,[h,h,h,h,h,h,h,h,h,h,h,h,h,h]),true).
evidence(data(c2,[h,t,h,h,h,h,h,t,t,h,t,t,h]),true).
```

Complexity of Querying

Complexity of querying

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.96x0.99=0.95

0.9x0.93=0.83

0.87x0.93=0.80

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

Complexity of querying

ProducesProduct

Company	Product	P
...	...	...

HeadquarteredIn

Company	City	P
...	...	...

```
select distinct x.Product, x.Company,
                x.P * y.P as P
from ProducesProduct x, HeadquarteredIn y
where x.Company = y.Company
      and y.City = 'san_jose'
order by P desc
```

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.96x0.99=0.95

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Product	Company	P
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egyptair	cairo	0.93
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...	...	...

```
result(Product,Company) :-
   producesProduct(Company,Product),
   headquarteredIn(Company,san_jose).
query(result(_,_)).
```

Complexity of querying

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sony	walkman	0.96
microsoft	mac_os_x	0.96
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microsoft	mac_os	0.9
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```
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query(result(_,_)).
```

each ground query has a **single proof**

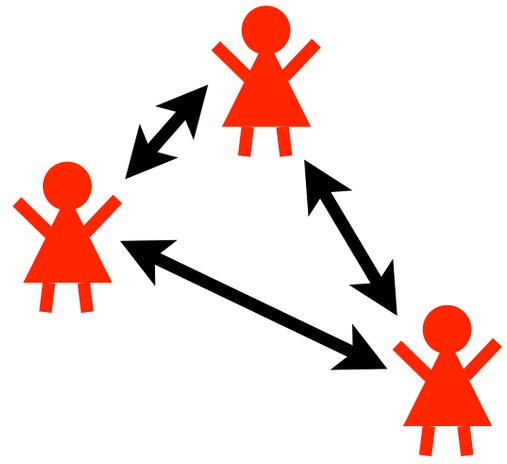
→ no disjoint-sum-problem,

easy evaluation

Complexity of querying in probabilistic databases

- queries have fixed size (no recursion)
- size of query << size of database
- complexity of evaluating given query measured in size of database (data complexity)
- previous example: polynomial
  (as all standard relational database queries)

How hard is evaluating q?



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```

```
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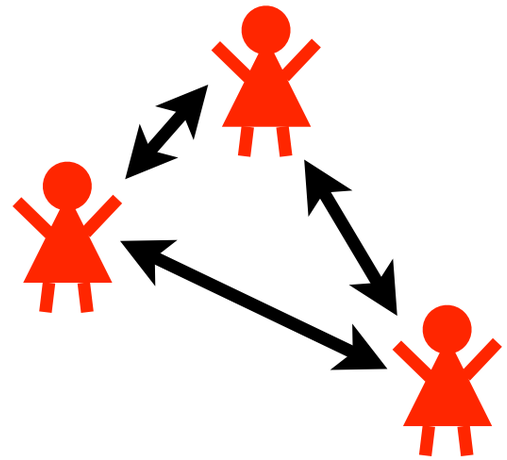

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```

proofs

```
s(1),i(1,2)

s(1),i(1,3)

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s(3),i(3,1)

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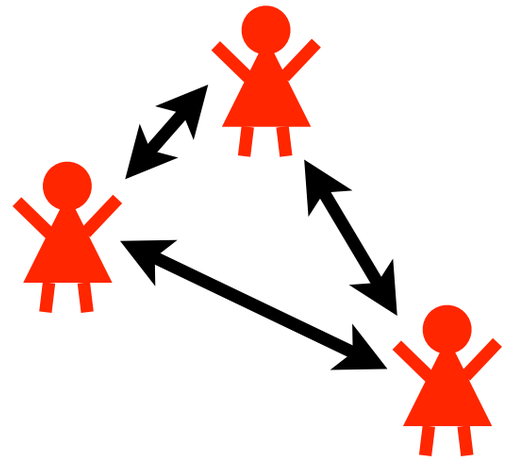

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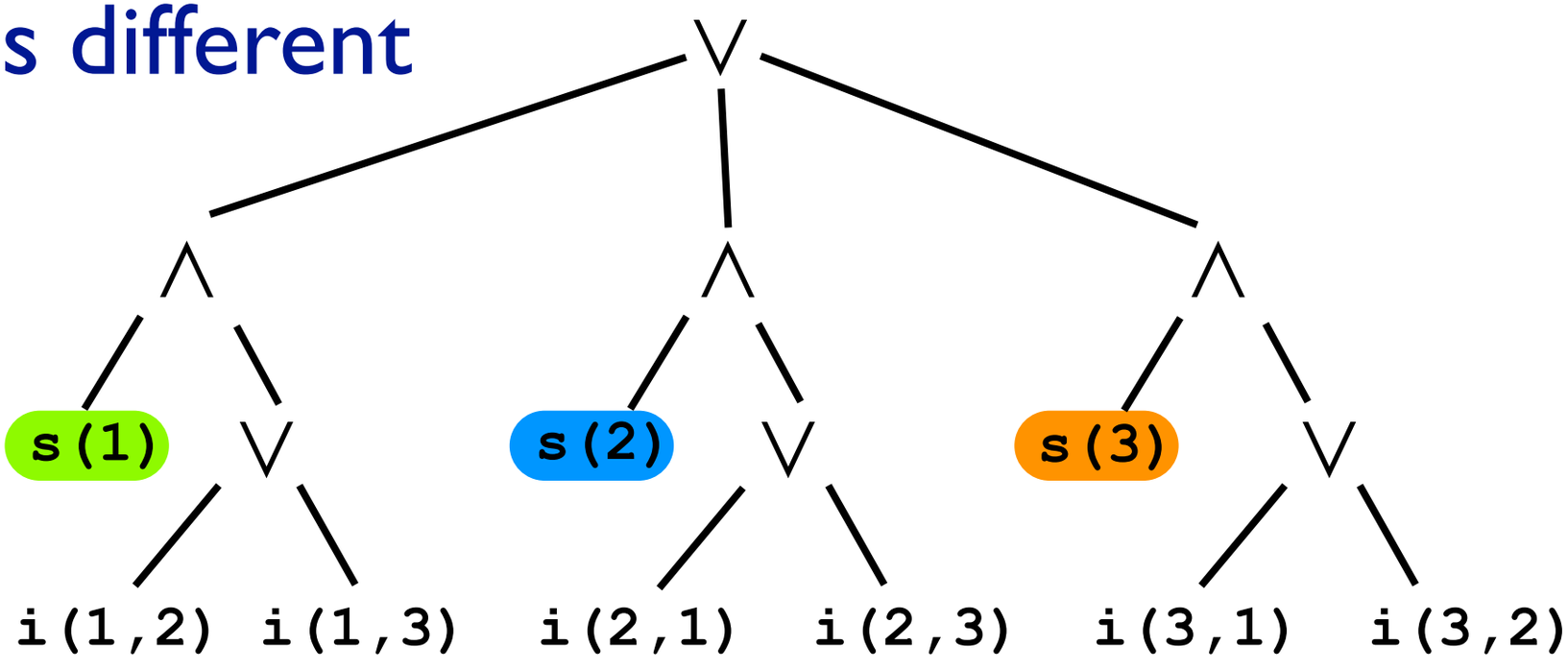
tree structure, all leaves different



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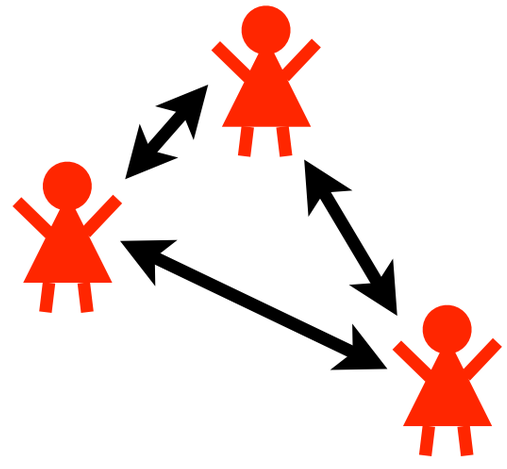

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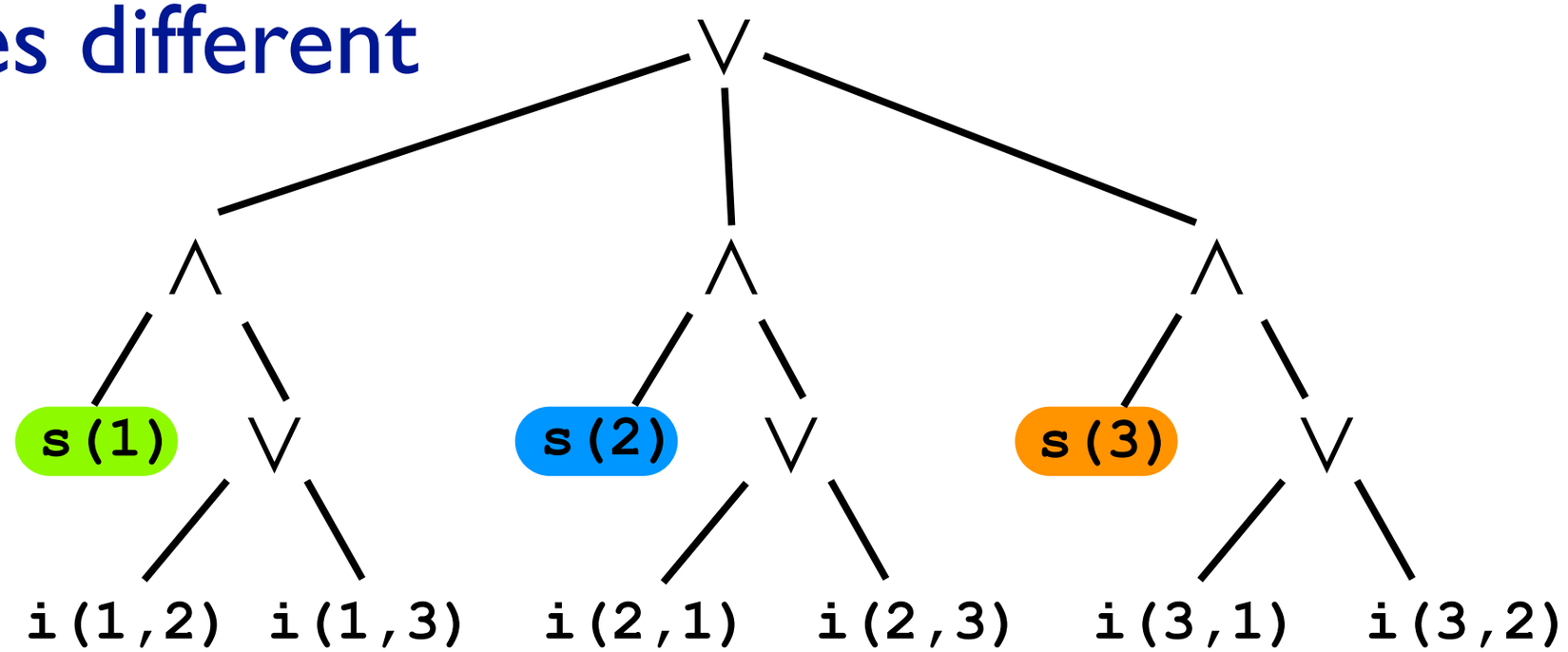
tree structure, all leaves different

$$P(\varphi_1 \wedge \varphi_2) = P(\varphi_1) \cdot P(\varphi_2)$$
$$P(\varphi_1 \vee \varphi_2) = 1 - (1 - P(\varphi_1)) \cdot (1 - P(\varphi_2))$$



$$P(\mathtt{q}) = 1 - \prod_{j=1..n} (1 - \left(P(\mathtt{s}(\mathtt{X_j})) (1 - \prod_{k=1..n, k \neq j} (1 - P(\mathtt{i}(\mathtt{X_j}, \mathtt{Y_k}))))\right))$$

polynomial in database size / number of persons n

proofs

- s(1),i(1,2)
- s(1),i(1,3)
- s(2),i(2,1)
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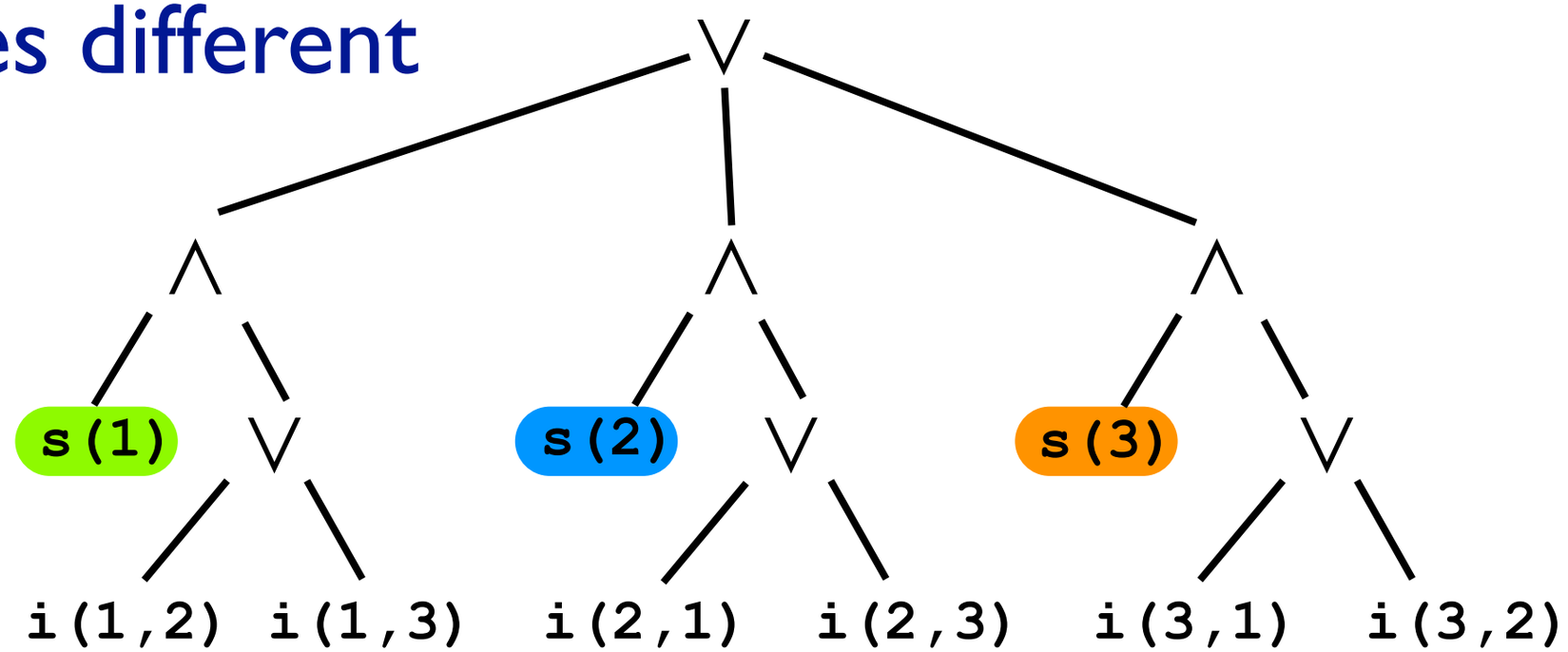

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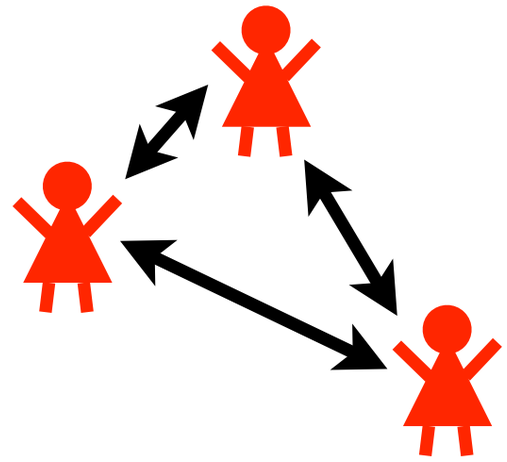

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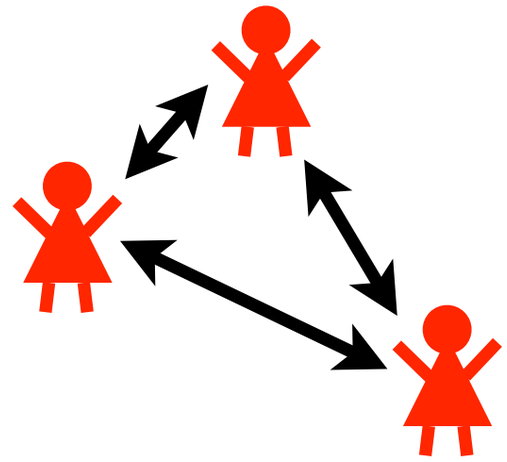

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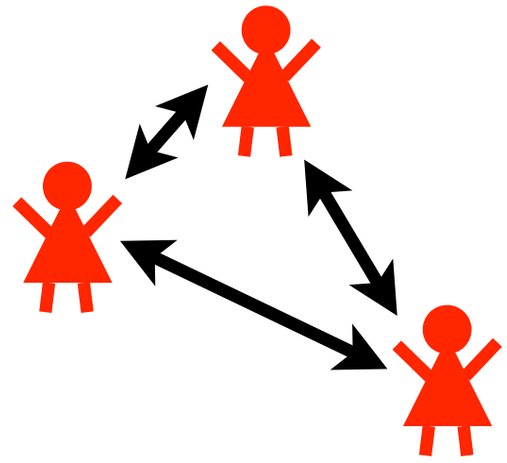

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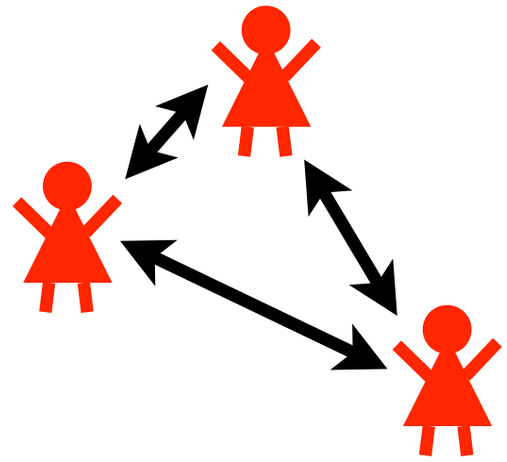

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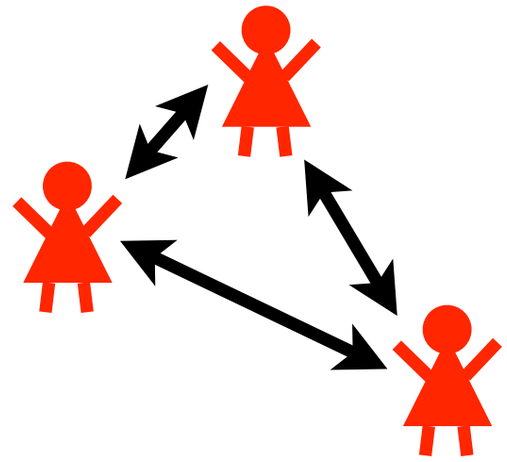

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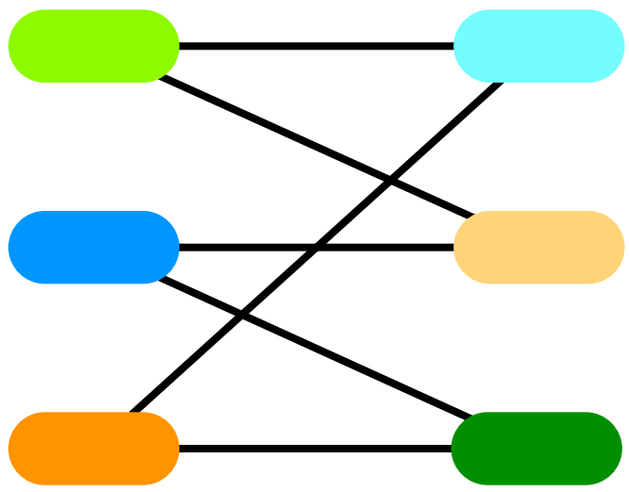

cannot build tree structure with all leaves different

Read-Once Formulas

- Propositional formulas that can be rewritten such that each variable occurs at most once (= as tree with all leaves different)

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no variable appears both pos & neg

$X \vee Y$ is unate

$(X \vee Y) \wedge (\neg X \vee Z)$ not

P4-free and normal

- represent DNF formula as graph:
  - a node for each variable
  - edge (X,Y) ↔ X,Y in same conjunct

P4-free and normal

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Our first example

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s(1),i(1,2)
s(1),i(1,3)
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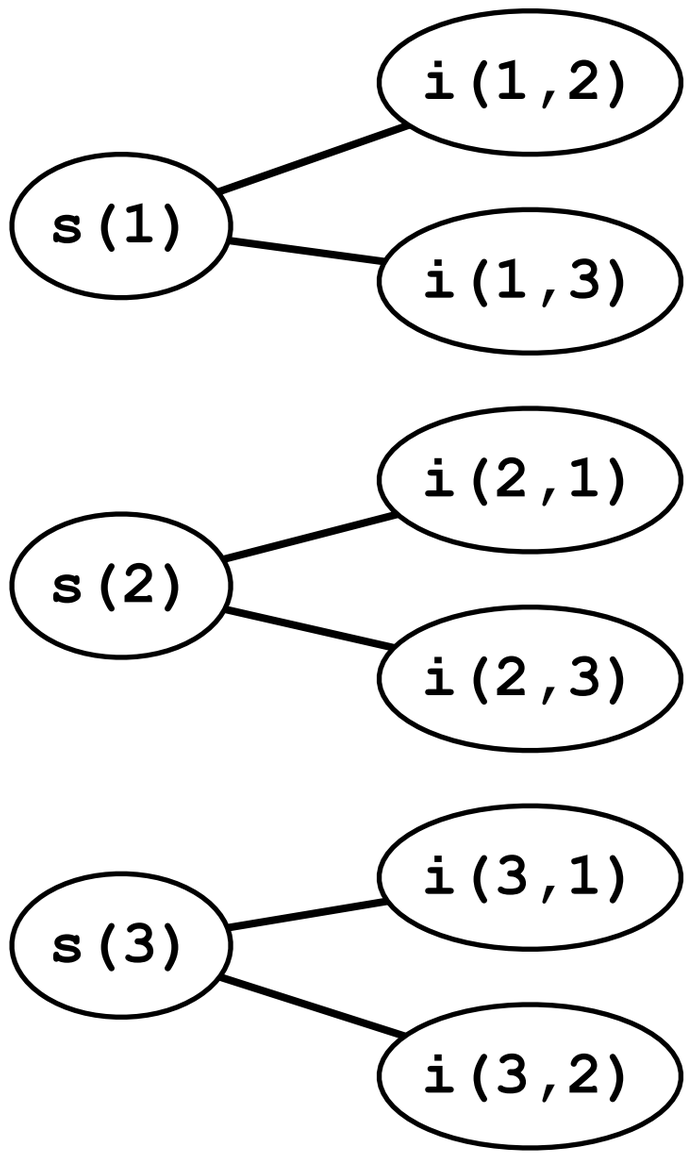

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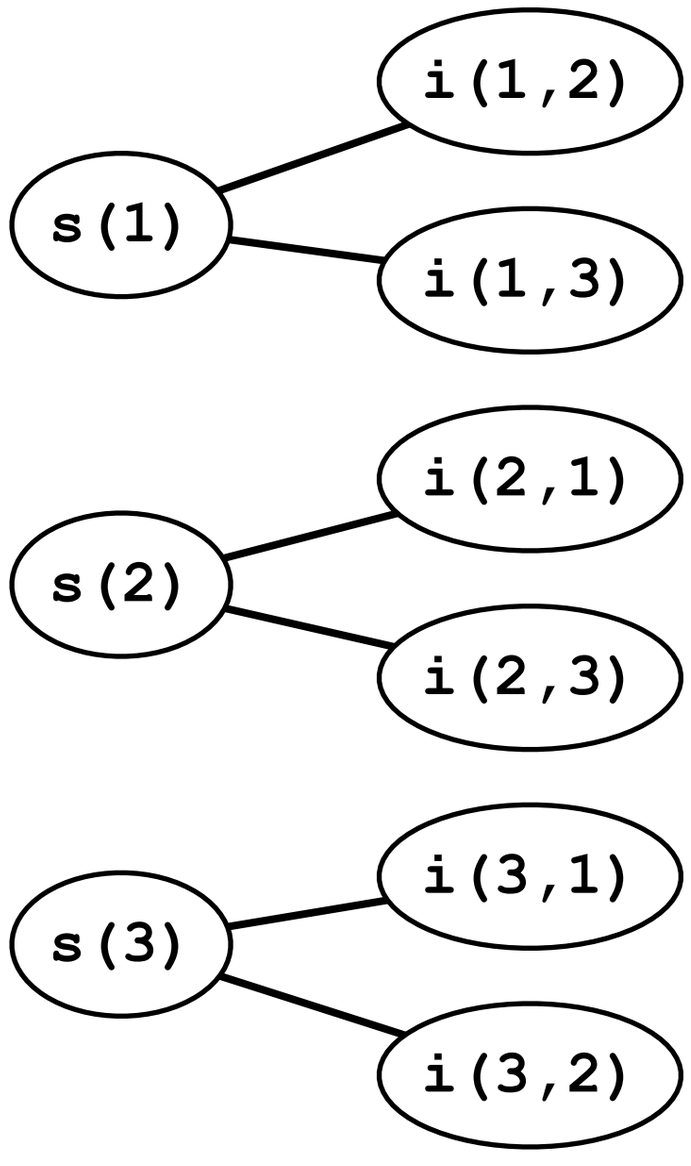
s(3),i(3,1)

s(3),i(3,2)



- **normal**: each clique is part of a conjunct ✓
- **P4-free**: no induced 4 node subgraph is a path

Our first example

s(1),i(1,2)

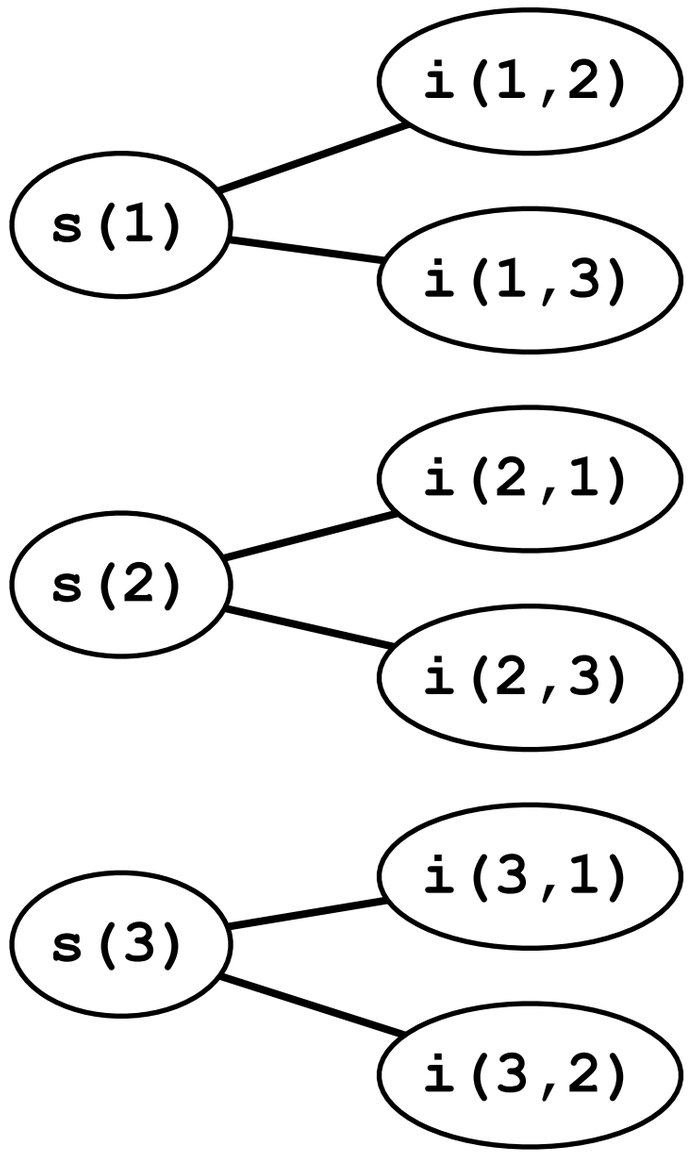
s(1),i(1,3)

s(2),i(2,1)

s(2),i(2,3)

s(3),i(3,1)

s(3),i(3,2)



- **normal**: each clique is part of a conjunct ✓
- **P4-free**: no induced 4 node subgraph is a path ✓

read-once

Our second example

```
s(1),i(1,2),m(2)
s(1),i(1,3),m(3)
s(2),i(2,1),m(1)
s(2),i(2,3),m(3)
s(3),i(3,1),m(1)
s(3),i(3,2),m(2)
```

- **normal**: each clique is part of a conjunct
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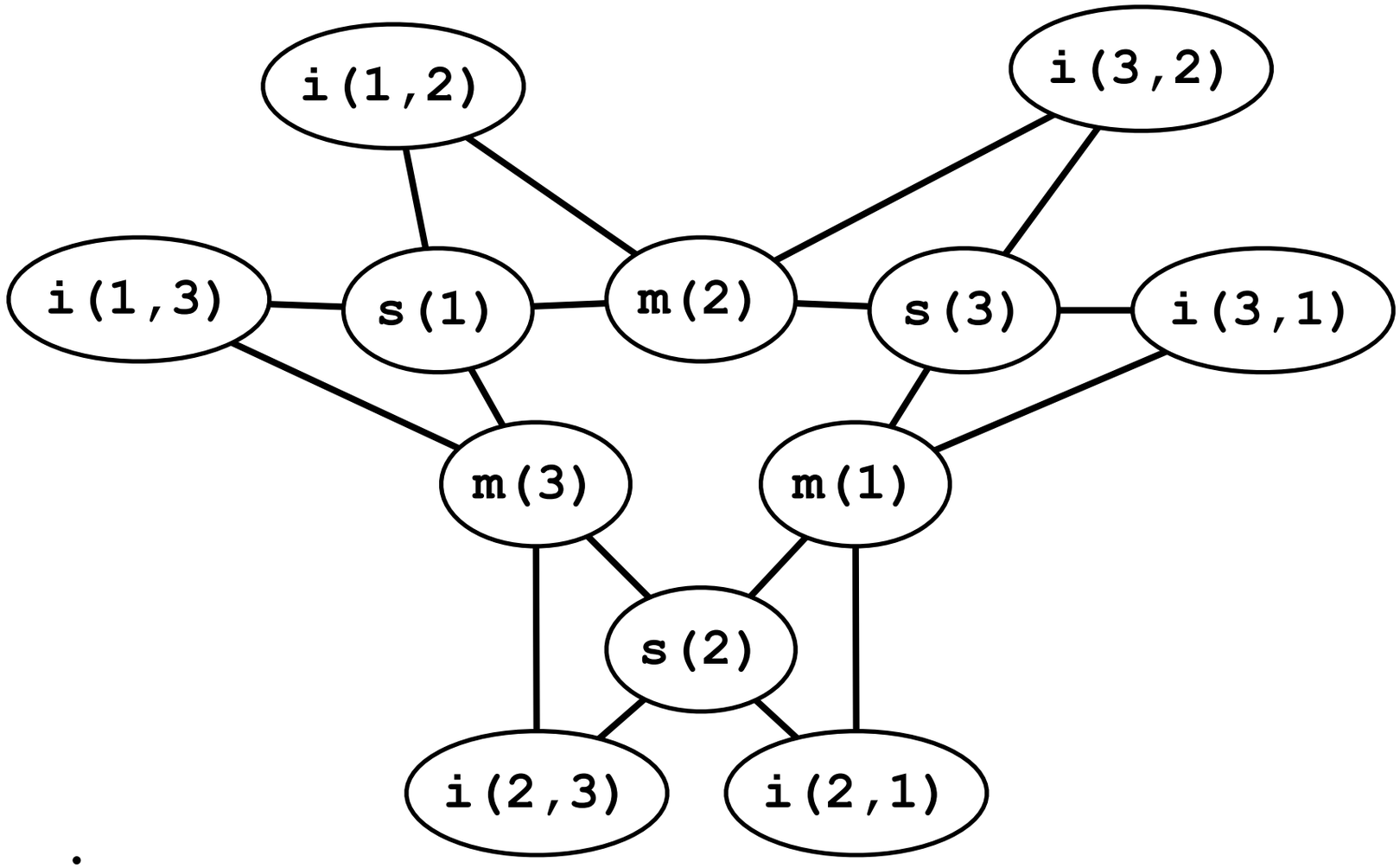
Our second example

```
s(1),i(1,2),m(2)
s(1),i(1,3),m(3)
s(2),i(2,1),m(1)
s(2),i(2,3),m(3)
s(3),i(3,1),m(1)
s(3),i(3,2),m(2)
```



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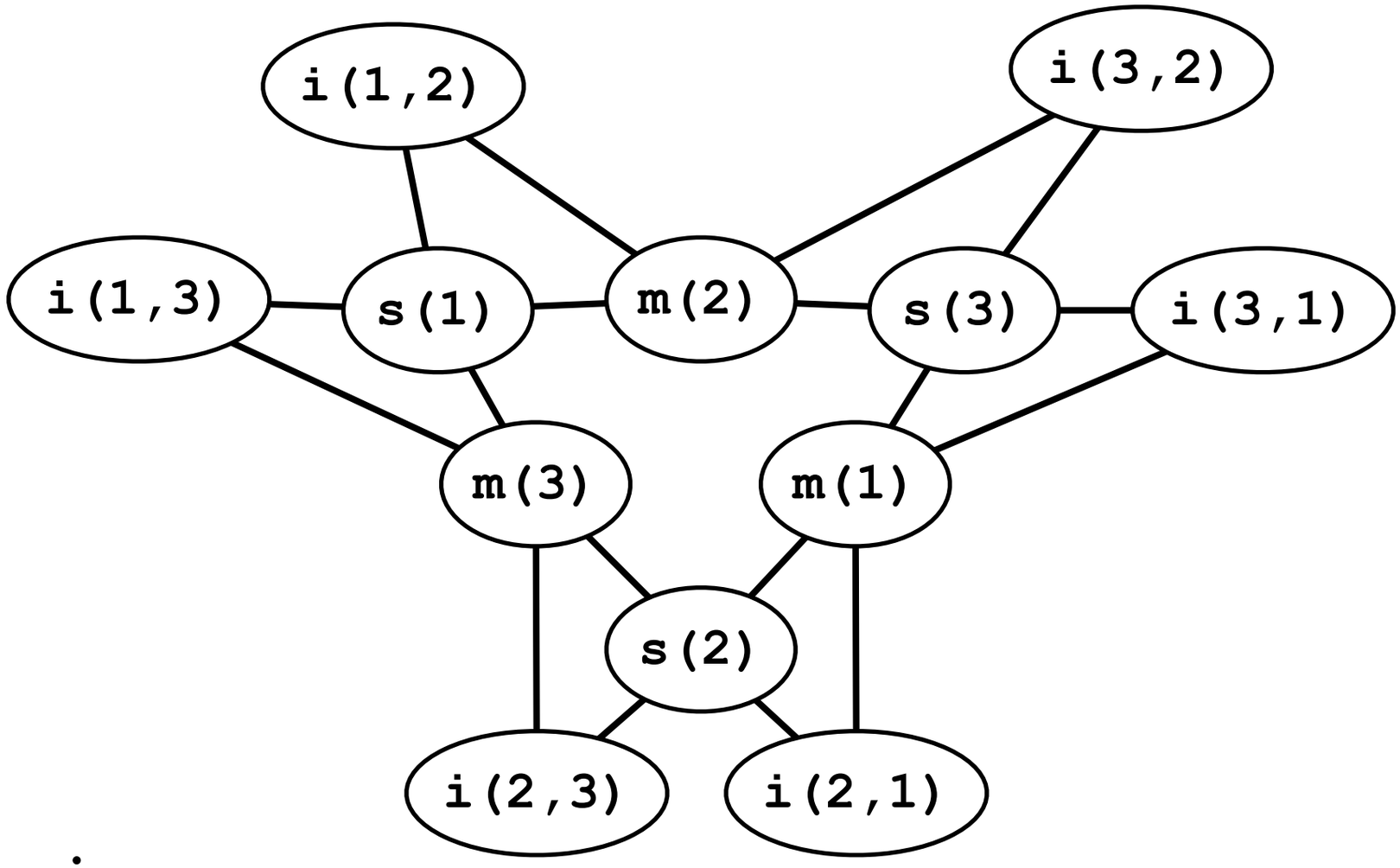
Our second example

```
s(1),i(1,2),m(2)
s(1),i(1,3),m(3)
s(2),i(2,1),m(1)
s(2),i(2,3),m(3)
s(3),i(3,1),m(1)
s(3),i(3,2),m(2)
```



- **normal**: each clique is part of a conjunct ✔
- **P4-free**: no induced 4 node subgraph is a path

Our second example

```
s(1),i(1,2),m(2)
s(1),i(1,3),m(3)
s(2),i(2,1),m(1)
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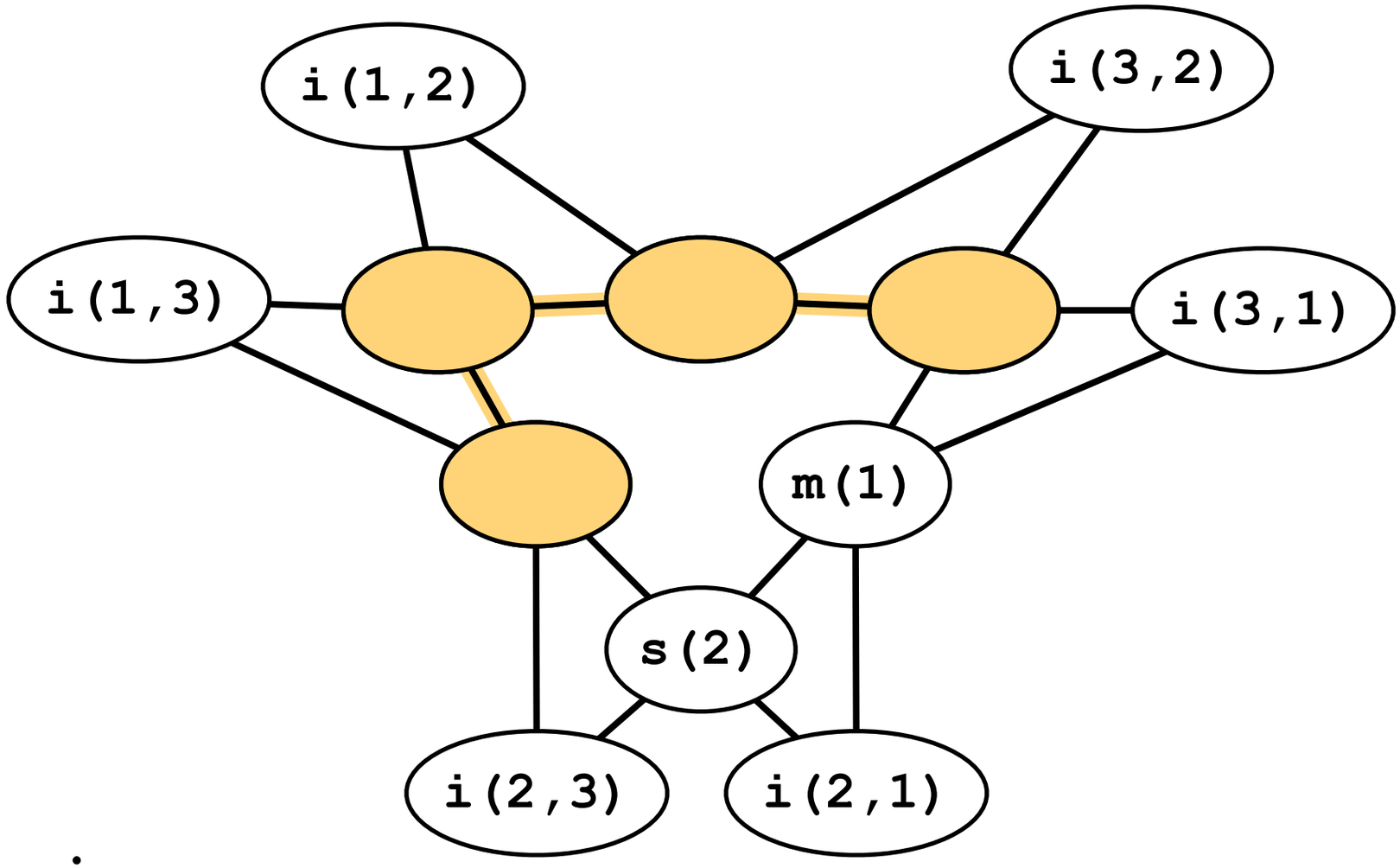
s(3),i(3,1),m(1)
s(3),i(3,2),m(2)
```



- **normal**: each clique is part of a conjunct ✔
- **P4-free**: no induced 4 node subgraph is a path ✗

not read-once
(and in fact known to be #P-hard, cf. PDB-book)

Dichotomy of UCQ Evaluation

- **U**nion of **C**onjunctive **Q**ueries
  ≈ Datalog without recursion and negation
- Theorem: UCQ evaluation is either polynomial in database size or #P-hard

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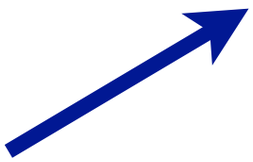
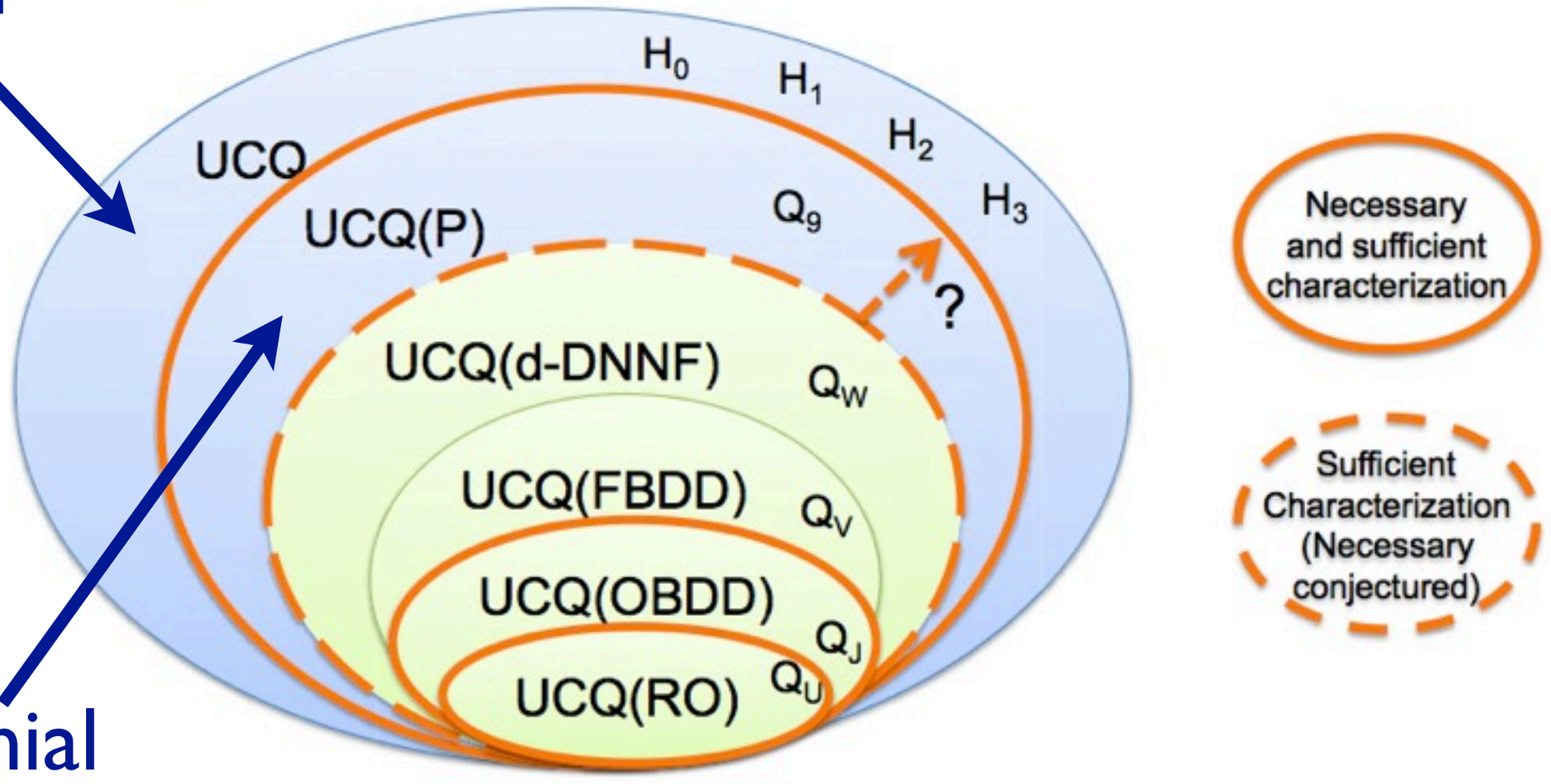


counting version of NP decision problems, e.g., model counting

#P-hard

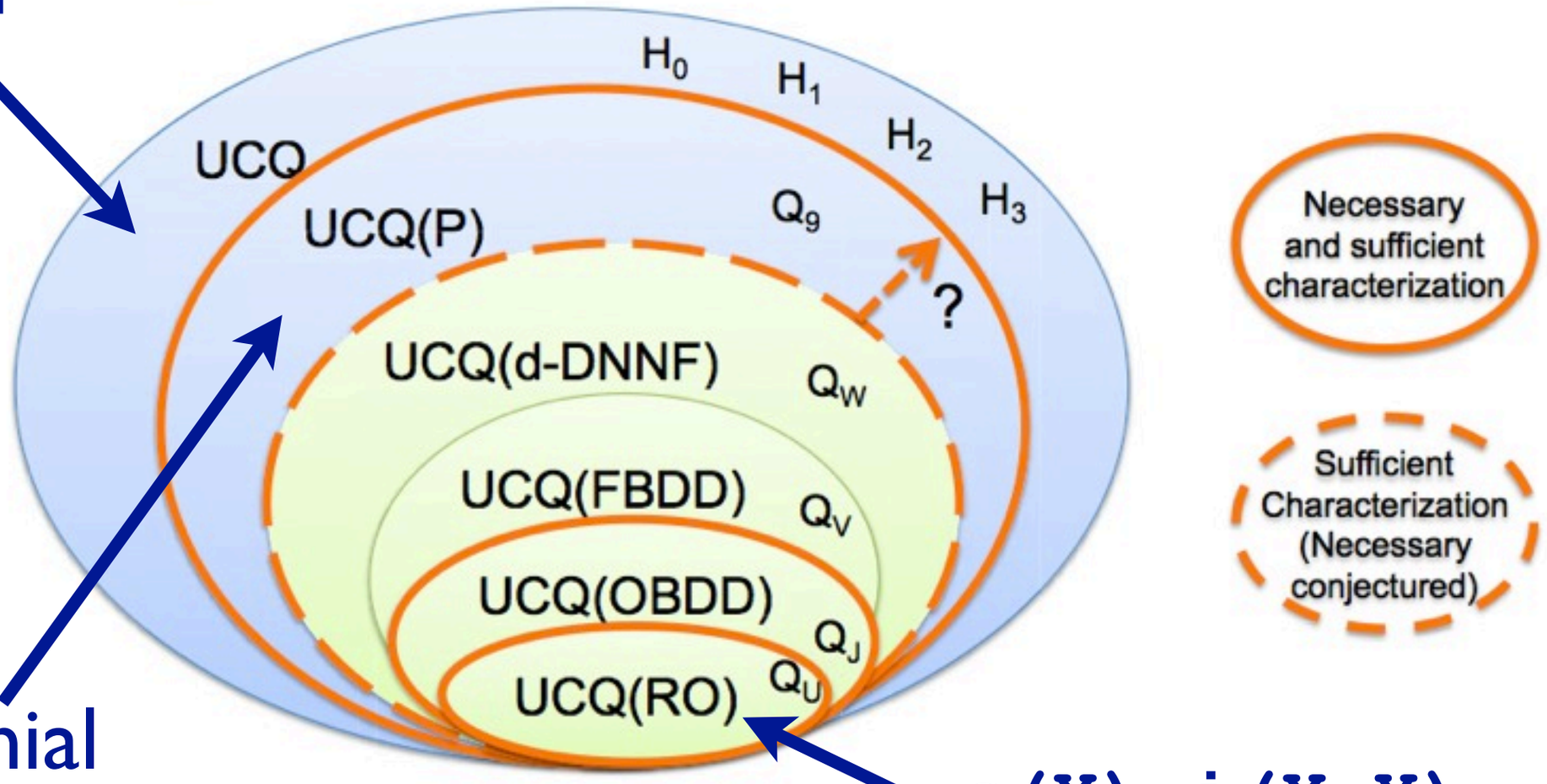


polynomial

Figure 5.4: The query compilation hierarchy for Unions of Conjunctive Queries (UCQ).

Fig. from [Suciu et al 2011]

#P-hard



polynomial

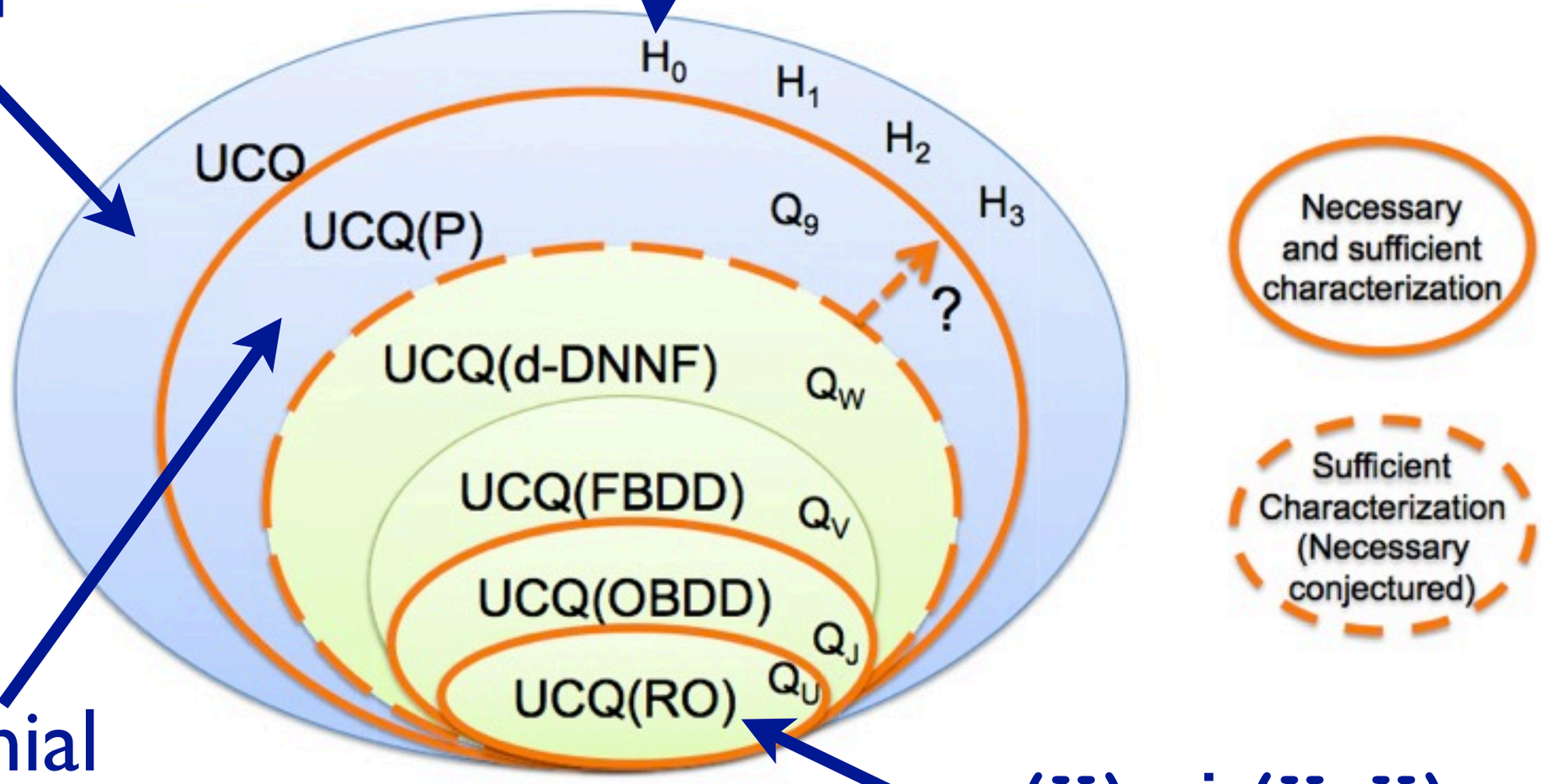
s(X),i(X,Y)

Figure 5.4: The query compilation hierarchy for Unions of Conjunctive Queries (UCQ).

Fig. from [Suciu et al 2011]

$H_0 = s(X), i(X,Y), m(Y)$

#P-hard



polynomial

$s(X), i(X,Y)$

Figure 5.4: The query compilation hierarchy for Unions of Conjunctive Queries (UCQ).

Fig. from [Suciu et al 2011]

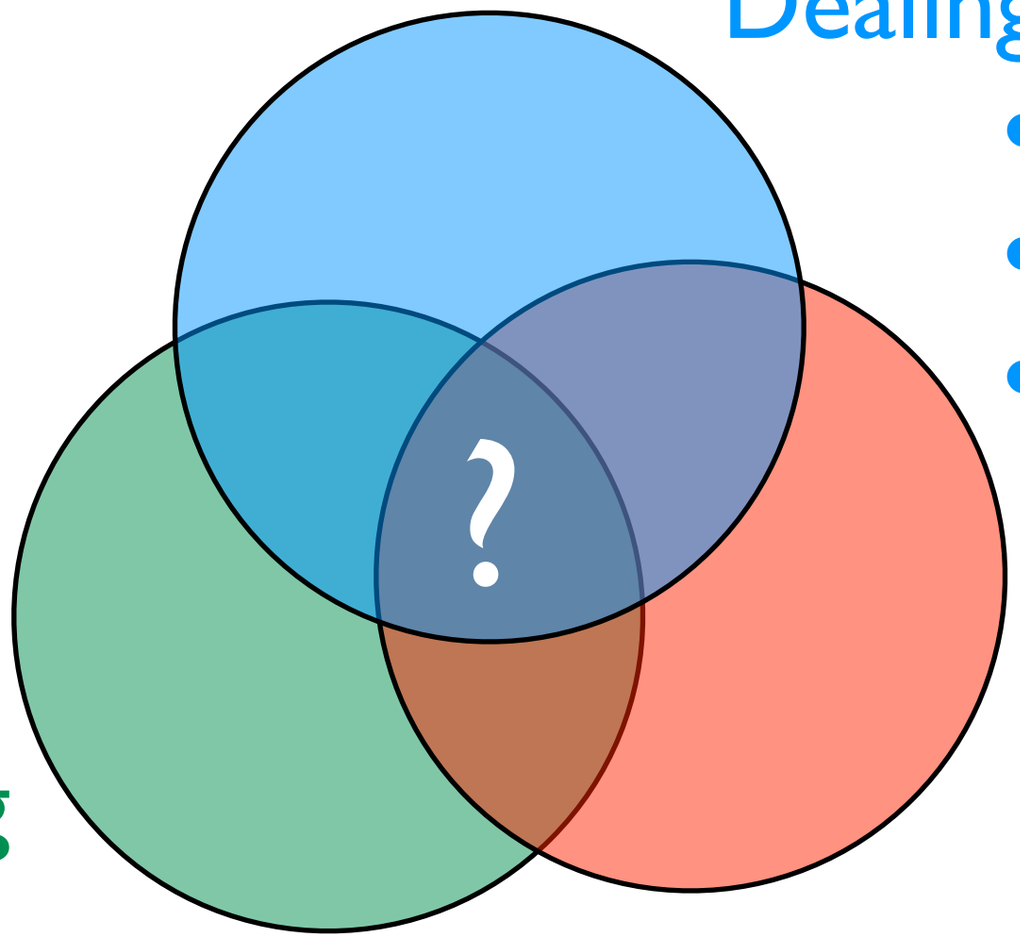
A key question in AI:

Dealing with uncertainty

- probability theory
- graphical models
- ...

Reasoning with relational data

- logic
- databases
- programming
- ...



Learning

- parameters
- structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

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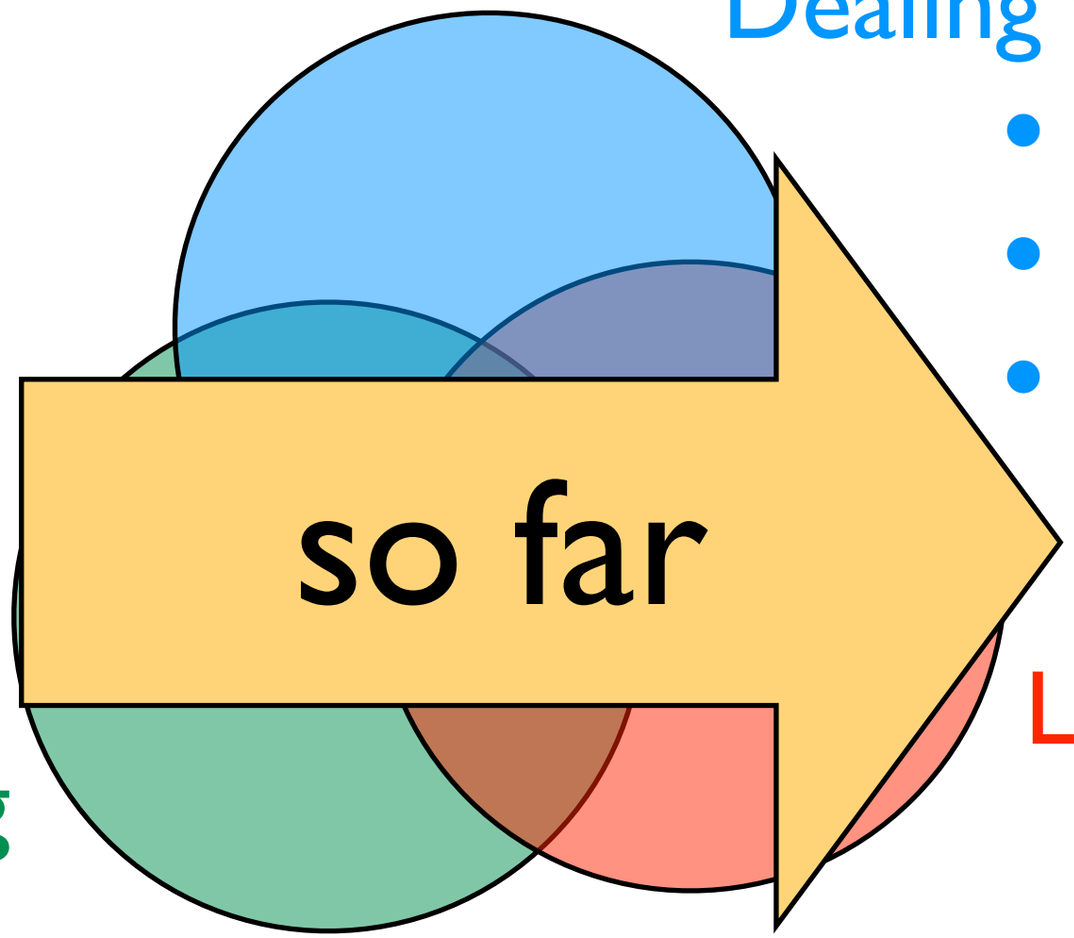

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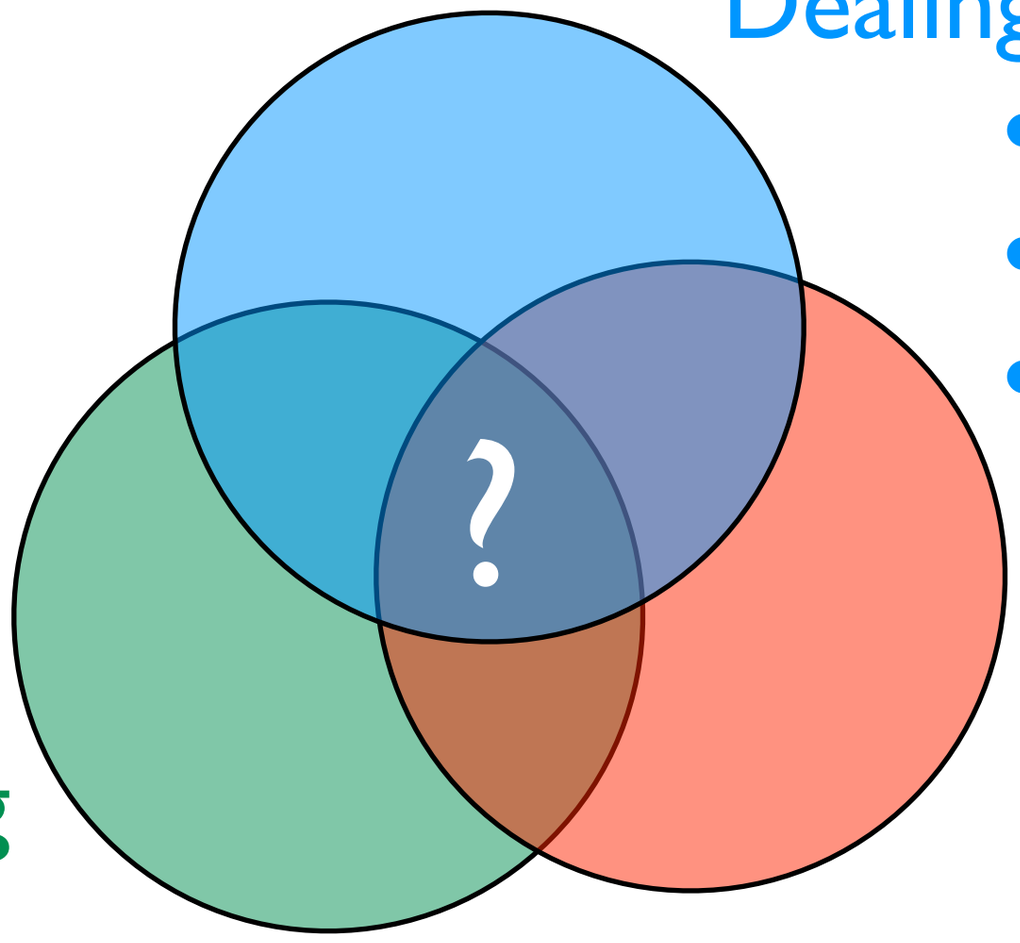

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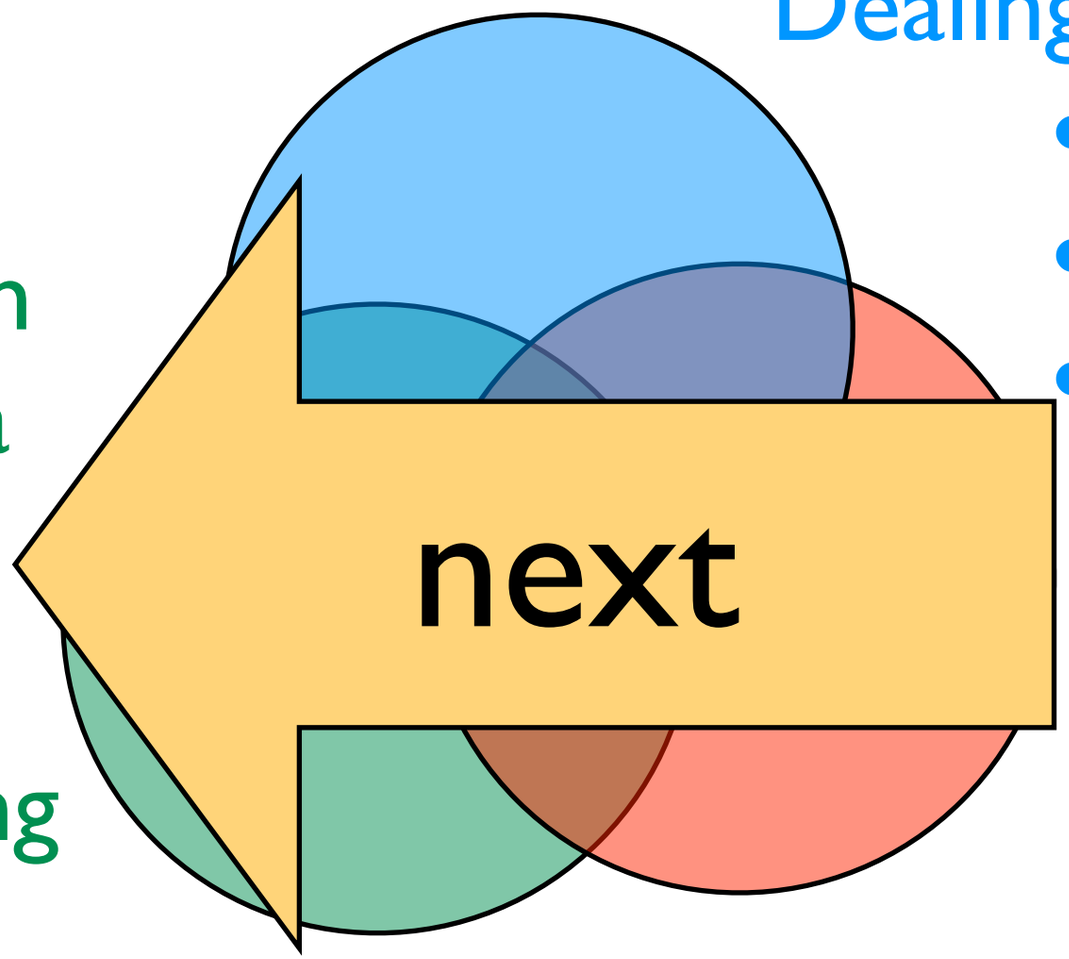


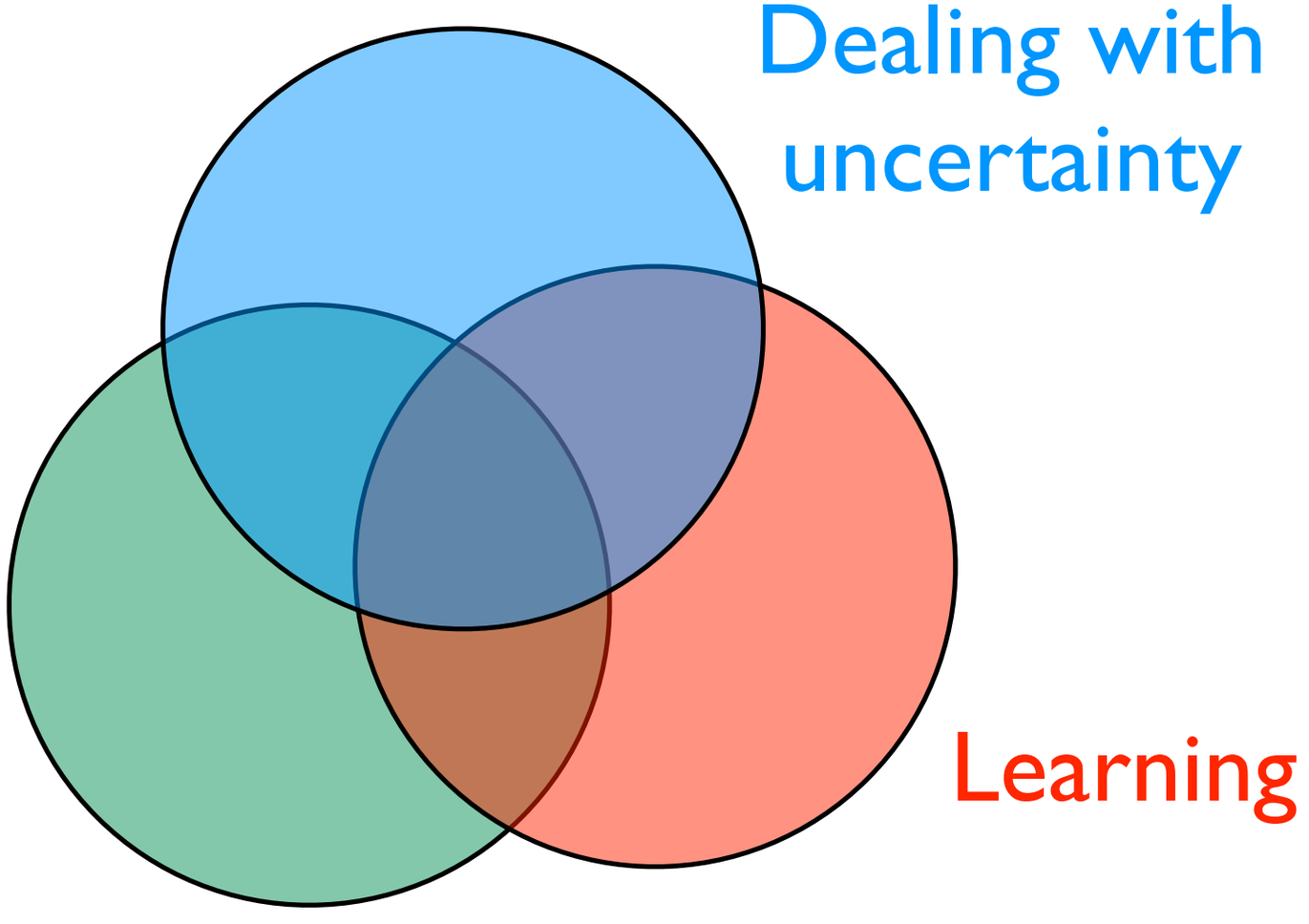
Learning

- parameters
- structure

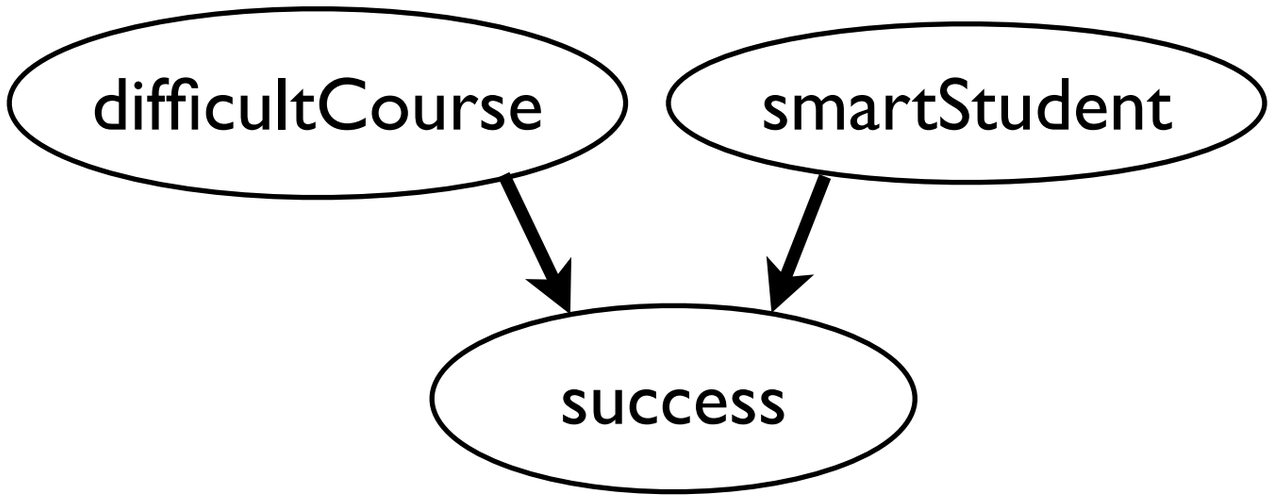
Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

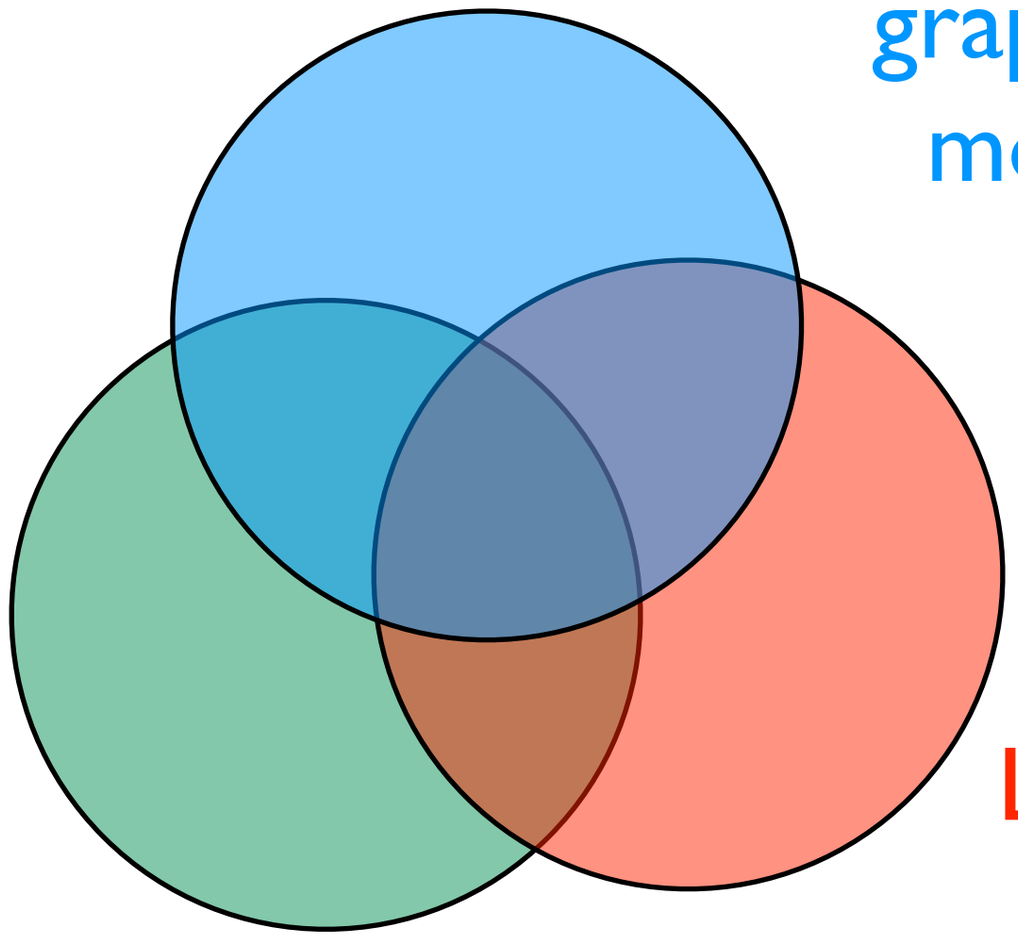
(lifted) graphical models

Lifted graphical models



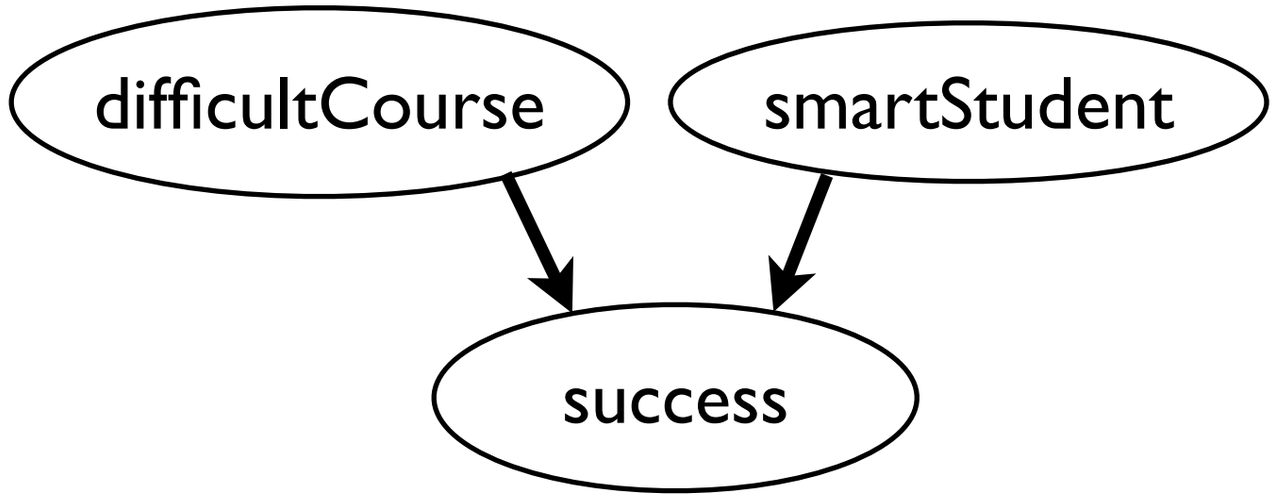
Lifted graphical models

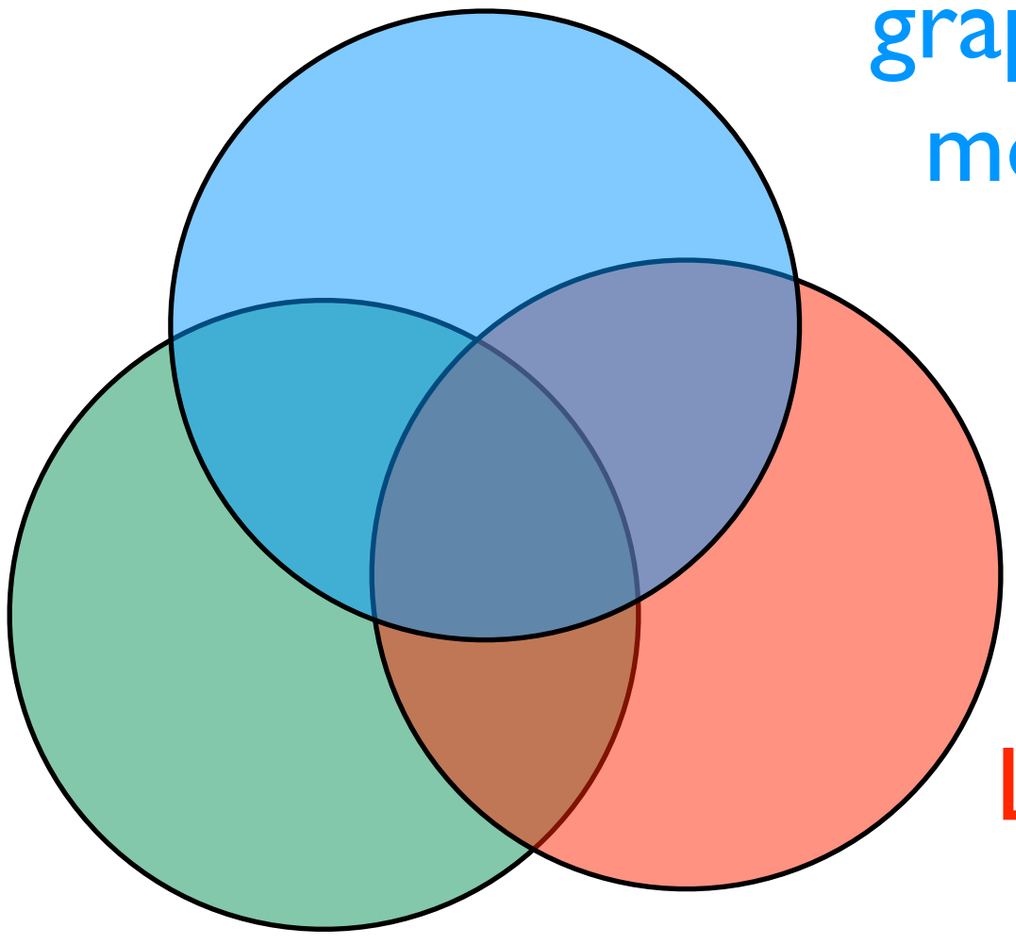




Lifted graphical models

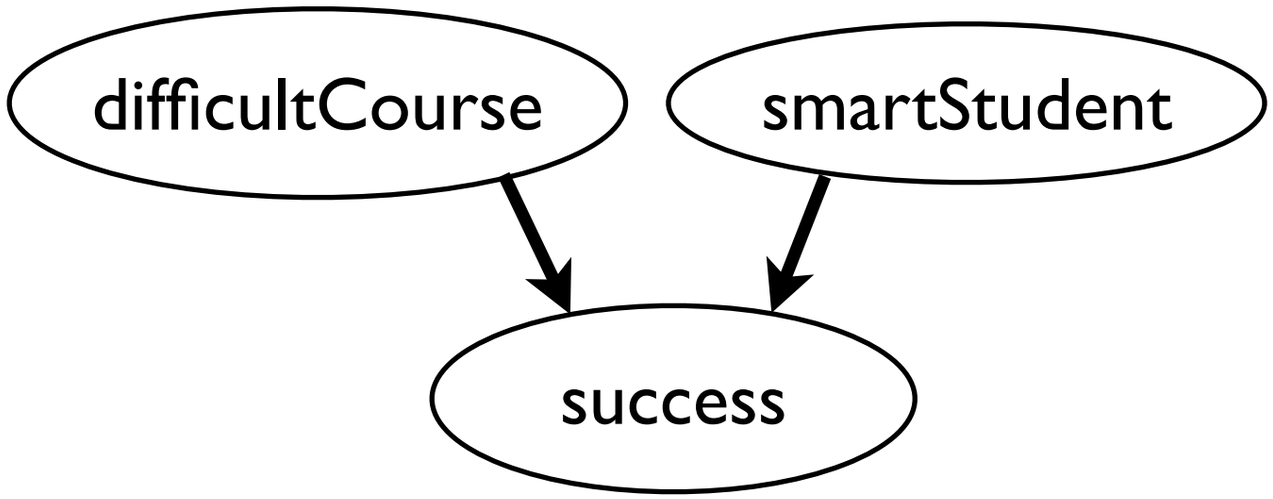
fixed set of random variables



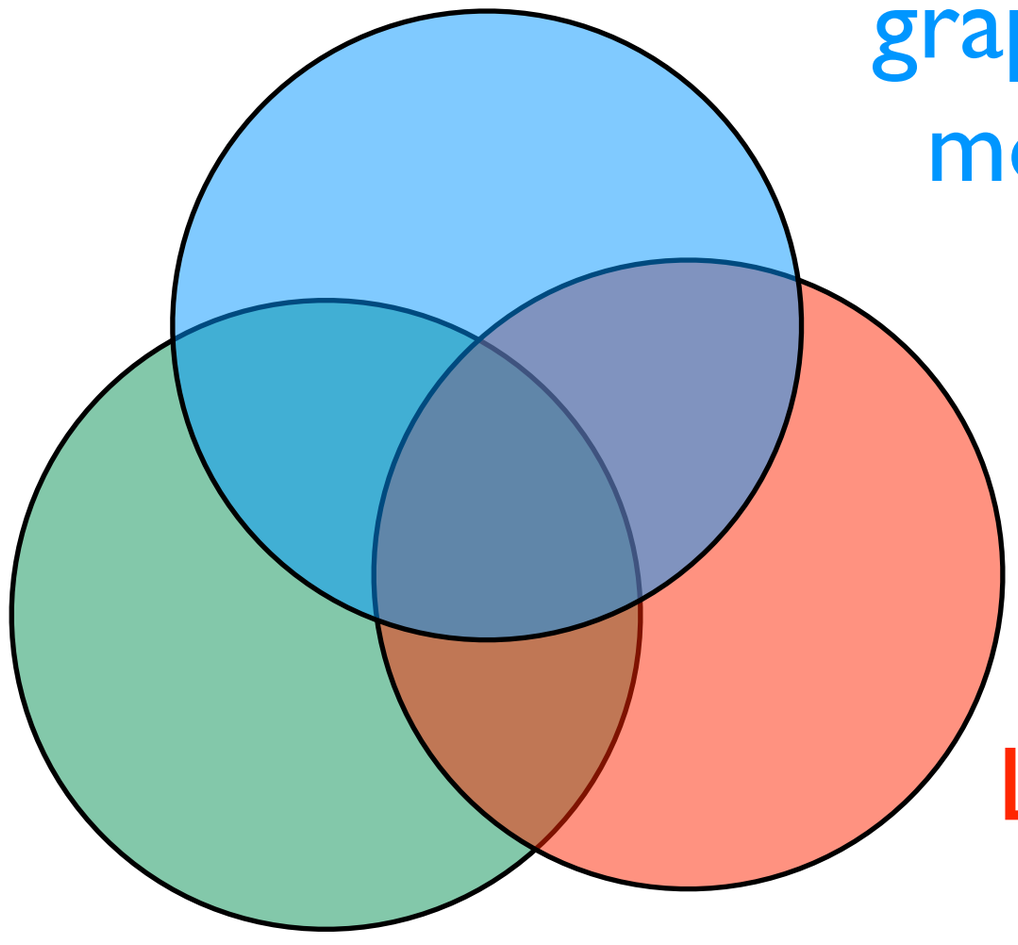


Lifted graphical models

fixed set of random variables

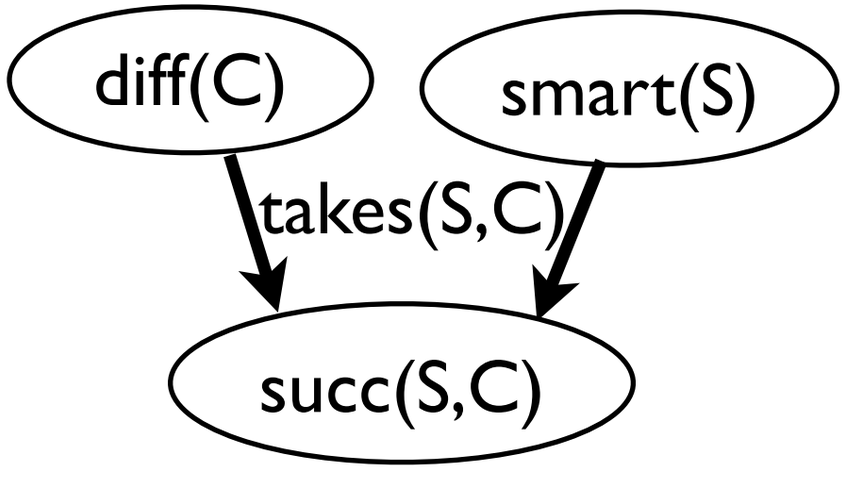


graphical model



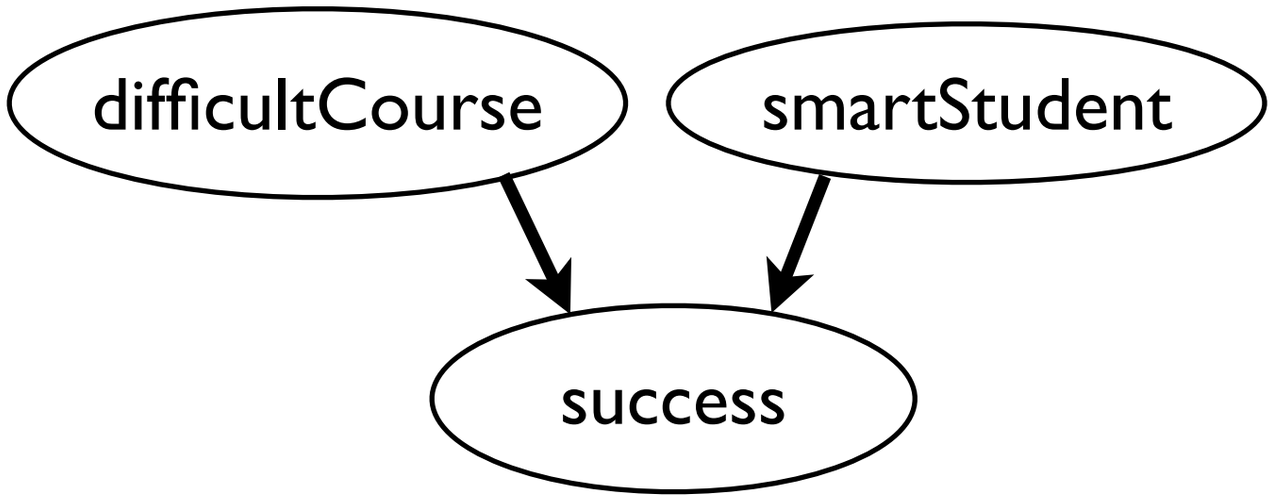
Learning

relational definition of graphical model

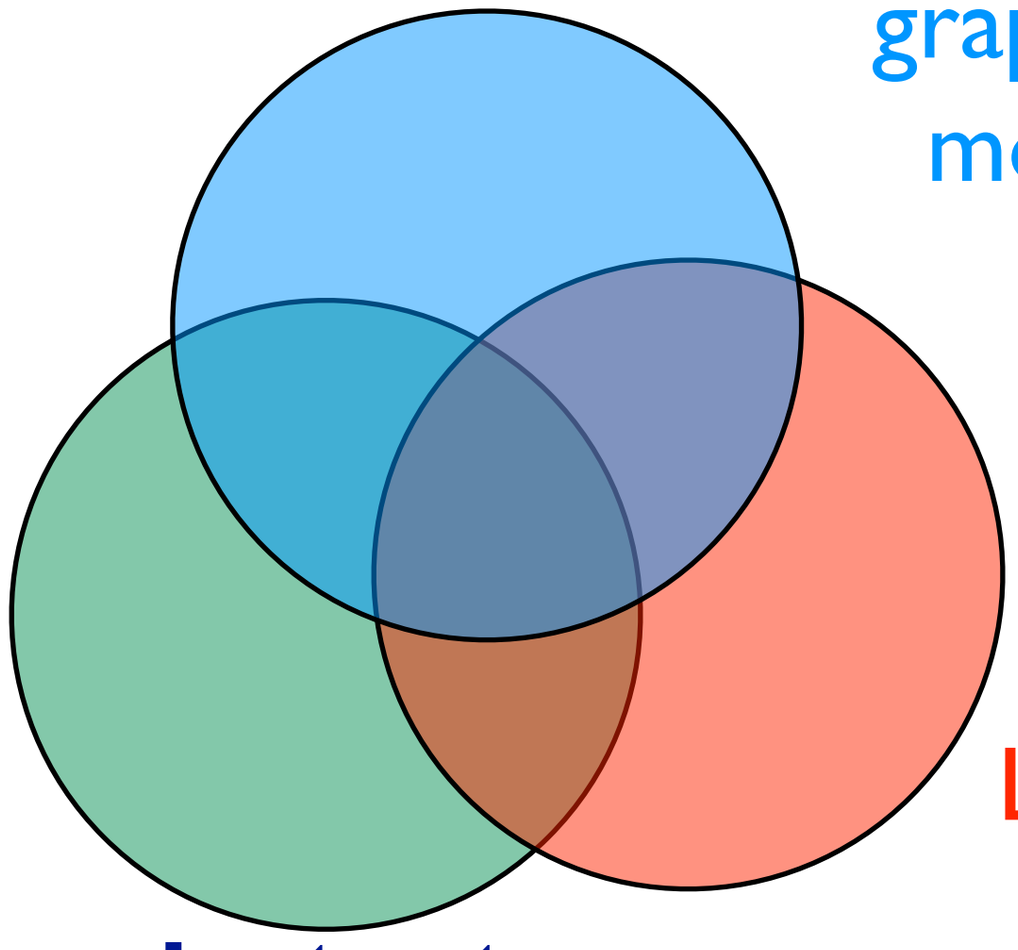


Lifted graphical models

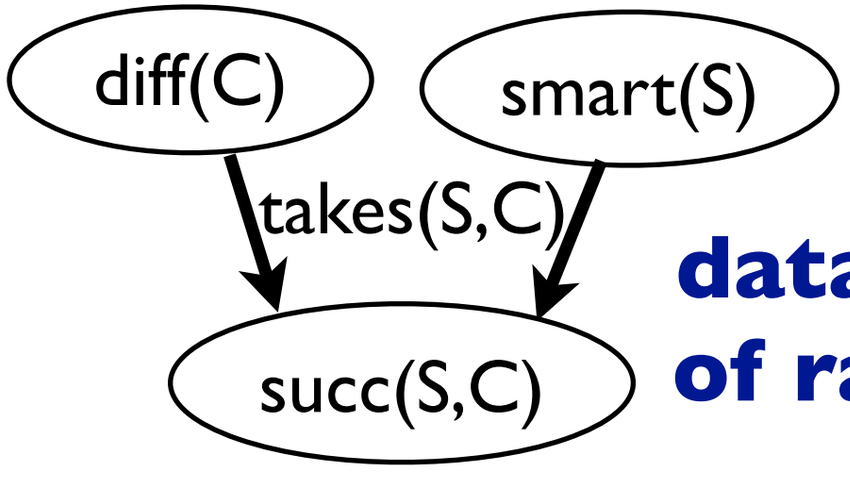
fixed set of random variables



graphical model



relational definition of graphical model

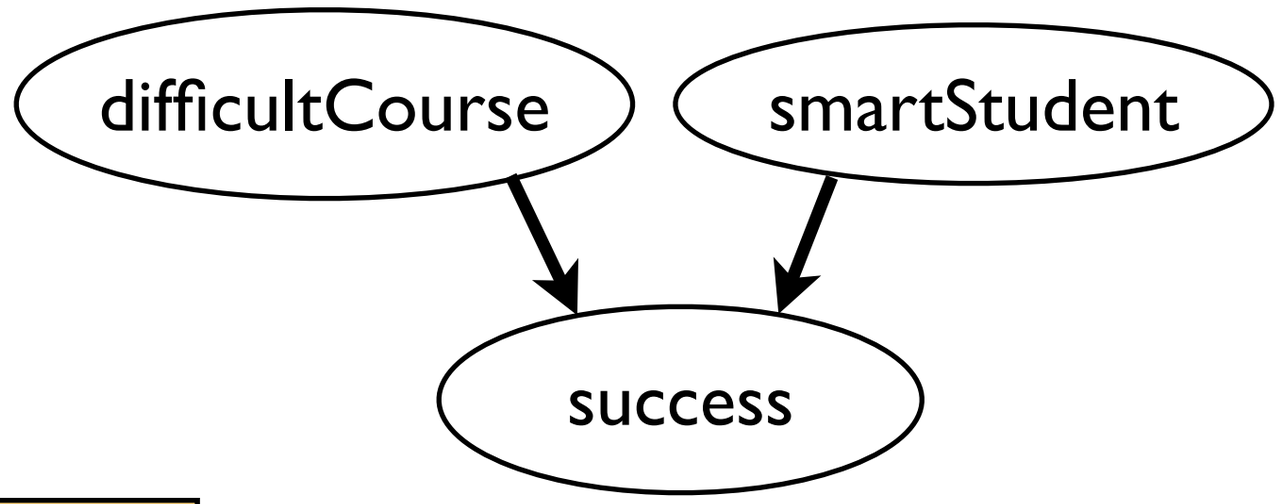


Learning

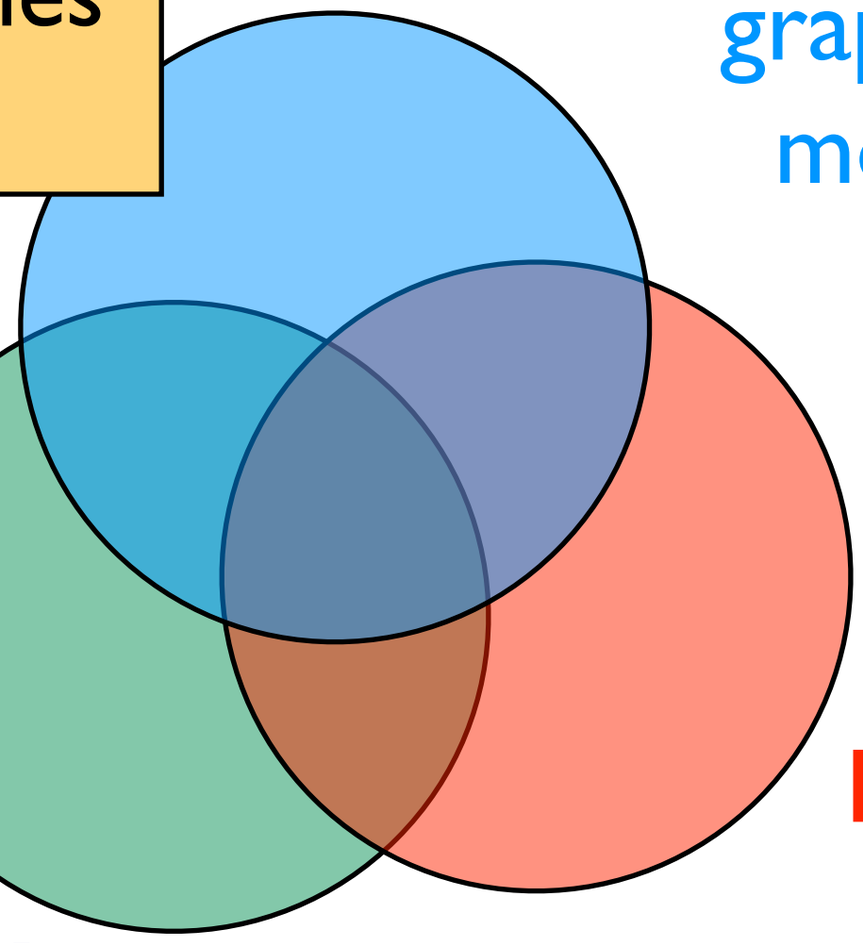
data-dependent set of random variables

Lifted graphical models

fixed set of random variables



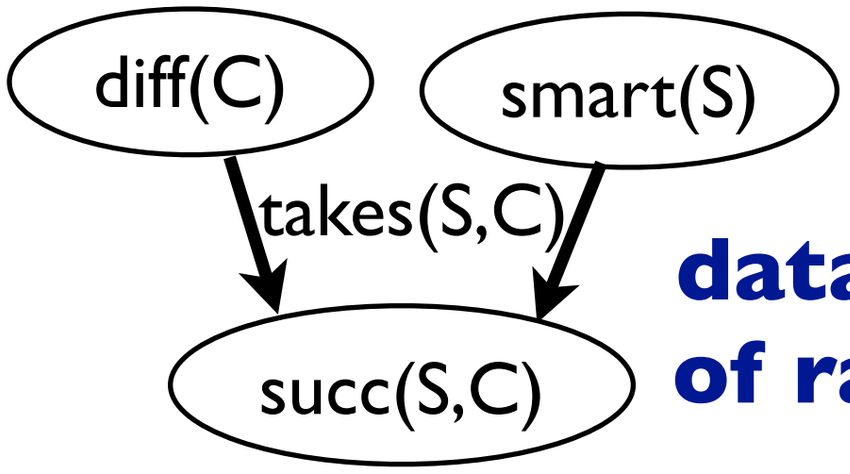
relational language defines graphical model



graphical model

relational definition of graphical model

Learning



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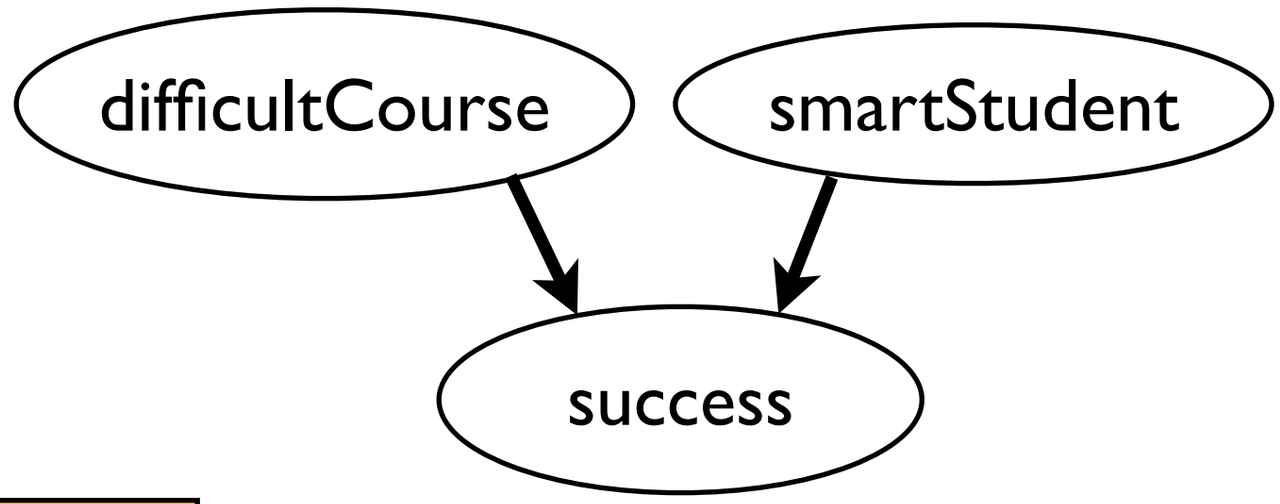

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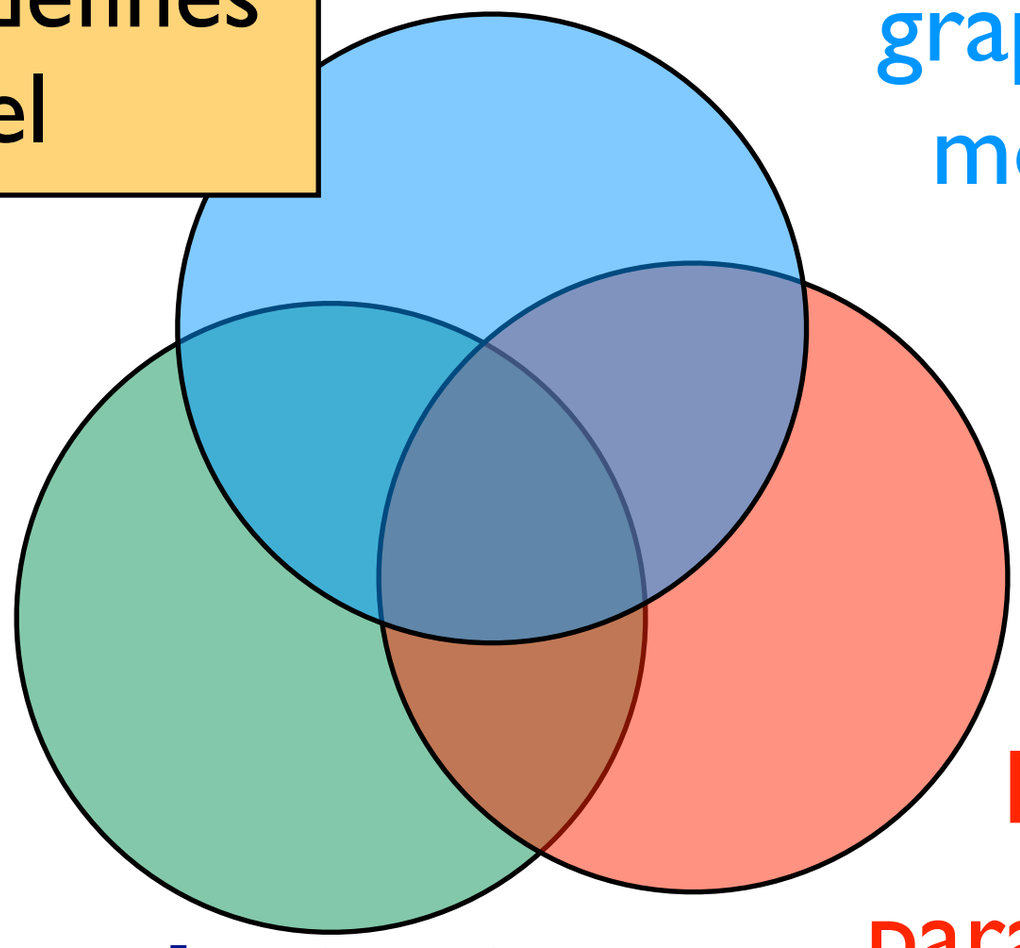
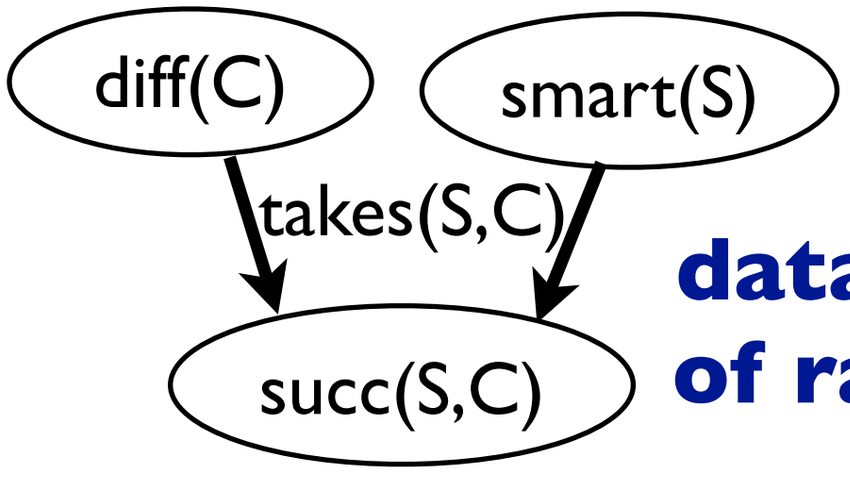
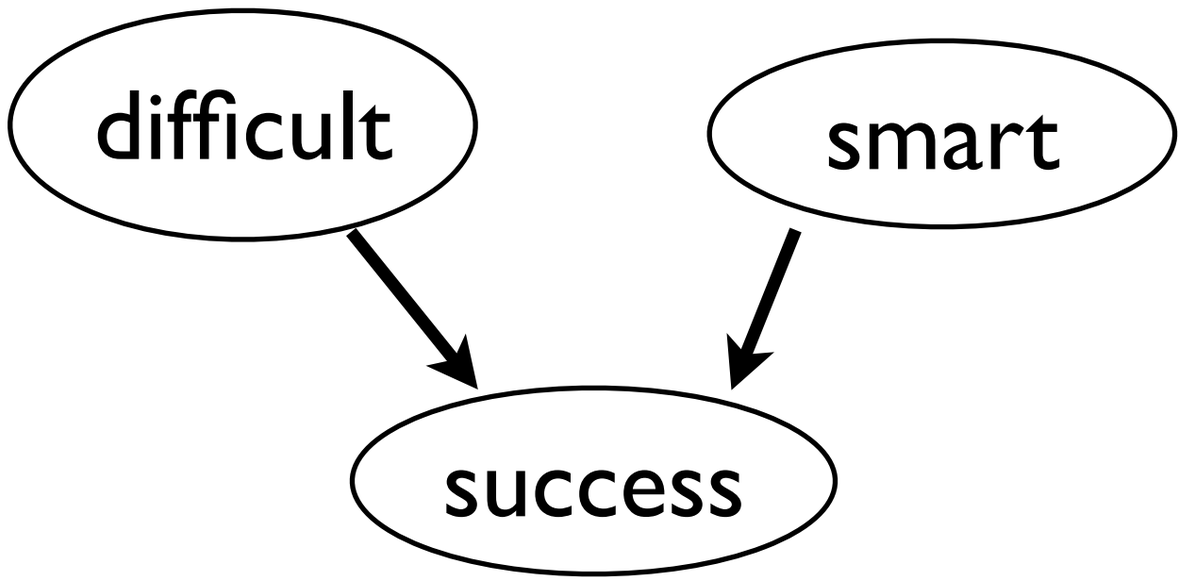


Learning parameters & structure



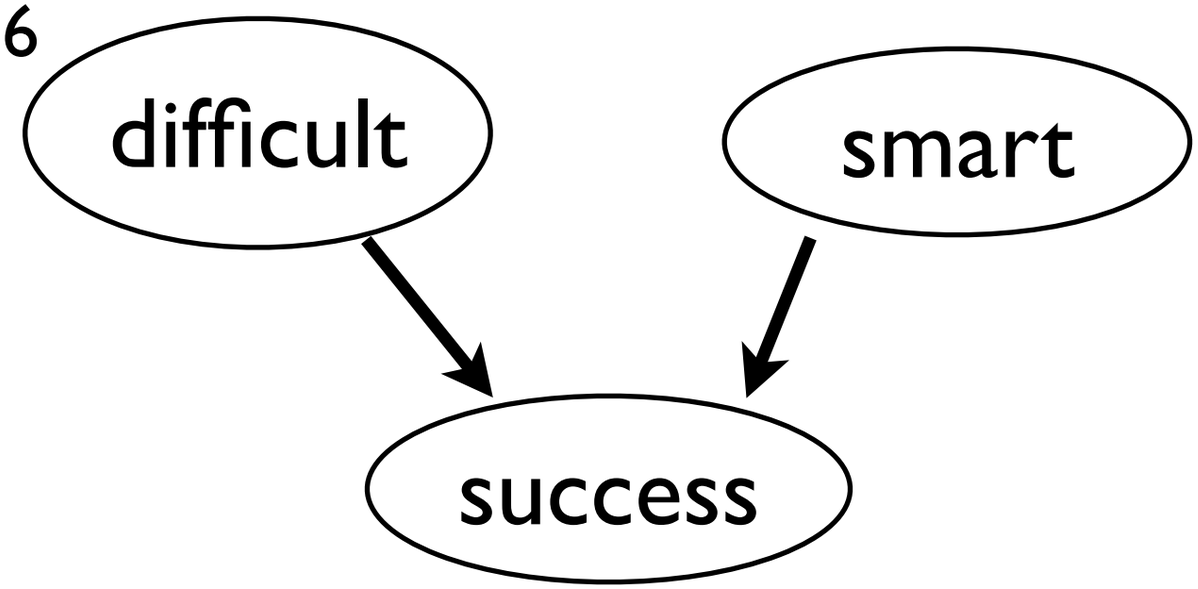
data-dependent set of random variables

Bayesian Network

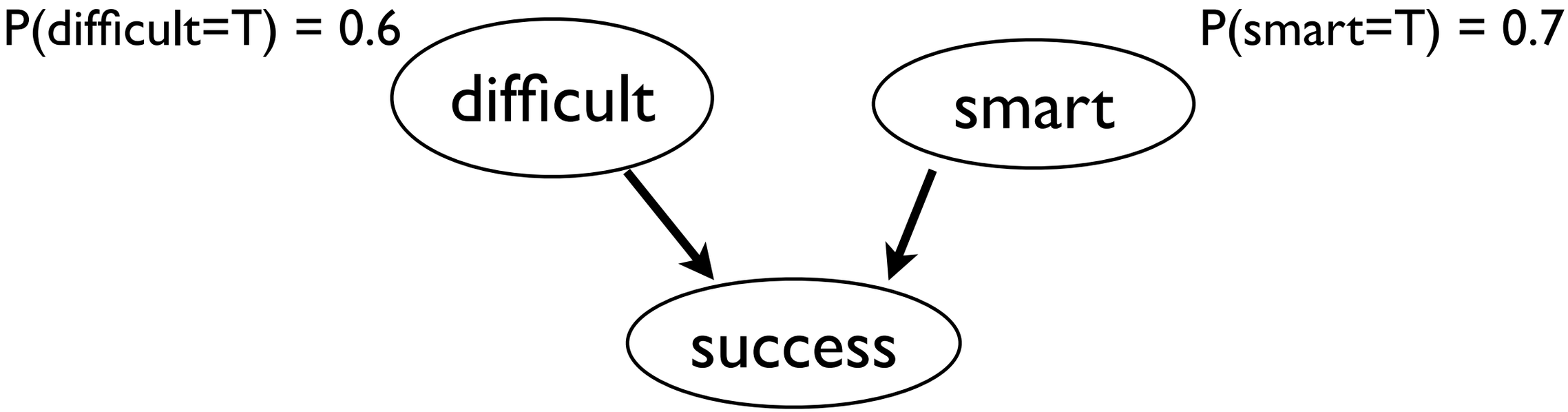


Bayesian Network

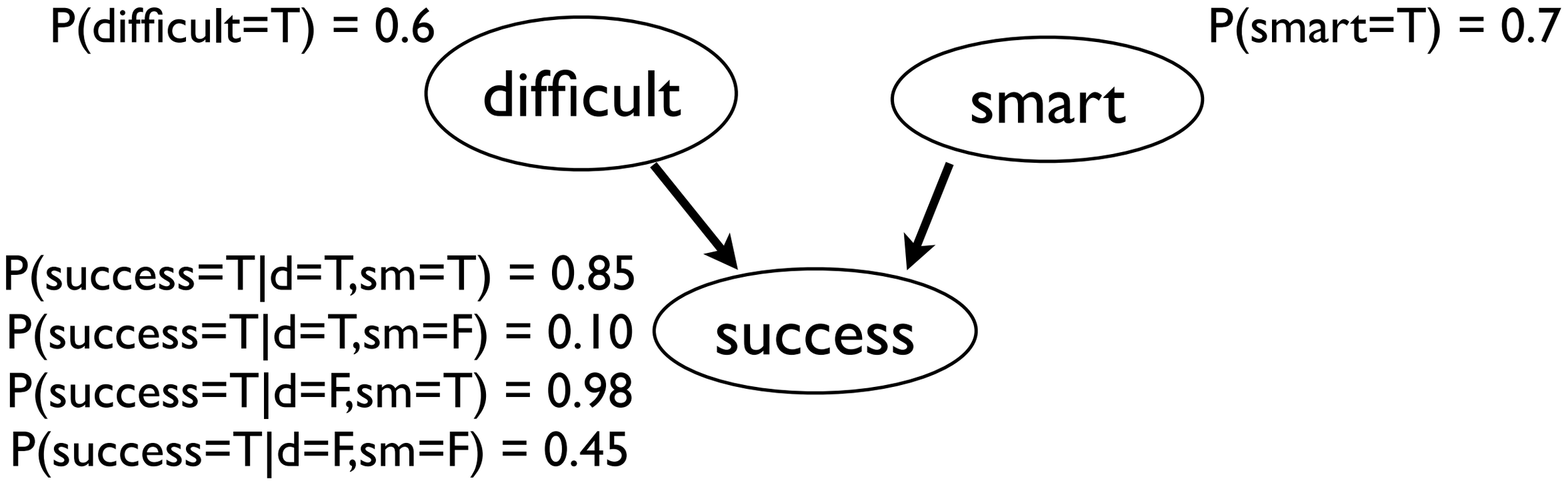
P(difficult=T) = 0.6



Bayesian Network



Bayesian Network



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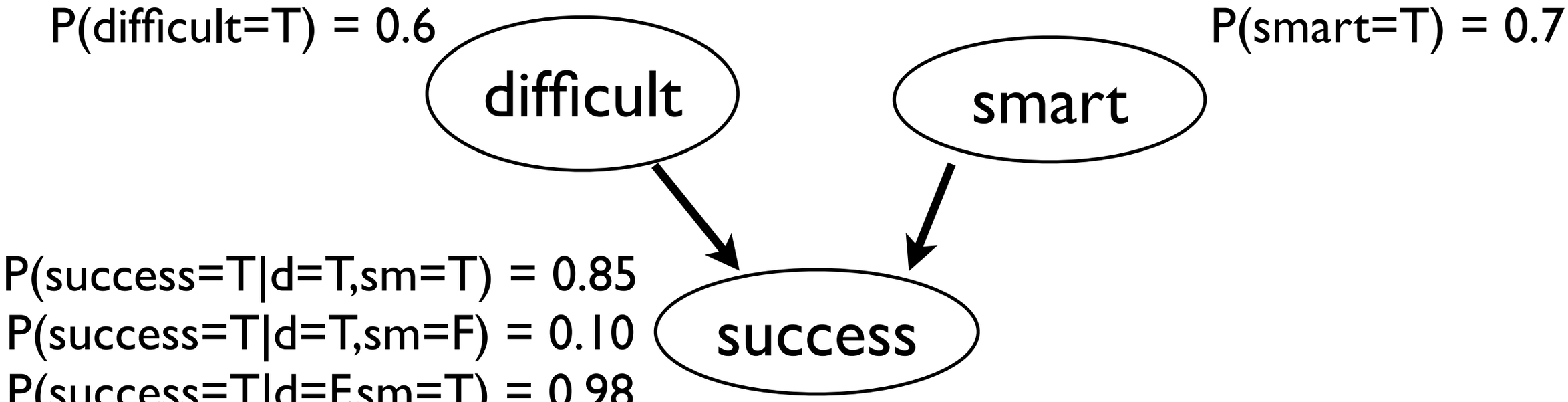
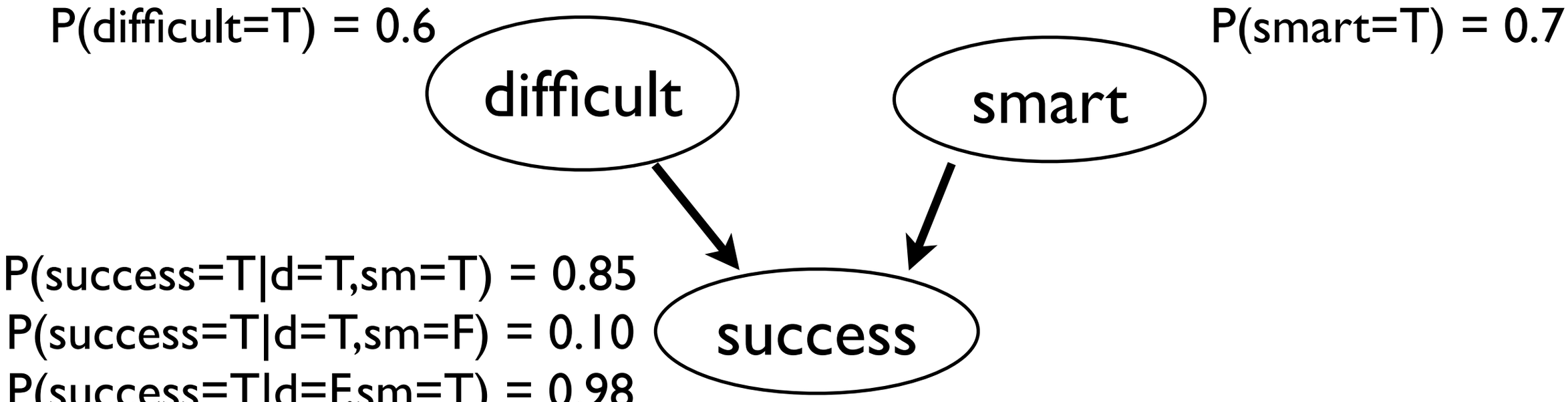


joint distribution

$$P(\text{difficult}) \times P(\text{smart}) \times P(\text{success}|\text{difficult},\text{smart})$$

Bayesian Network



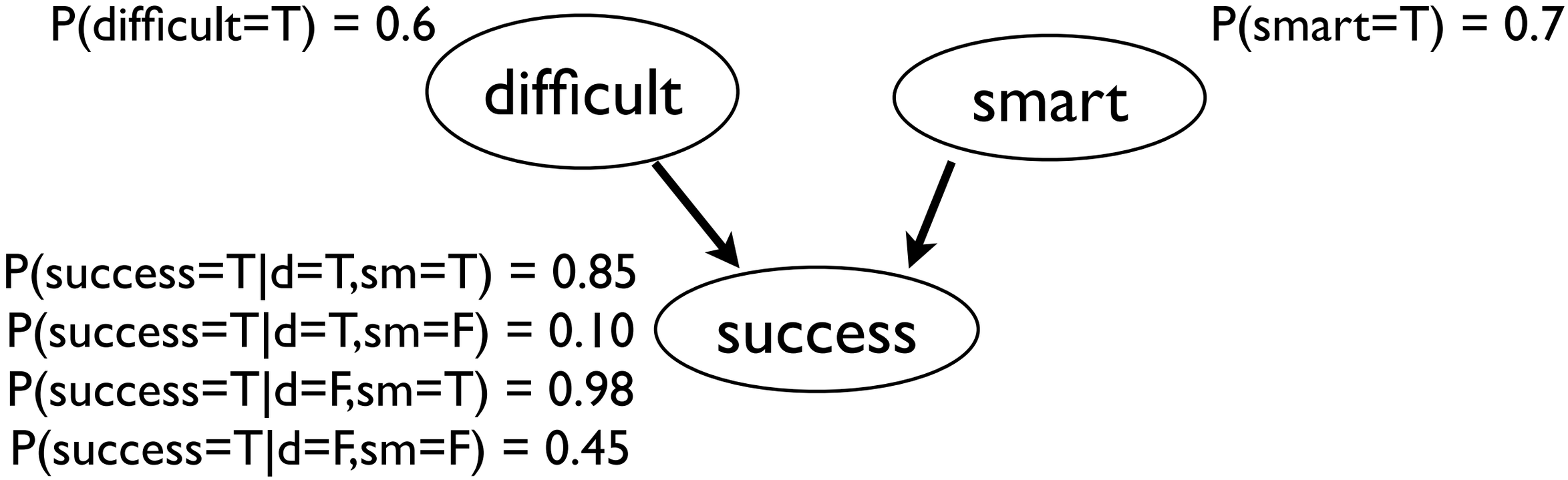
joint distribution

P(difficult)×P(smart)×P(success|difficult,smart)

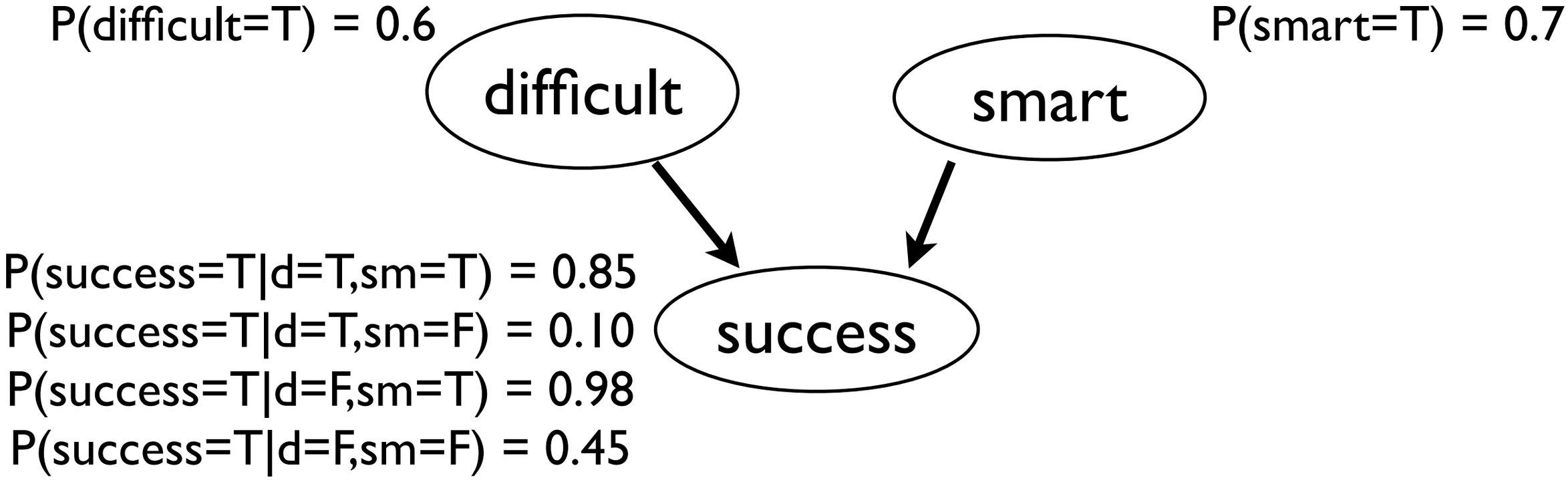
directed acyclic graph

$$P(X_1, \ldots, X_n) = \prod_{i=1,\ldots,n} P(X_i|\mathrm{par}(X_i))$$

Bayesian Network in ProbLog?

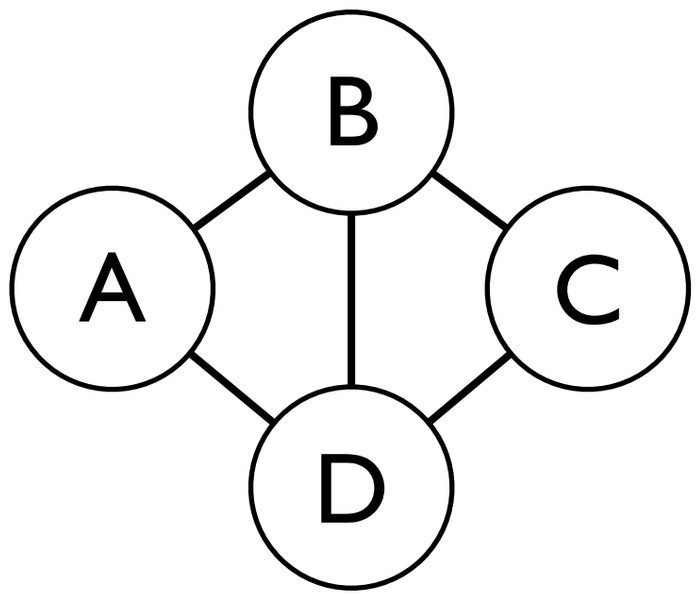


Bayesian Network in ProbLog?



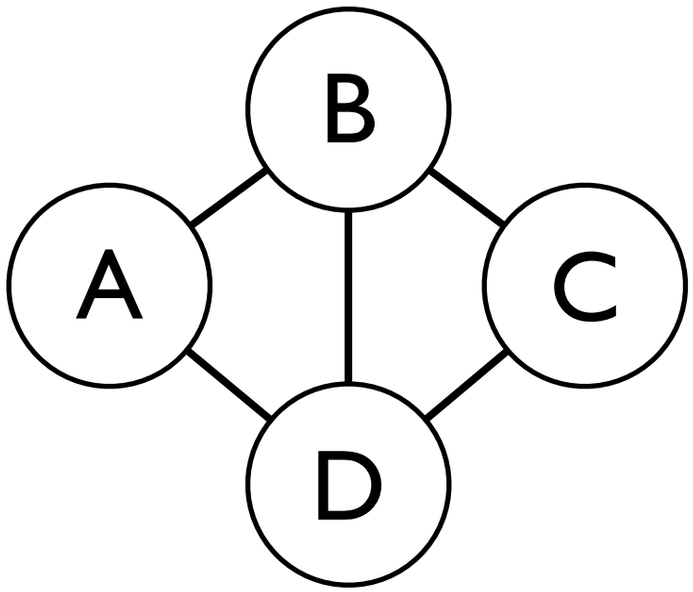

```
0.6::difficult.
0.7::smart.
0.85::success <- difficult, smart.
0.10::success <- difficult, \+smart.
0.98::success <- \+difficult, smart.
0.45::success <- \+difficult, \+smart.
```

Markov Network



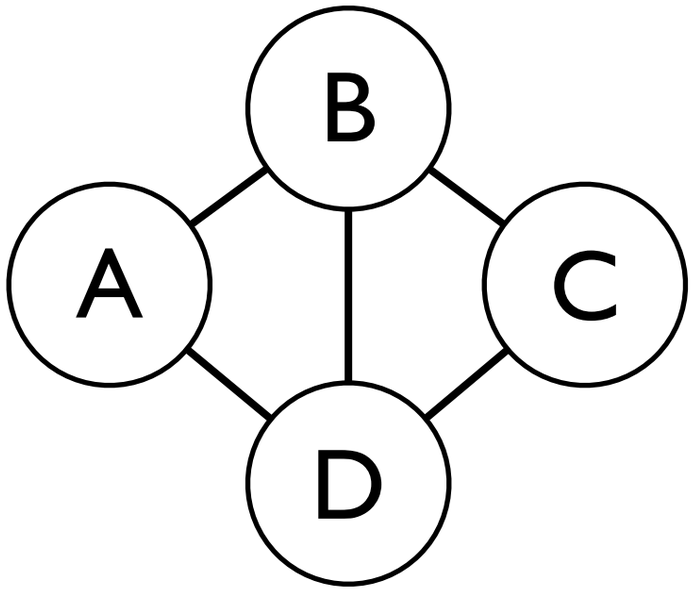
Markov Network

A	B	D	$\phi_1(A, B, D)$
T	T	T	5
T	T	F	1
T	F	T	1
T	F	F	5
F	T	T	1
F	T	F	5
F	F	T	5
F	F	F	1



Markov Network

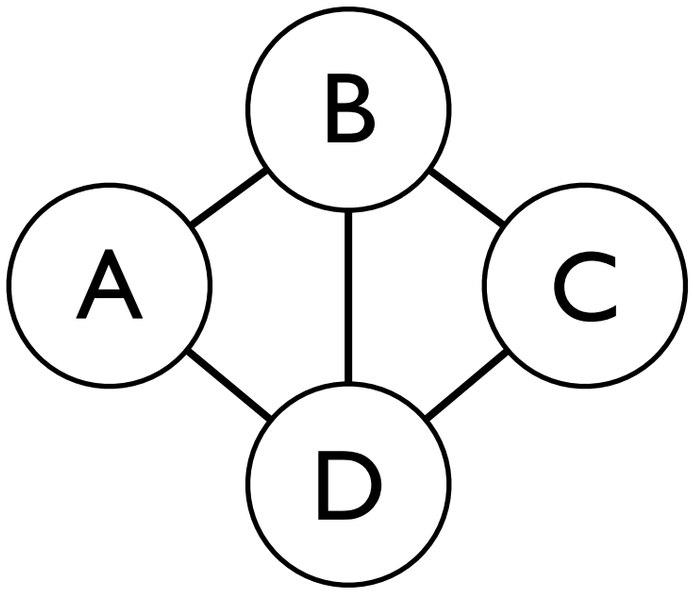
A	B	D	$\phi_1(A, B, D)$
T	T	T	5
T	T	F	1
T	F	T	1
T	F	F	5
F	T	T	1
F	T	F	5
F	F	T	5
F	F	F	1



B	C	D	$\phi_2(B, C, D)$
T	T	T	4
T	T	F	2
T	F	T	3
T	F	F	1
F	T	T	2
F	T	F	4
F	F	T	1
F	F	F	3

Markov Network

A	B	D	$\phi_1(A, B, D)$
T	T	T	5
T	T	F	1
T	F	T	1
T	F	F	5
F	T	T	1
F	T	F	5
F	F	T	5
F	F	F	1

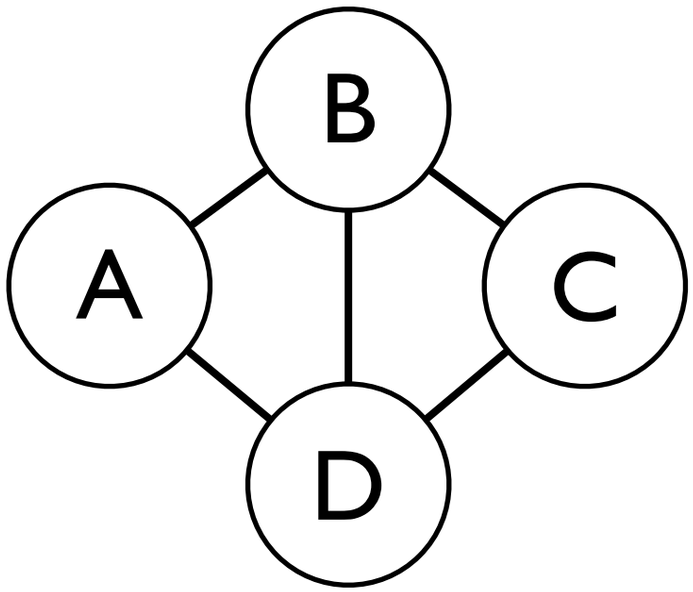


B	C	D	$\phi_2(B, C, D)$
T	T	T	4
T	T	F	2
T	F	T	3
T	F	F	1
F	T	T	2
F	T	F	4
F	F	T	1
F	F	F	3

$$P(A = T, B = F, C = T, D = F) = \frac{1}{Z}\phi_1(A = T, B = F, D = F)\phi_2(B = F, C = T, D = F)$$
$$= \frac{1}{Z} \cdot 5 \cdot 4$$

Markov Network

A	B	D	$\phi_1(A,B,D)$
T	T	T	5
T	T	F	1
T	F	T	1
T	F	F	5
F	T	T	1
F	T	F	5
F	F	T	5
F	F	F	1



B	C	D	$\phi_2(B,C,D)$
T	T	T	4
T	T	F	2
T	F	T	3
T	F	F	1
F	T	T	2
F	T	F	4
F	F	T	1
F	F	F	3

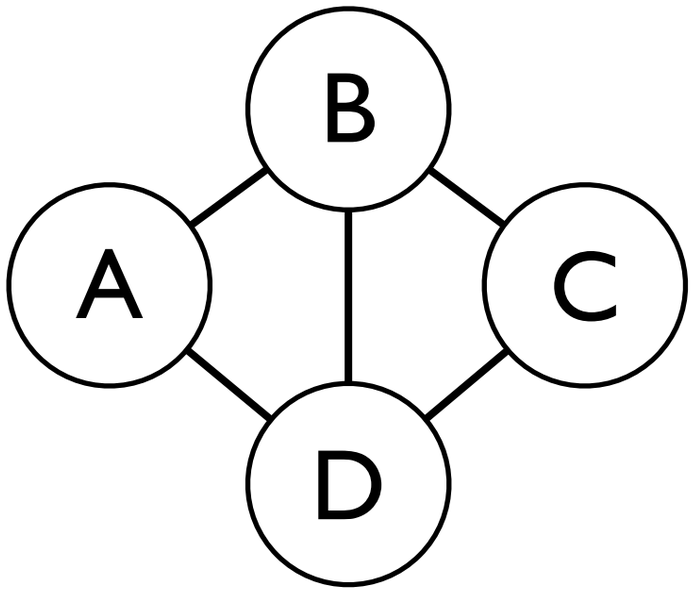
$$P(A=T,B=F,C=T,D=F) = \frac{1}{Z}\phi_1(A=T,B=F,D=F)\phi_2(B=F,C=T,D=F)$$
$$= \frac{1}{Z}\cdot 5 \cdot 4$$

undirected graph

$$P(\mathbf{X}=\mathbf{x}) = \frac{1}{Z}\prod_{\phi_i \in F}\phi_i(\mathbf{X}_{\phi_i}=\mathbf{x}_{\phi_i})$$
$$Z = \sum_{\mathbf{x}'}\prod_{\phi_i \in F}\phi_i(\mathbf{X}_{\phi_i}=\mathbf{x'}_{\phi_i})$$

Markov Network in ProbLog?

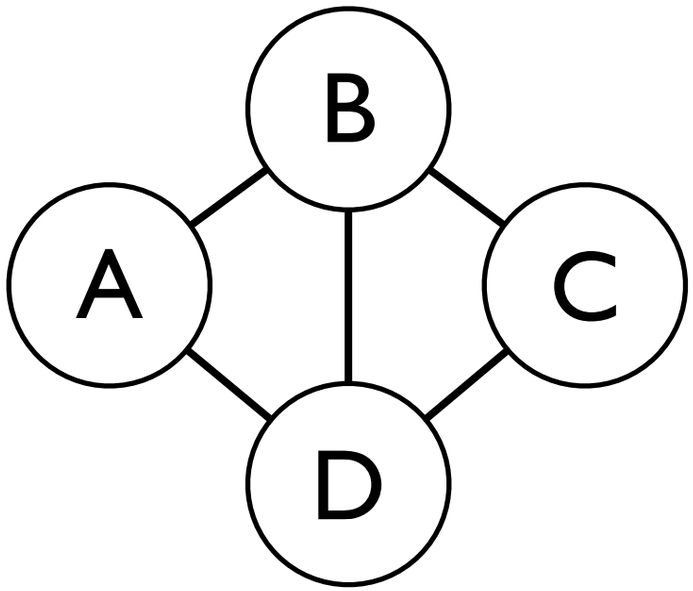
A	B	D	$\phi_1(A,B,D)$
T	T	T	5
T	T	F	1
T	F	T	1
T	F	F	5
F	T	T	1
F	T	F	5
F	F	T	5
F	F	F	1



B	C	D	$\phi_2(B,C,D)$
T	T	T	4
T	T	F	2
T	F	T	3
T	F	F	1
F	T	T	2
F	T	F	4
F	F	T	1
F	F	F	3

Markov Network in ProbLog?

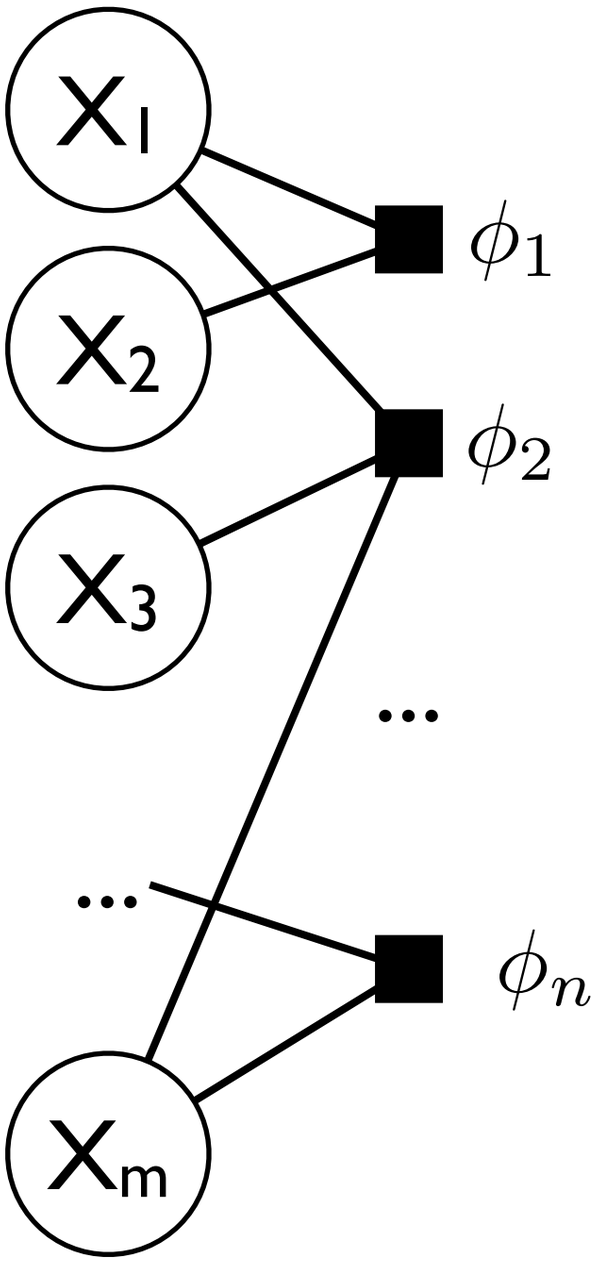
A	B	D	$\phi_1(A,B,D)$
T	T	T	5
T	T	F	1
T	F	T	1
T	F	F	5
F	T	T	1
F	T	F	5
F	F	T	5
F	F	F	1



B	C	D	$\phi_2(B,C,D)$
T	T	T	4
T	T	F	2
T	F	T	3
T	F	F	1
F	T	T	2
F	T	F	4
F	F	T	1
F	F	F	3

no direct mapping due to general potential functions and normalization

Factor Graph

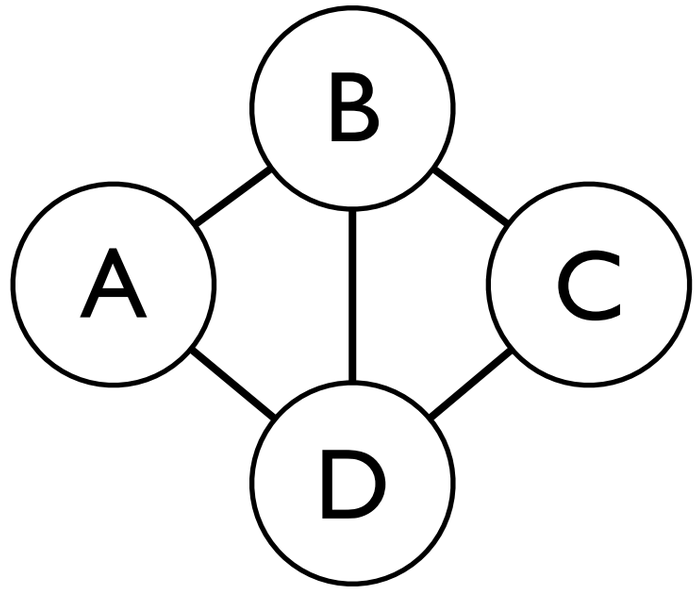


- bi-partite undirected graph
- variables $X_1, \ldots, X_m$
- factors $\phi_1, \ldots, \phi_n$ map interpretations of subsets of variables to non-negative reals
- joint distribution

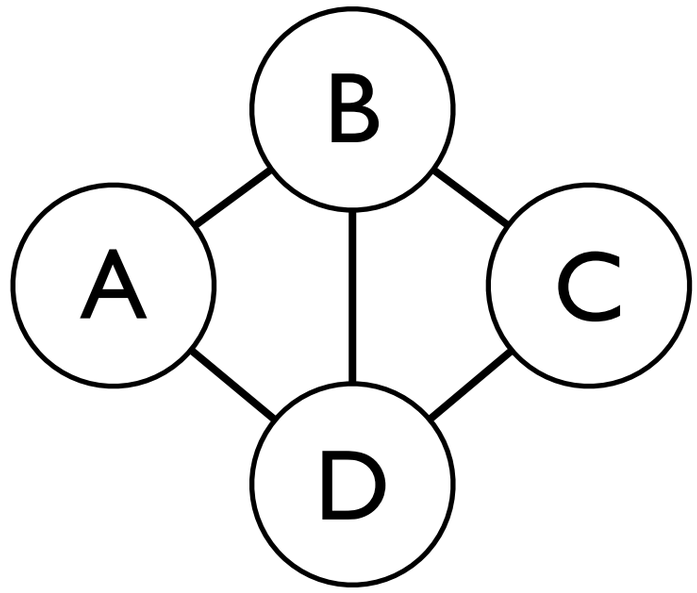
$$P(\mathbf{X} = \mathbf{x}) = \frac{1}{Z} \prod_{i=1..n} \phi_i(\mathbf{X}_{\phi_i} = \mathbf{x}_{\phi_i})$$

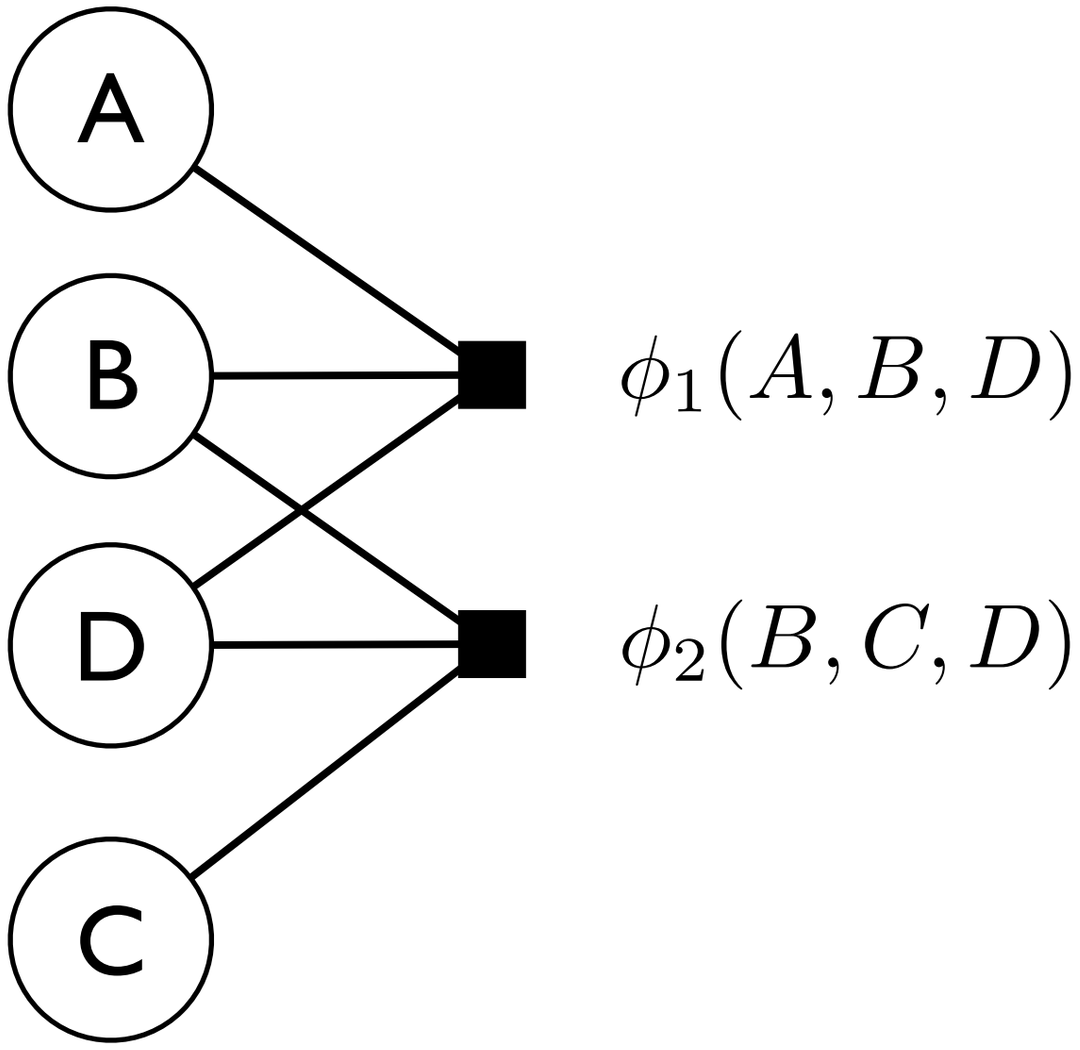
$$Z = \sum_{\mathbf{x}'} \prod_{i=1..n} \phi_i(\mathbf{X}_{\phi_i} = \mathbf{x}'_{\phi_i})$$

MN as Factor Graph?

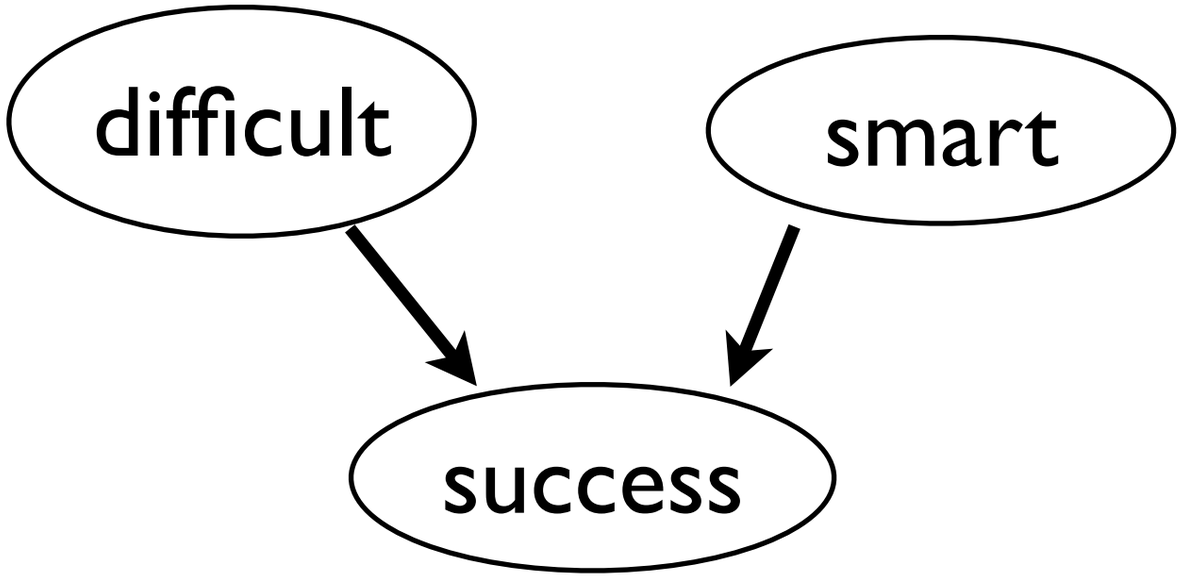


MN as Factor Graph?

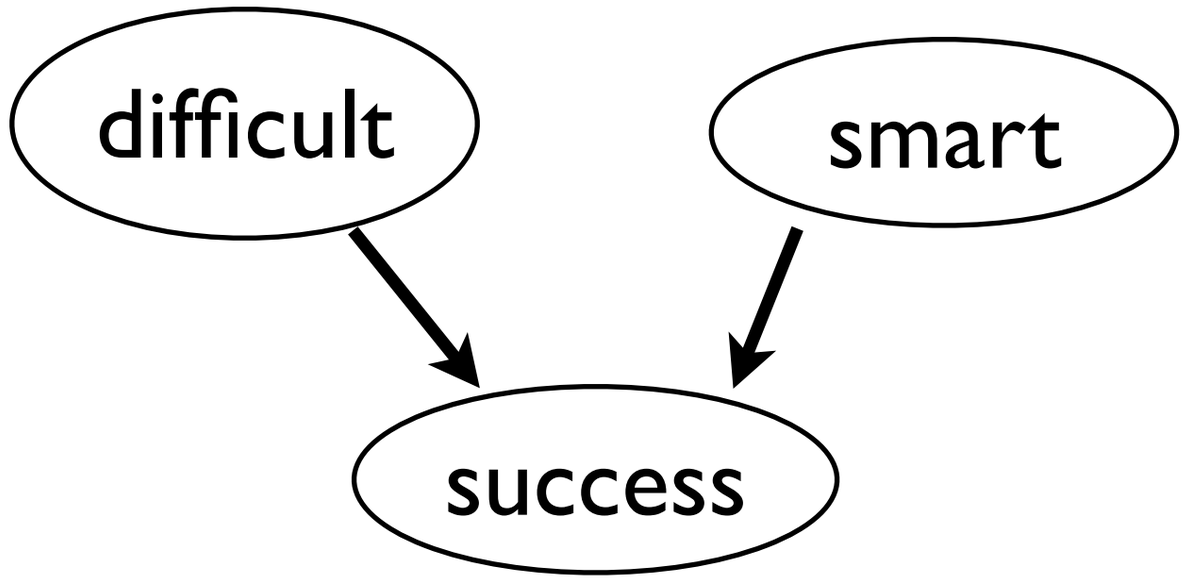


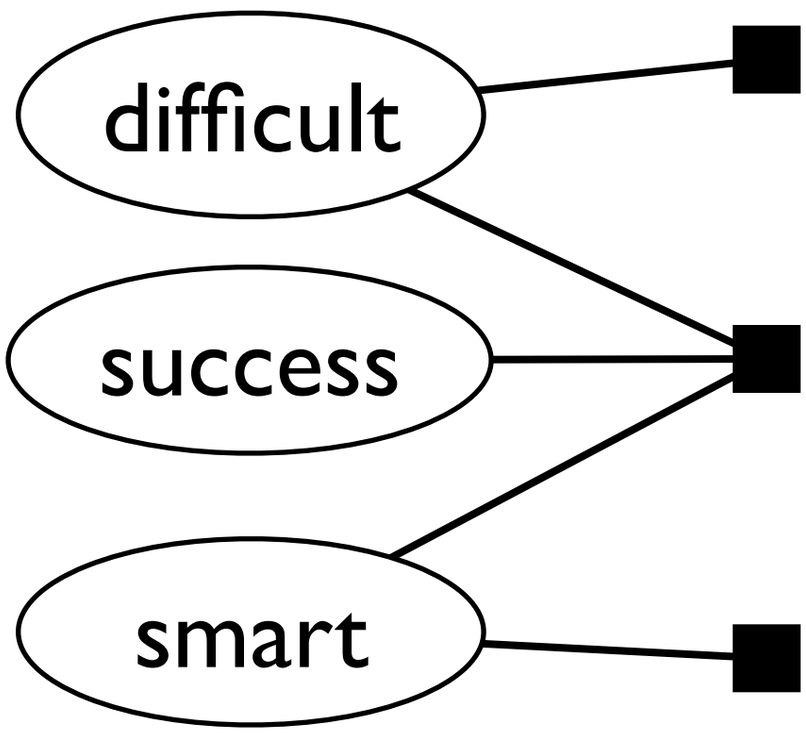


BN as Factor Graph?



BN as Factor Graph?





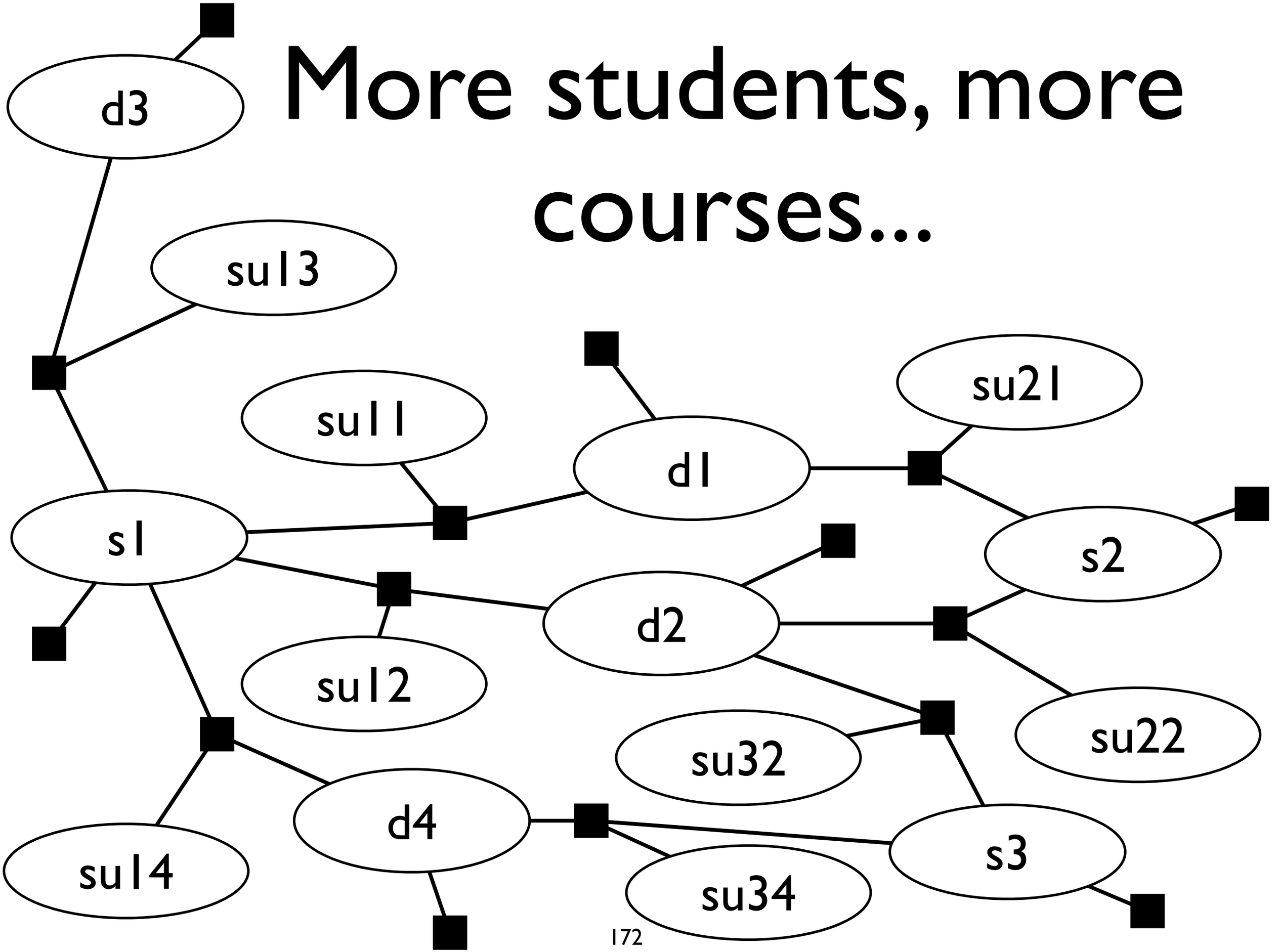
$$\phi_1(d) = P(d)$$

$$\phi_2(su, d, sm) = P(su|d, sm)$$

$$\phi_3(sm) = P(sm)$$

Z=1

More students, more courses...



More students, more

gets cumbersome quickly....
relational language to write template?

s(P)

su(P,C)

d(C)

d3

su13

s1

su12

su32

su22

d4

su14

su34

s3

Lots of proposals in the literature, e.g.

- relational Markov networks (RMNs) [Taskar et al 2002]
- Markov logic networks (MLNs) [Richardson & Domingos 2006]
- probabilistic soft logic (PSL) [Broecheler et al 2010]
- FACTORIE [McCallum et al 2009]
- Bayesian logic programs (BLPs) [Kersting & De Raedt 2001]
- relational Bayesian networks (RBNs) [Jaeger 2002]
- logical Bayesian networks (LBNs) [Fierens et al 2005]
- probabilistic relational models (PRMs) [Koller & Pfeffer 1998]
- Bayesian logic (BLOG) [Milch et al 2005]
- CLP(BN) [Santos Costa et al 2008]
- probabilistic programming languages such as ProbLog, PRISM, Church, ...
- and many more ...

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- relational Markov networks (RMNs) [Taskar et al 2002]
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- Bay
- rela
- logi
- probabilistic relational models (PRMs) [Koller & Pfeffer 1998]
- Bayesian logic (BLOG) [Milch et al 2005]
- CLP(BN) [Santos Costa et al 2008]
- probabilistic programming languages such as ProbLog, PRISM, Church, ...
- and many more ...

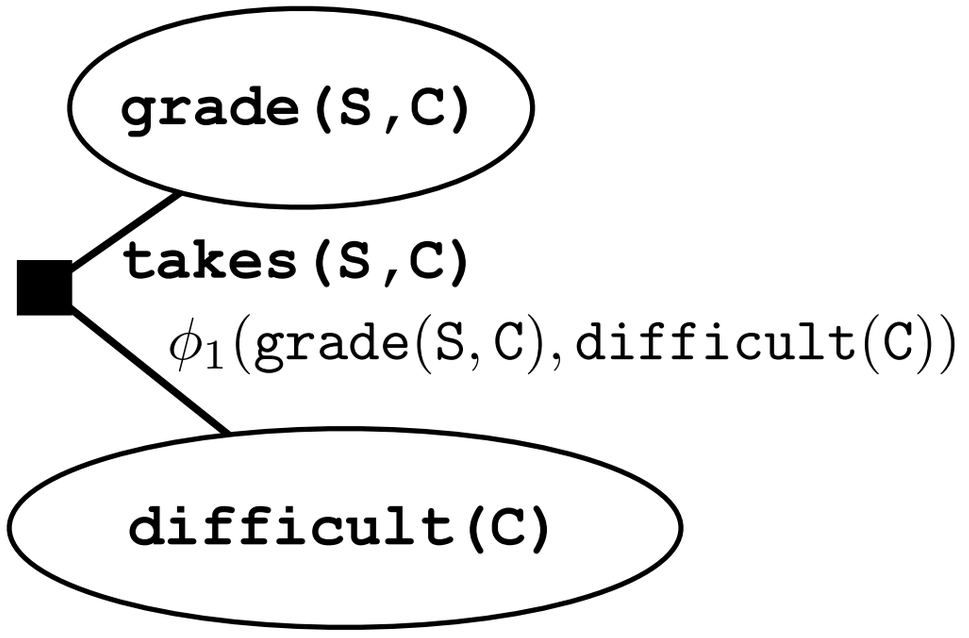
common principle:
parameterized factor graph
(par-factor graph)

Par-Factor Graph

[Poole 03]

Par-Factor Graph

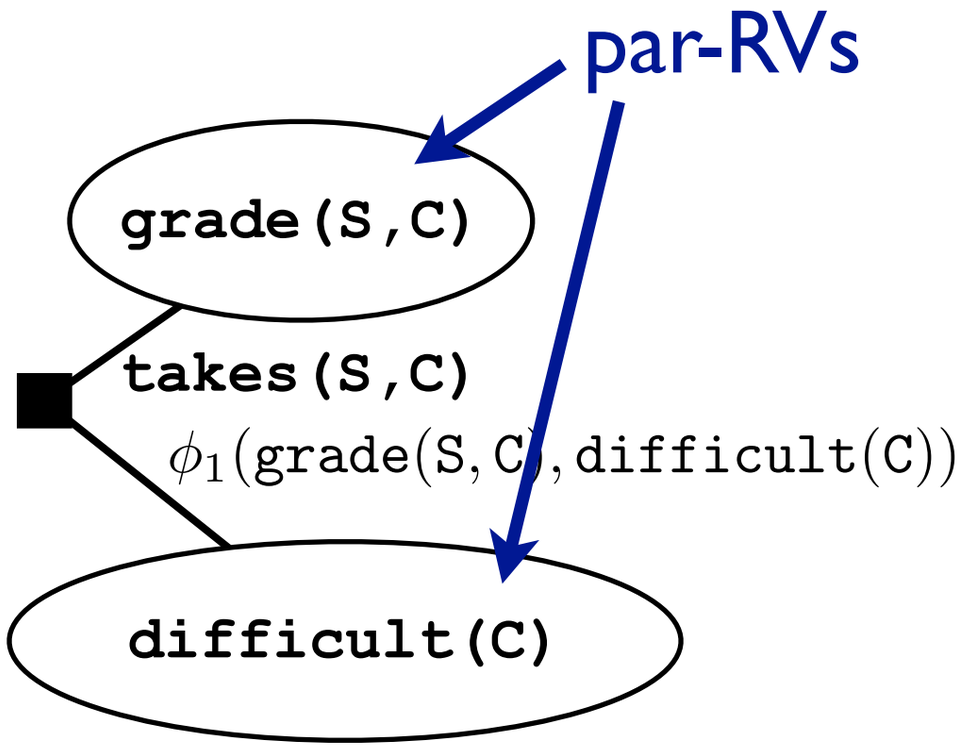
[Poole 03]



parameterized
factor (par-factor)

Par-Factor Graph

[Poole 03]

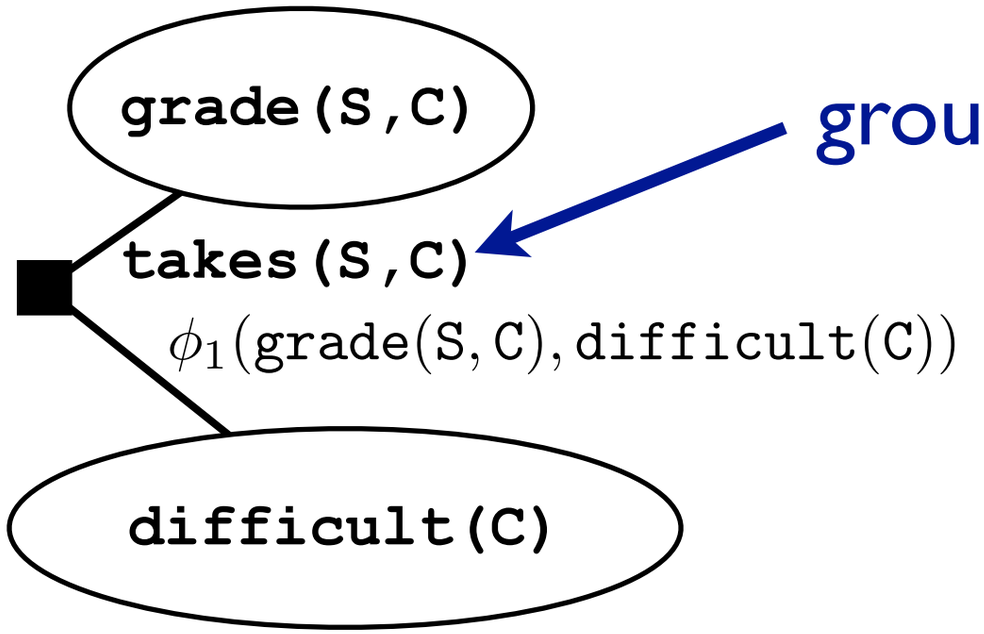


parameterized

factor (par-factor)

Par-Factor Graph

[Poole 03]



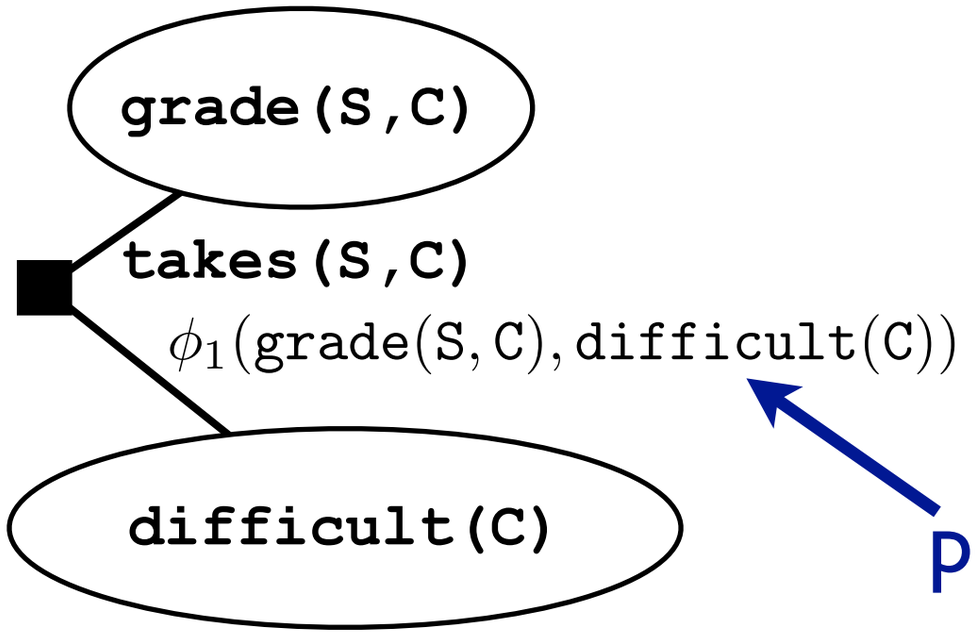
grounding constraint

parameterized

factor (par-factor)

Par-Factor Graph

[Poole 03]

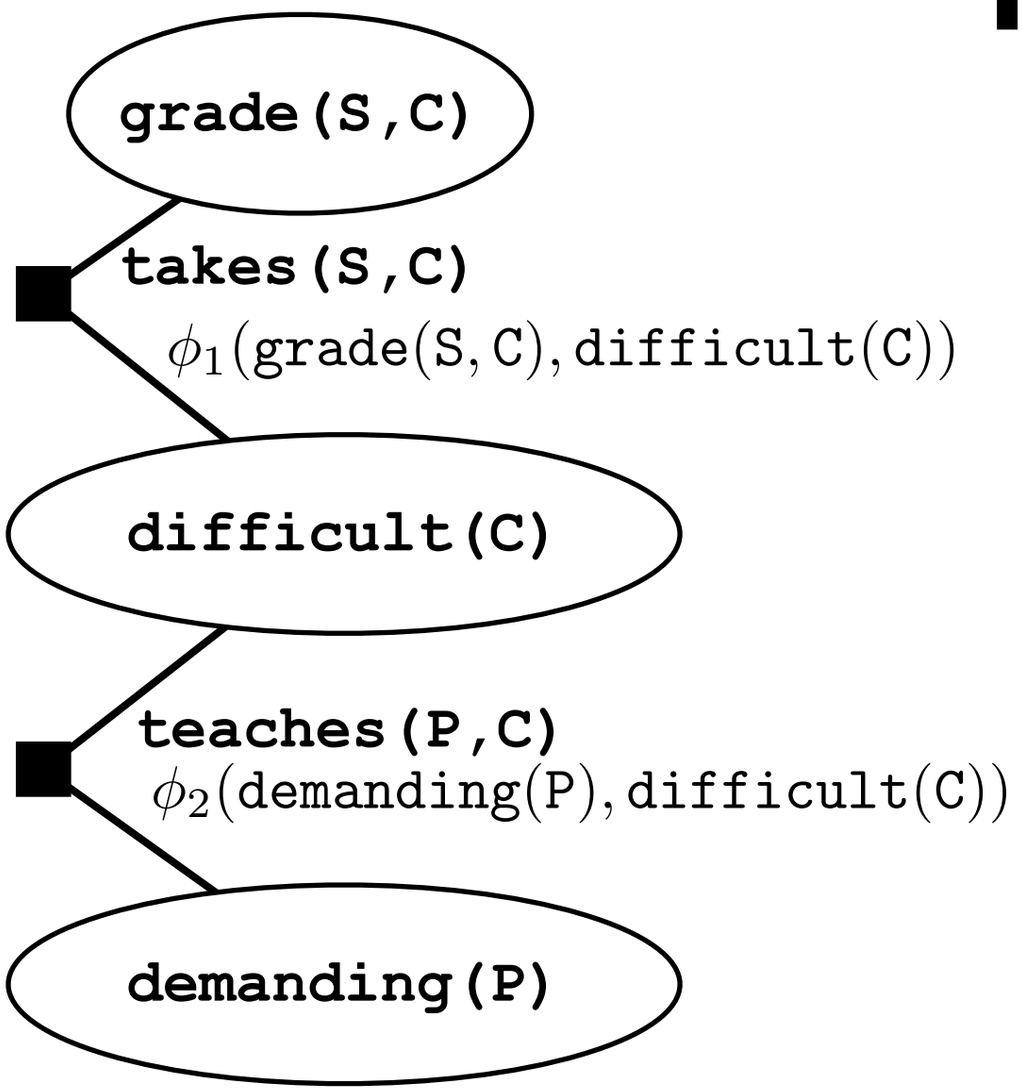


parameterized
factor (par-factor)

potential function

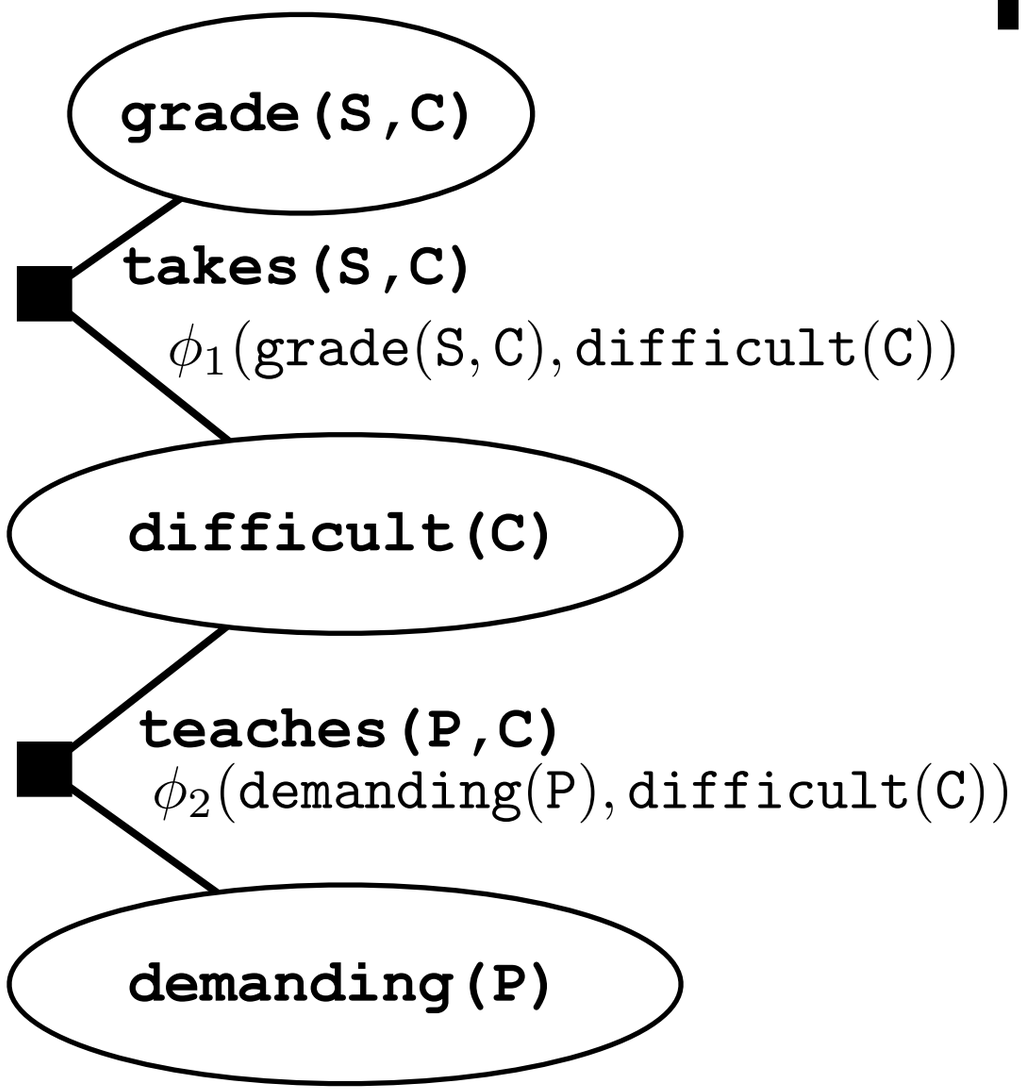
Par-Factor Graph

[Poole 03]



Par-Factor Graph

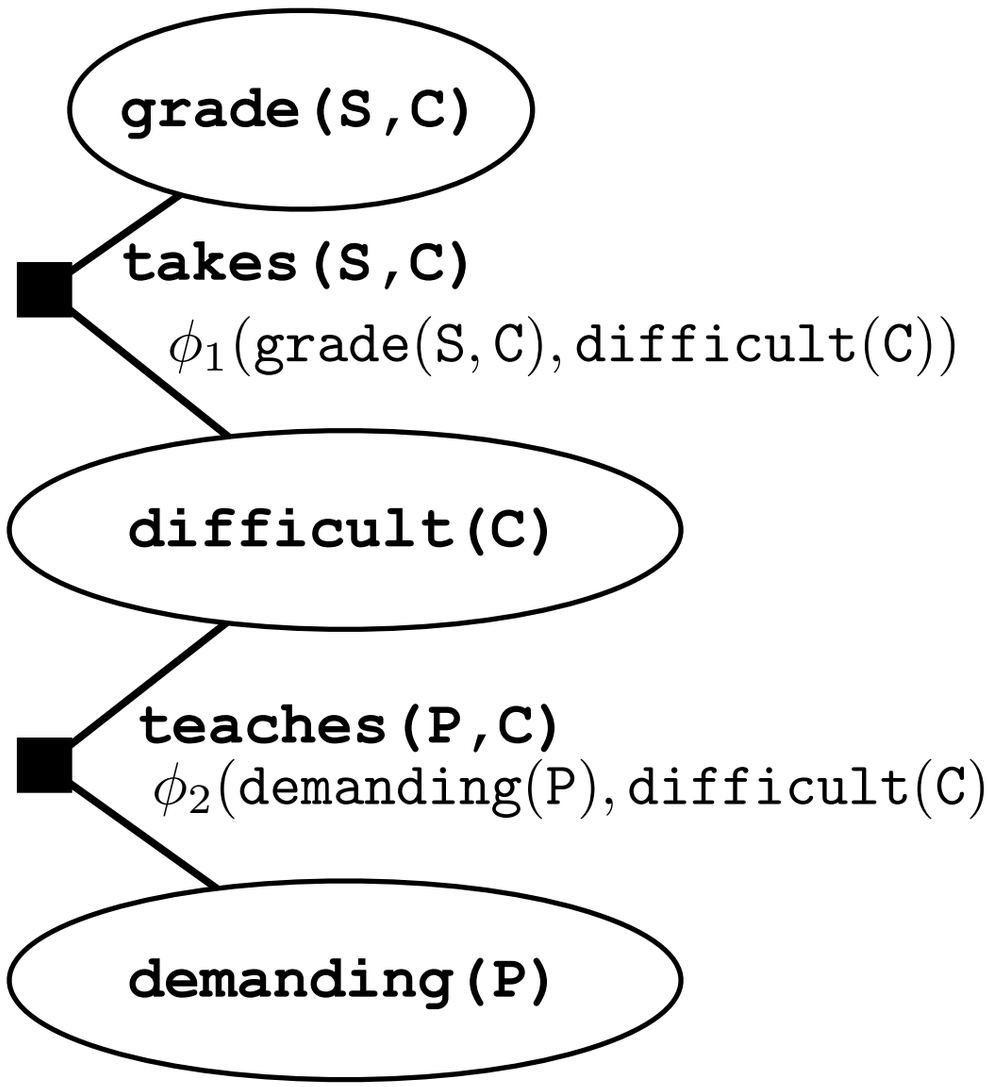
[Poole 03]



$$P(.) = \frac{1}{Z} \cdot \prod_{\mathtt{takes(s,c)}} \phi_1(\mathtt{grade(s,c)},\mathtt{difficult(c)}) \cdot \prod_{\mathtt{teaches(p,c)}} \phi_2(\mathtt{demanding(p)},\mathtt{difficult(c)})$$

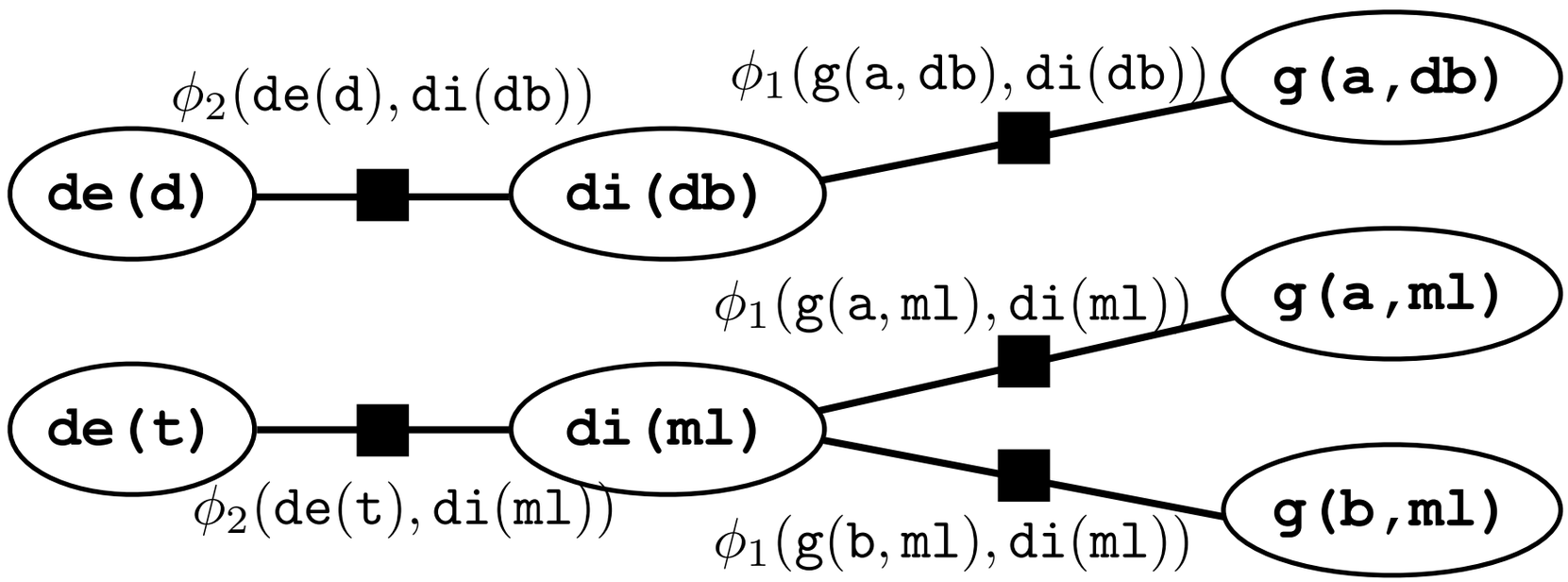
Par-Factor Graph

[Poole 03]

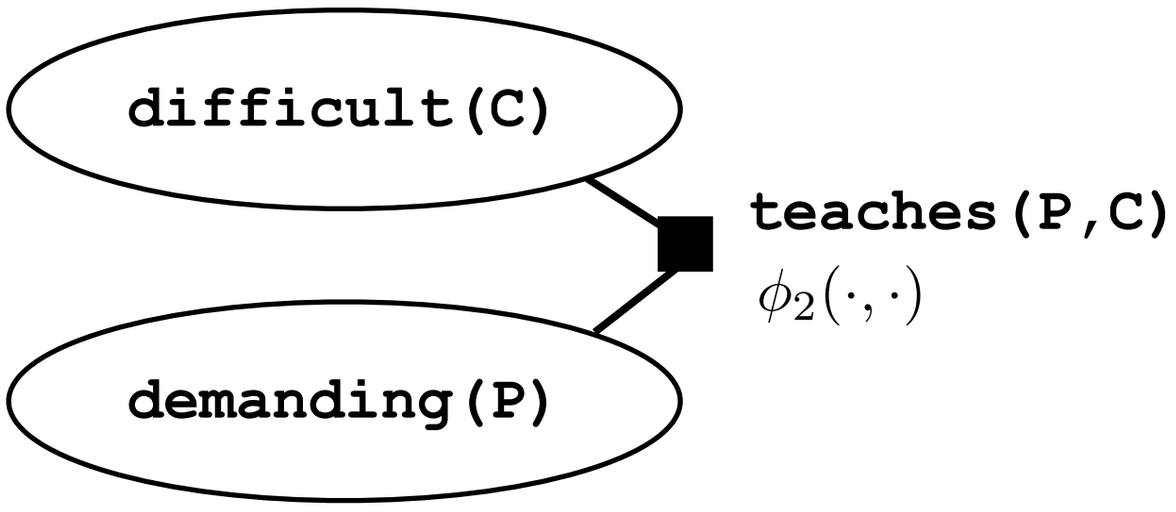


$$P(.) = \frac{1}{Z} \cdot \prod_{\mathtt{takes(s,c)}} \phi_1(\mathtt{grade}(\mathtt{s},\mathtt{c}),\mathtt{difficult}(\mathtt{c})) \cdot \prod_{\mathtt{teaches(p,c)}} \phi_2(\mathtt{demanding}(\mathtt{p}),\mathtt{difficult}(\mathtt{c}))$$

```
takes(ann,ml).
takes(bob,ml).
takes(ann,db).
teaches(dan,db).
teaches(tom,ml).
```

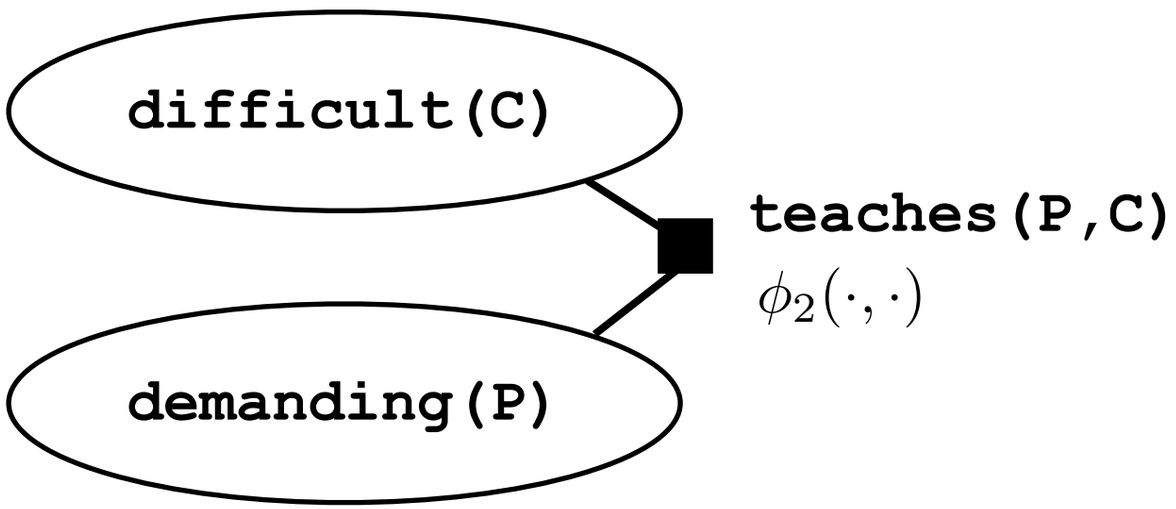


What if multiple professors teach the same course?




```
teaches(eve,db)
teaches(dan,db).
teaches(tom,ml).
teaches(bob,ai).
teaches(ann,ai).
teaches(ed,ai).
```

What if multiple professors teach the same course?




```
teaches(eve,db)
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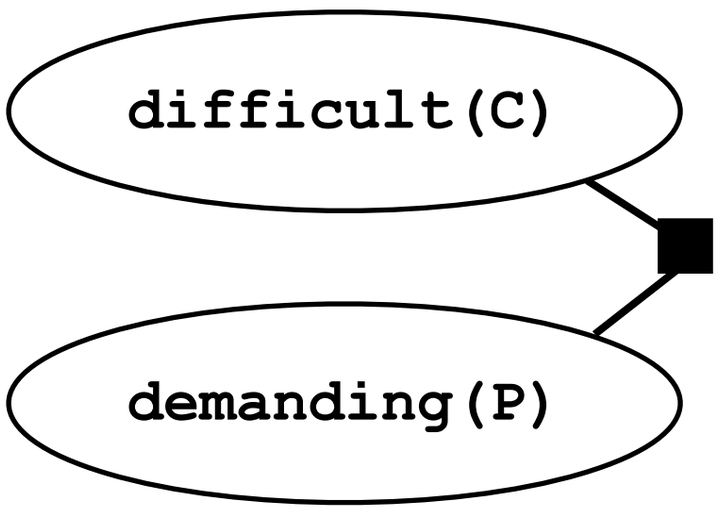
teaches(tom,ml).
teaches(bob,ai).
teaches(ann,ai).
teaches(ed,ai).
```

Option 1: aggregate values of RVs, then compute potential

$$\phi_2(\texttt{demanding(P)}, \texttt{difficult(C)}) = P(\texttt{difficult(C)}|\text{mean}\{\texttt{demanding(P)}\})$$

What if multiple professors teach the same course?

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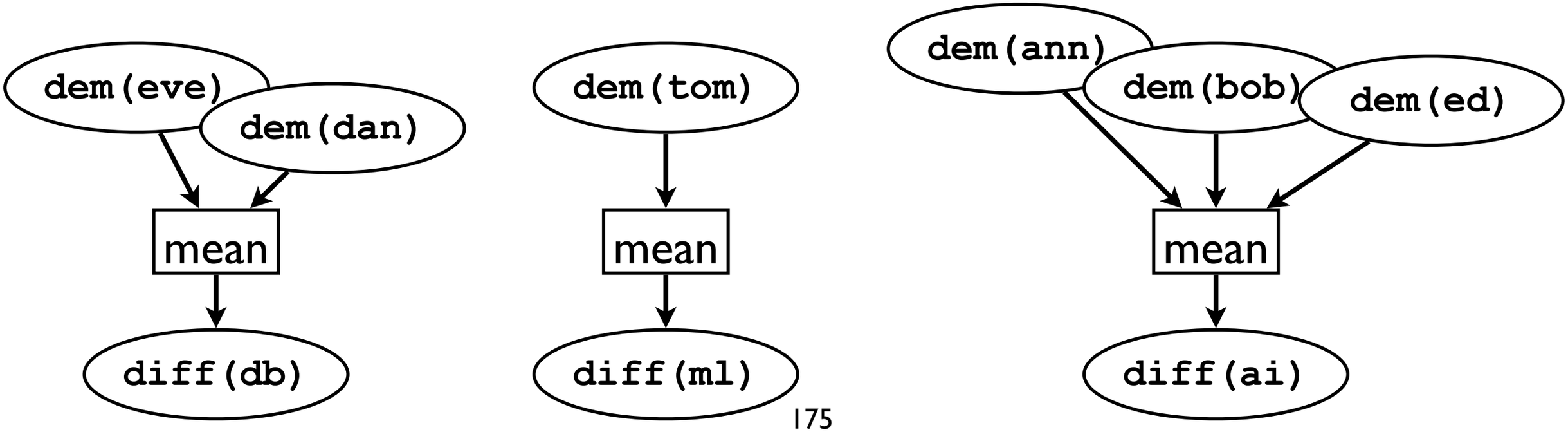
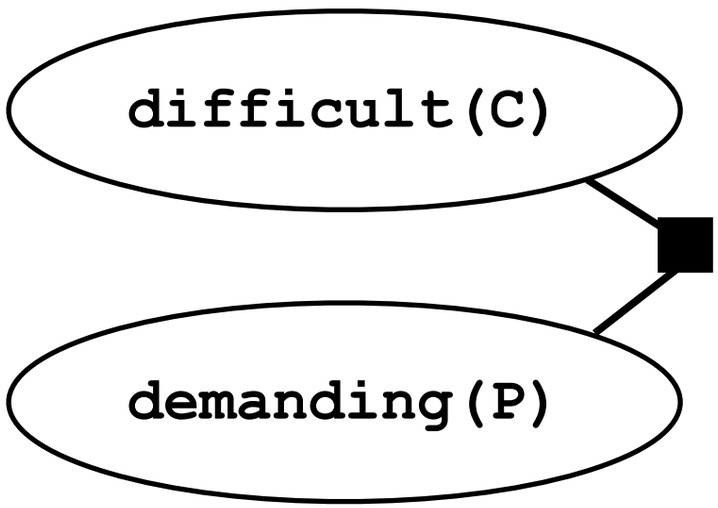


What if multiple professors teach the same course?

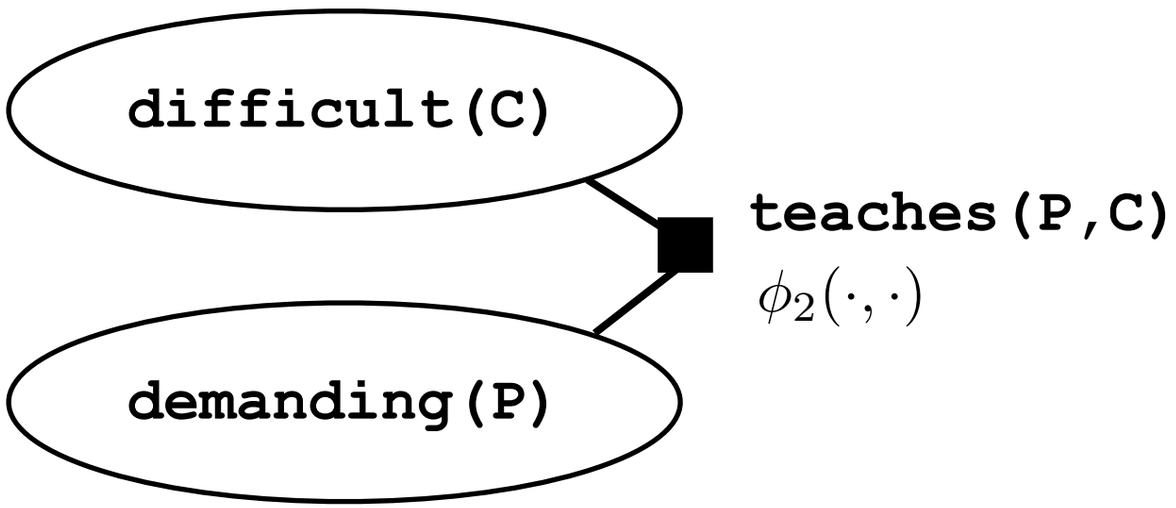



```
teaches(eve,db)
teaches(dan,db).
teaches(tom,ml).
teaches(bob,ai).
teaches(ann,ai).
teaches(ed,ai).
```

Option 2: compute potential for each RV, then combine

$$\phi_2(\texttt{demanding(P)}, \texttt{difficult(C)}) = 1 - \prod_{\texttt{P}} (1 - P(\texttt{difficult(C)} | \texttt{demanding(P)}))$$

What if multiple professors teach the same course?




```
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teaches(dan,db).
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```

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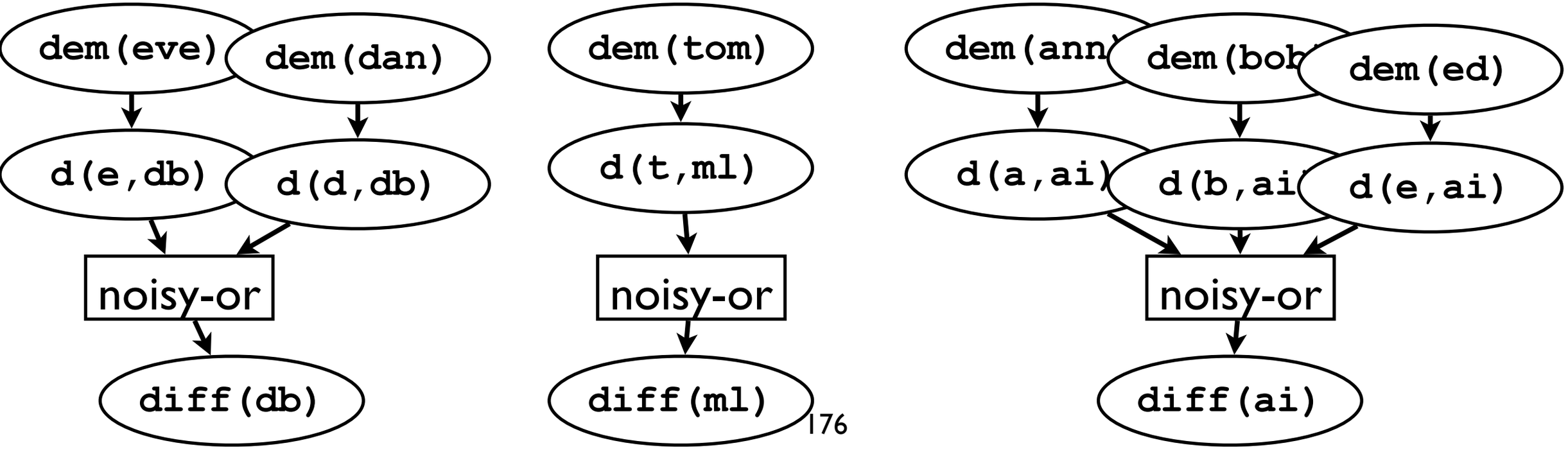
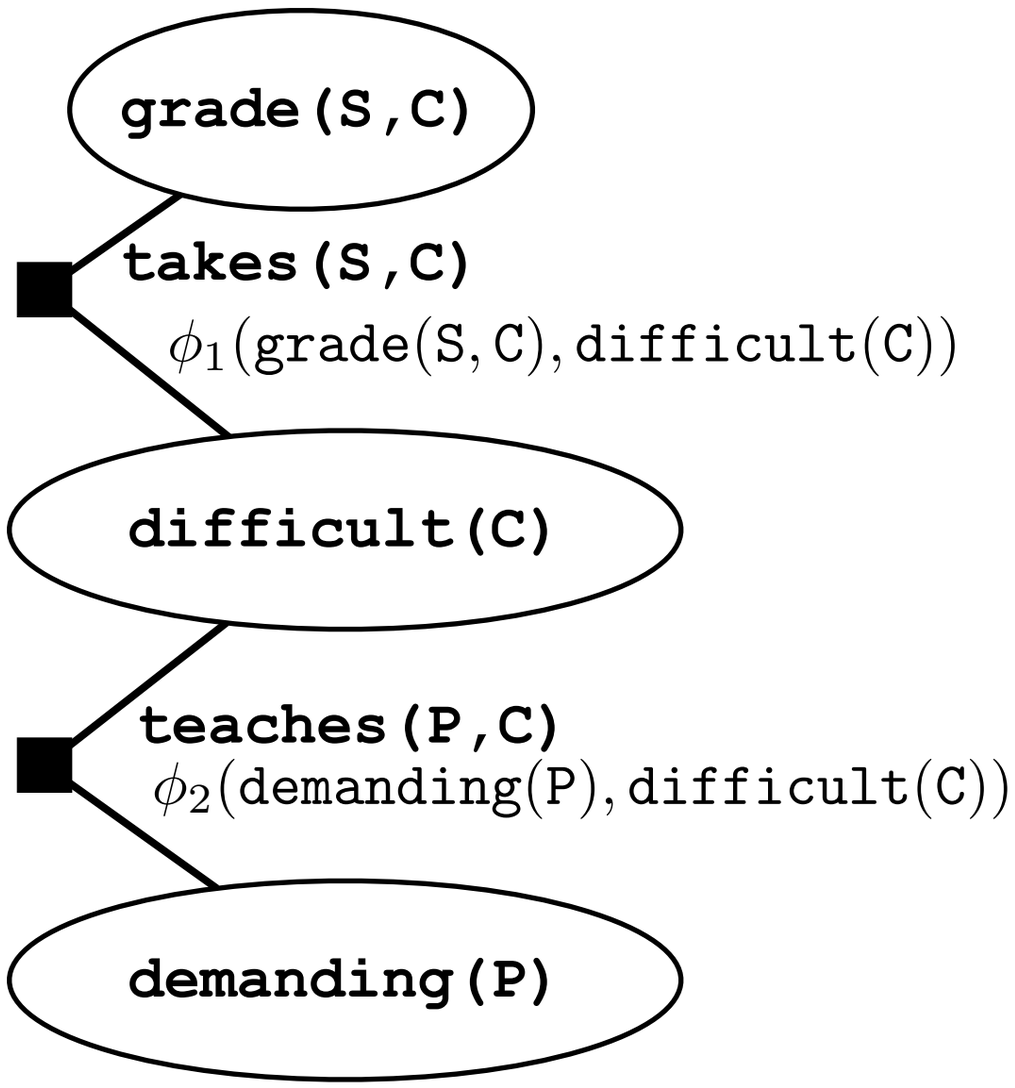
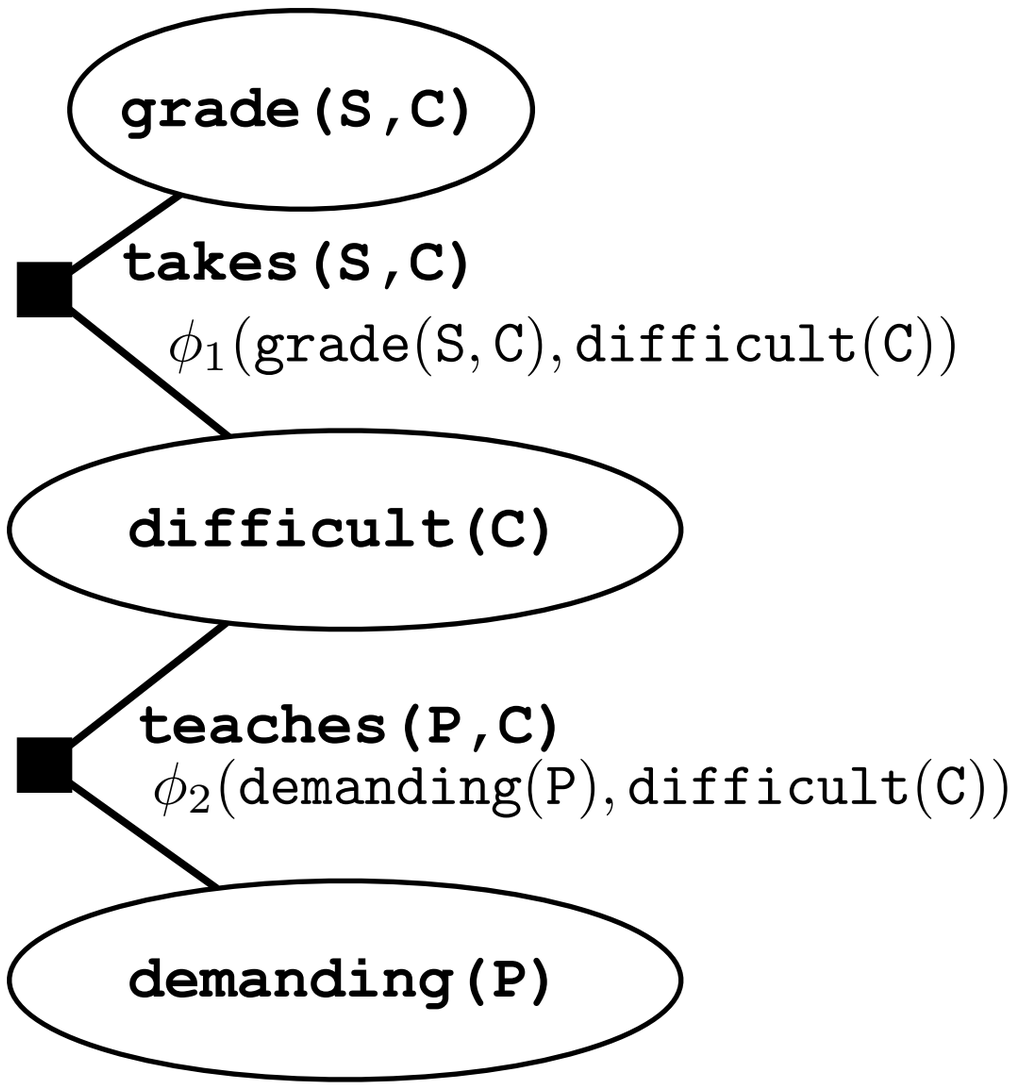


Inference by Grounding



demanding(tom)?

```
 takes(ann,ml).
 takes(bob,ml).
 takes(ann,db).
teaches(dan,db).
teaches(tom,ml).
```

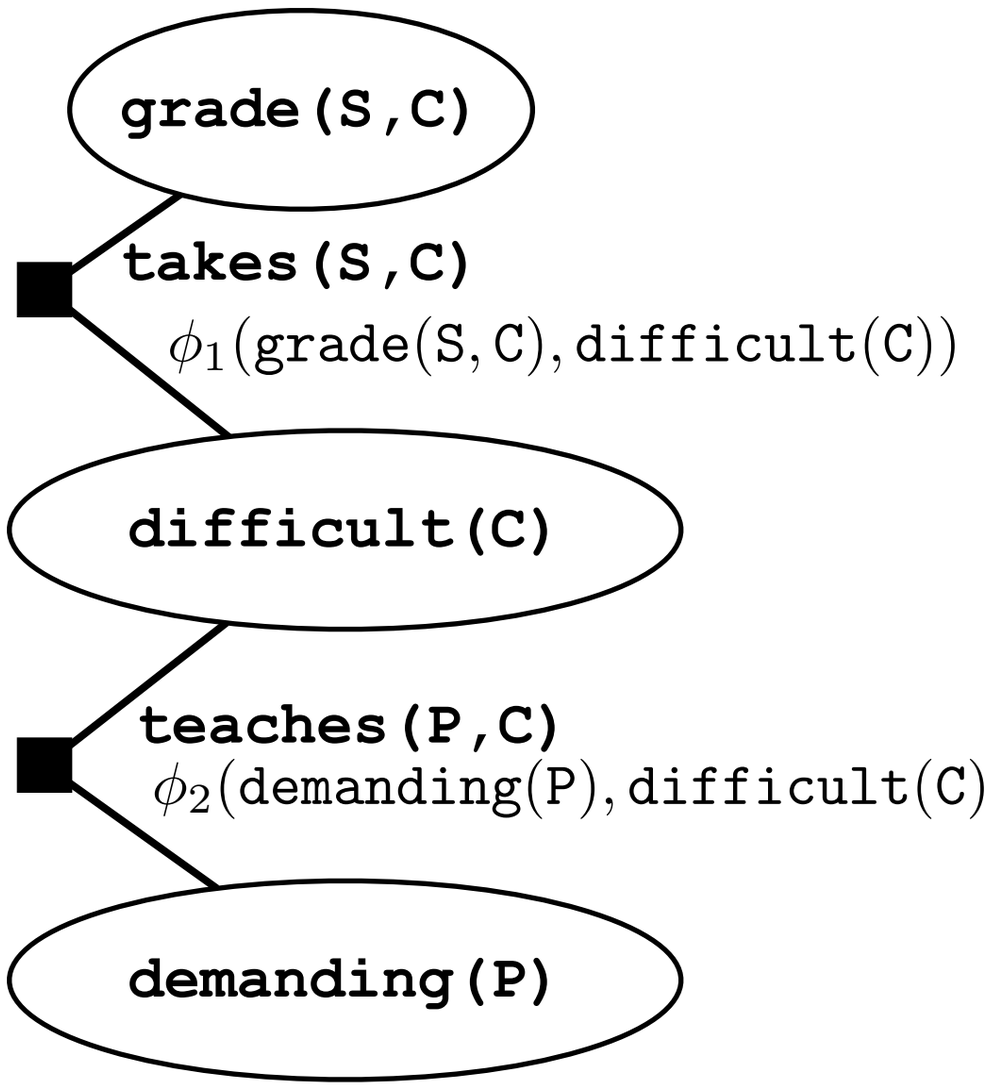



```
 takes(ann,ml).
 takes(bob,ml).
 takes(ann,db).
teaches(dan,db).
teaches(tom,ml).
```

Inference by Grounding

demanding(tom)?

1. construct factor graph
2. run any propositional inference technique

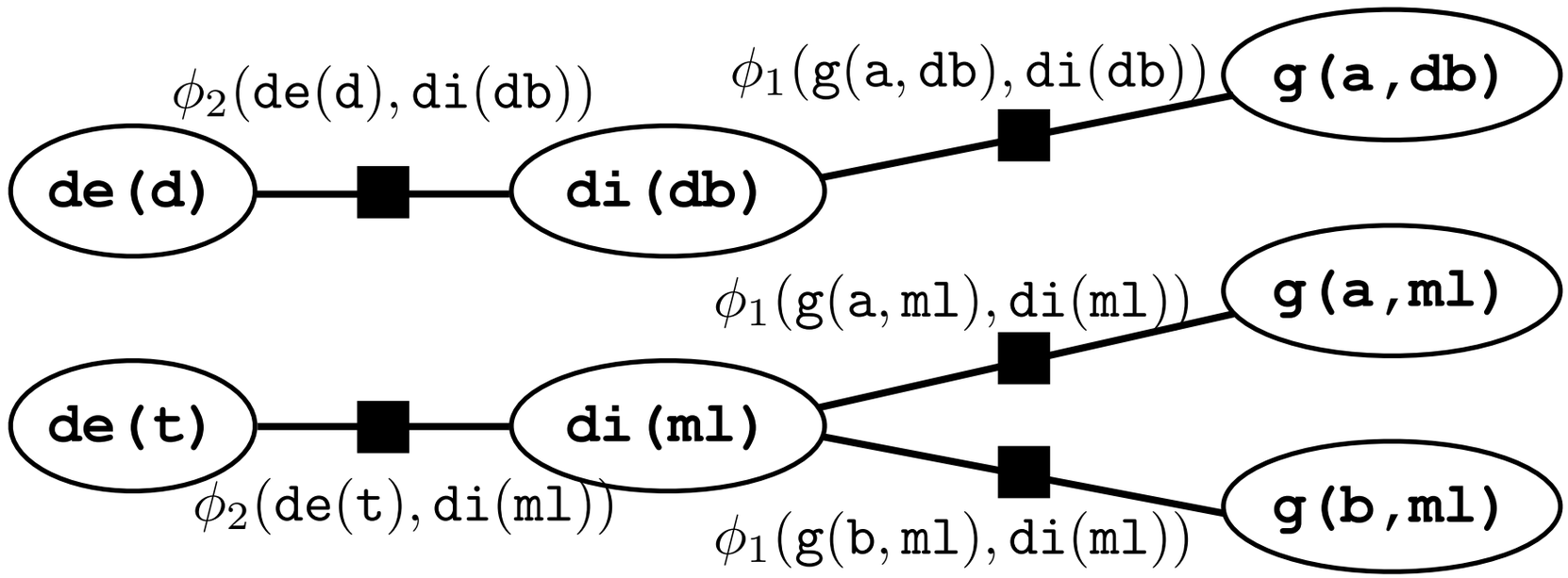



```
takes(ann,ml).
takes(bob,ml).
takes(ann,db).
teaches(dan,db).
teaches(tom,ml).
```

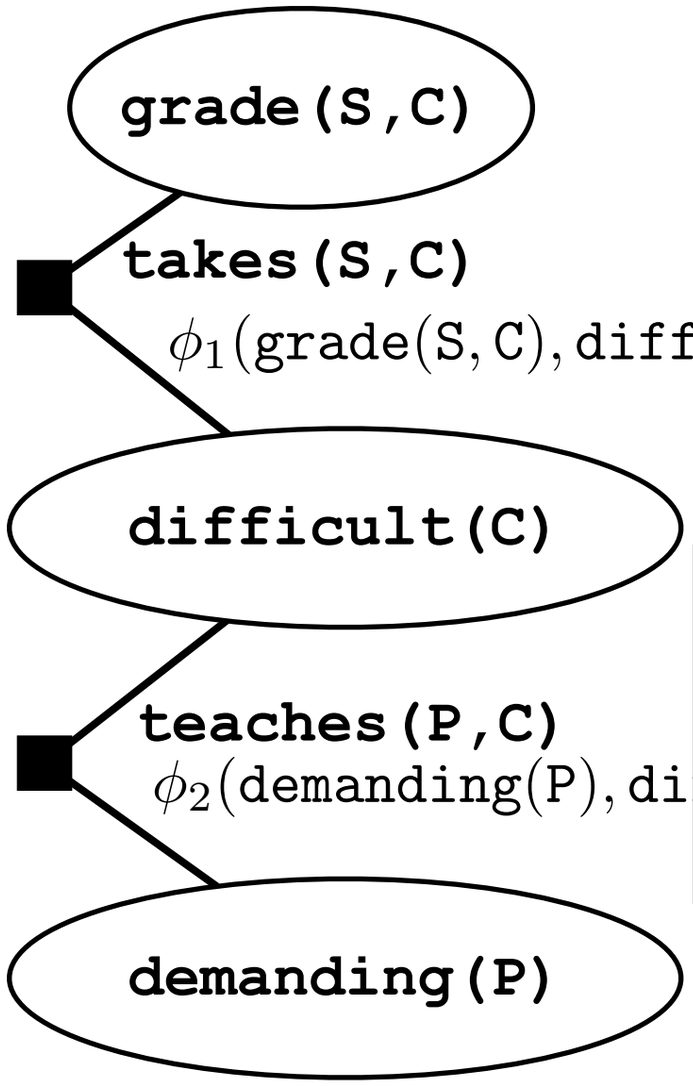
Inference by Grounding

demanding(tom)?

1. construct factor graph
2. run any propositional inference technique



Inference by Grounding

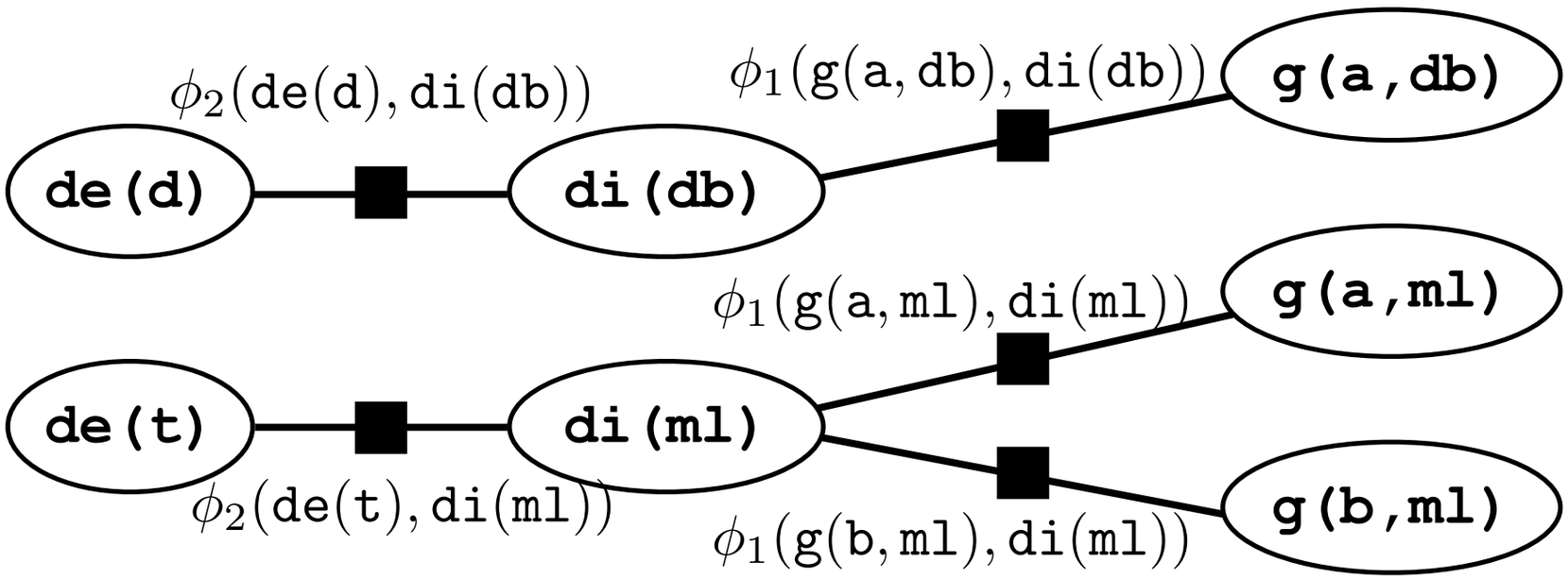


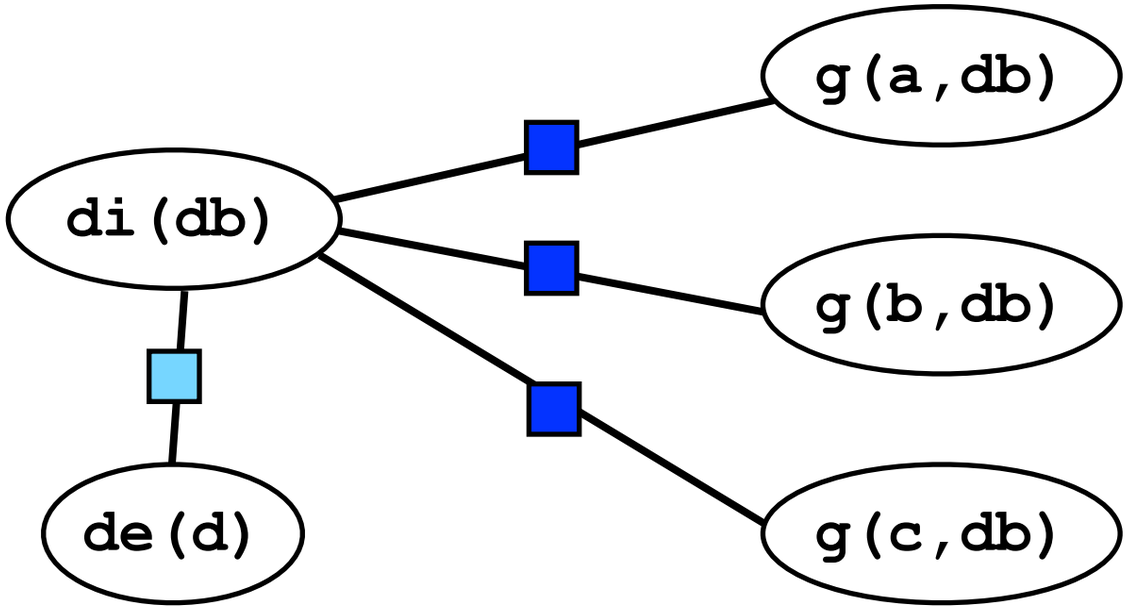
Problem: often huge factor graphs with many repetitions or symmetries (even if restricted to relevant parts)

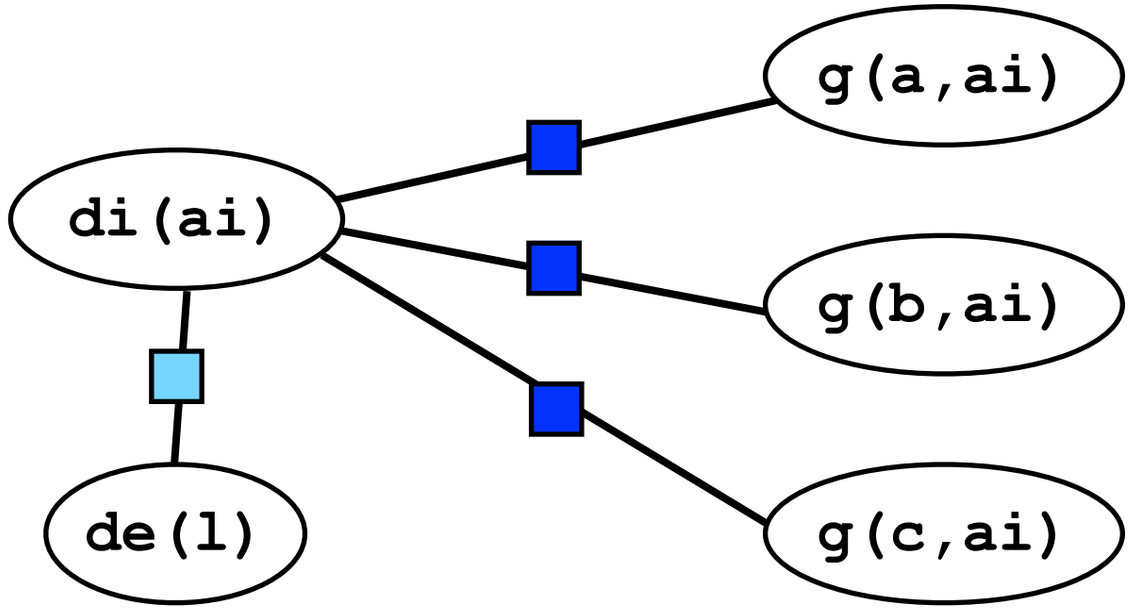
demanding(tom)?

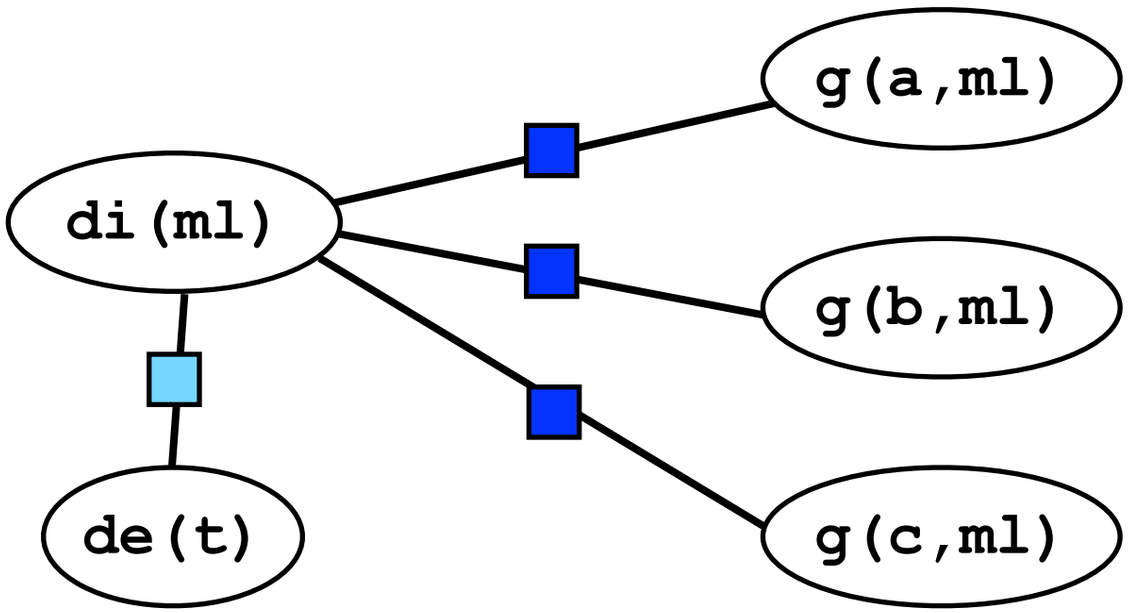
1. Unroll factor graph
2. Run any propositional inference technique

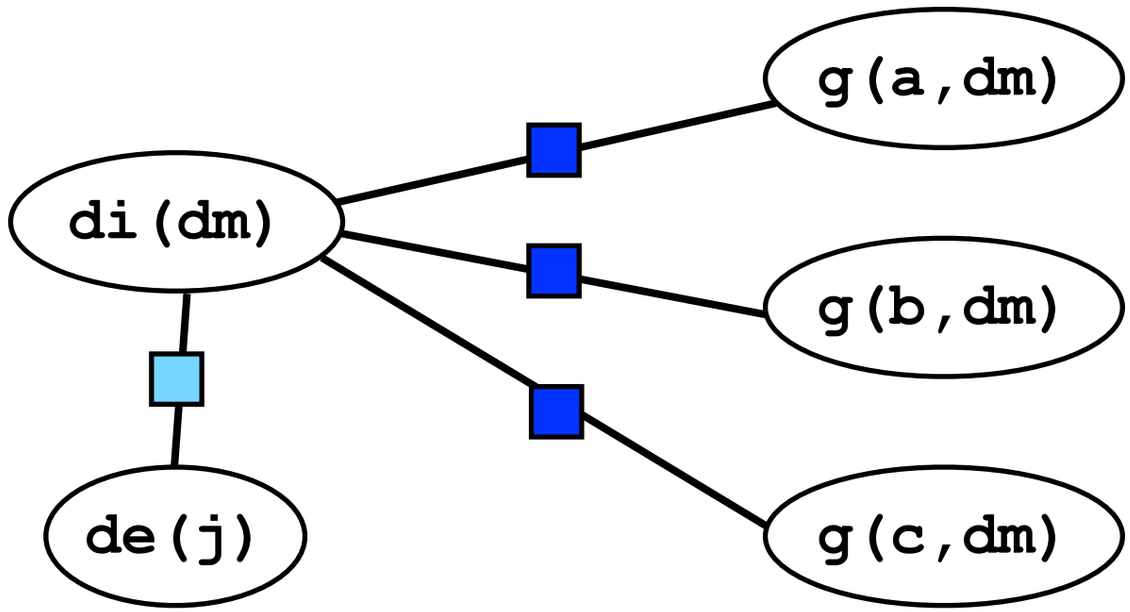
```
takes(ann,ml).
takes(bob,ml).
takes(ann,db).
teaches(dan,db).
teaches(tom,ml).
```

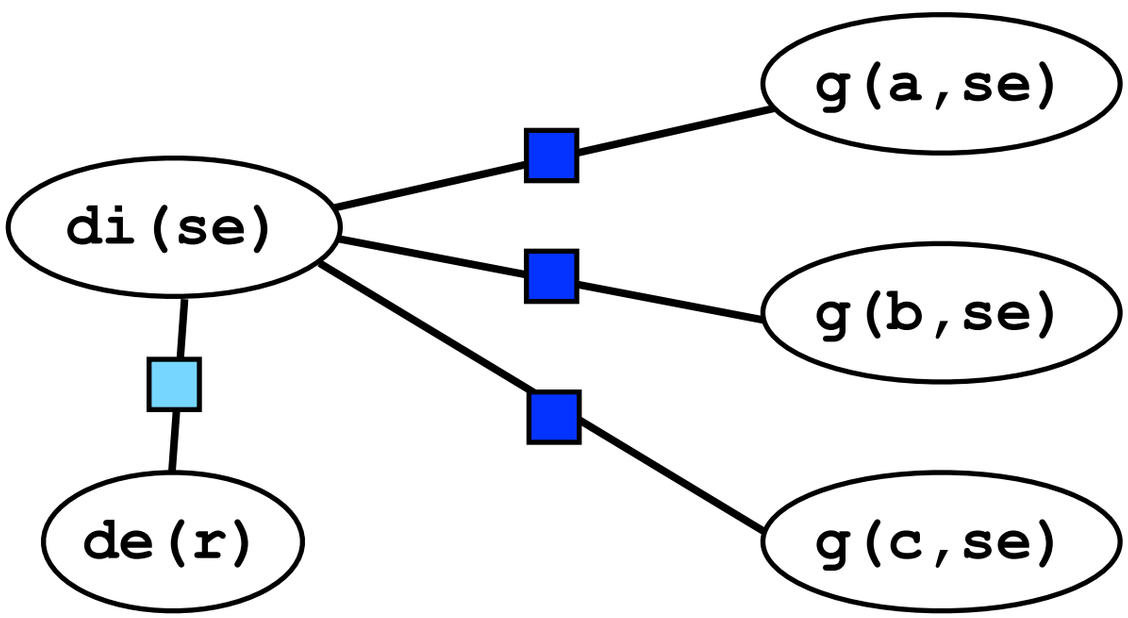


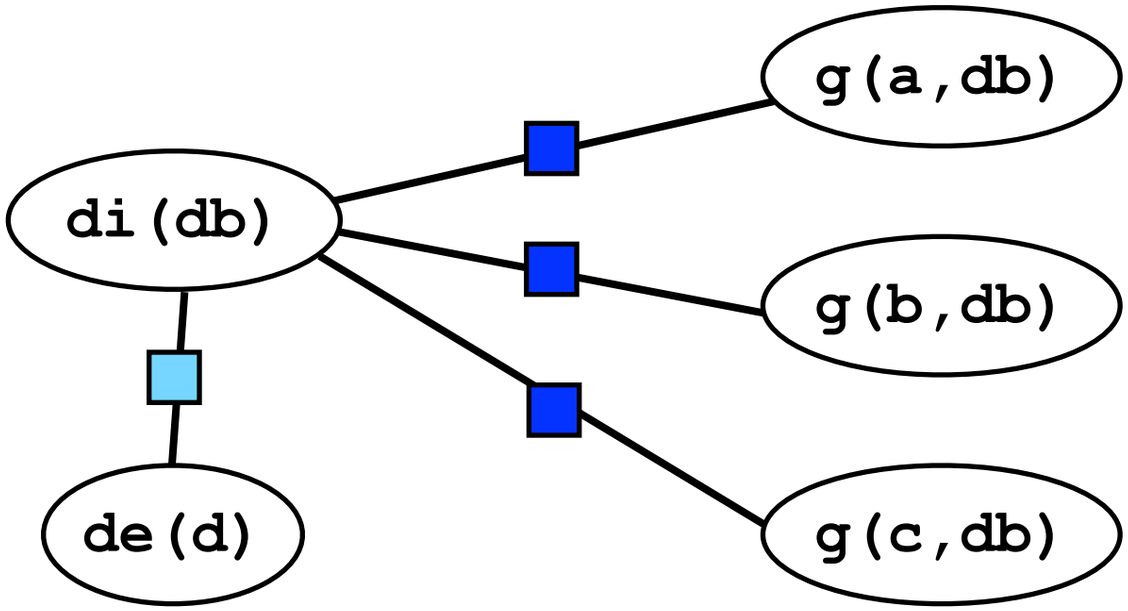

g(a,db)
di(db)
g(b,db)
de(d)
g(c,db)

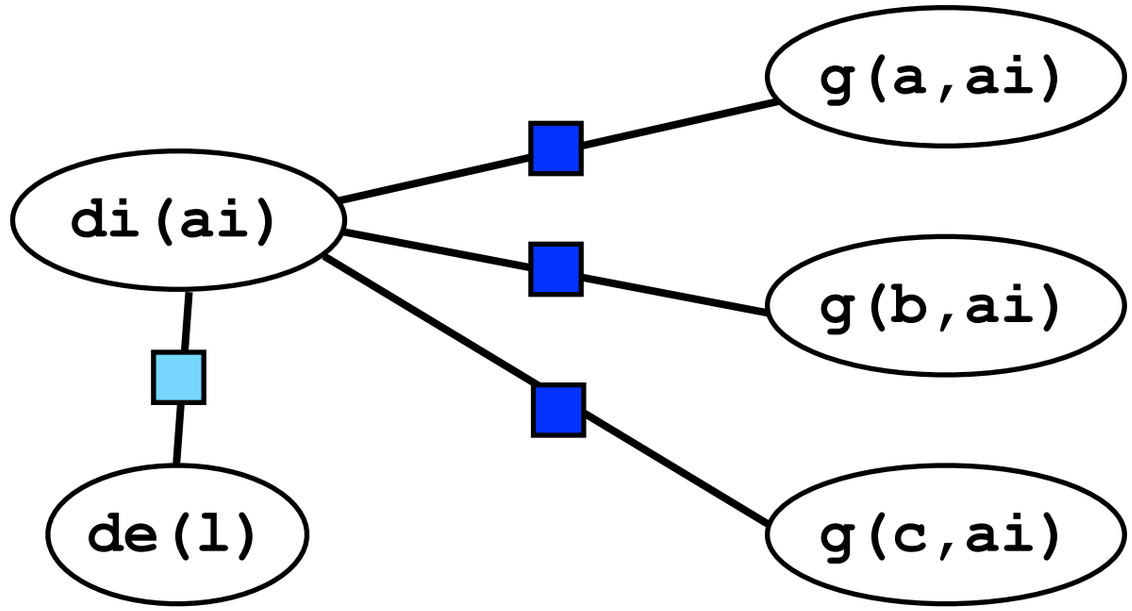

g(a,ai)
di(ai)
g(b,ai)
de(l)
g(c,ai)

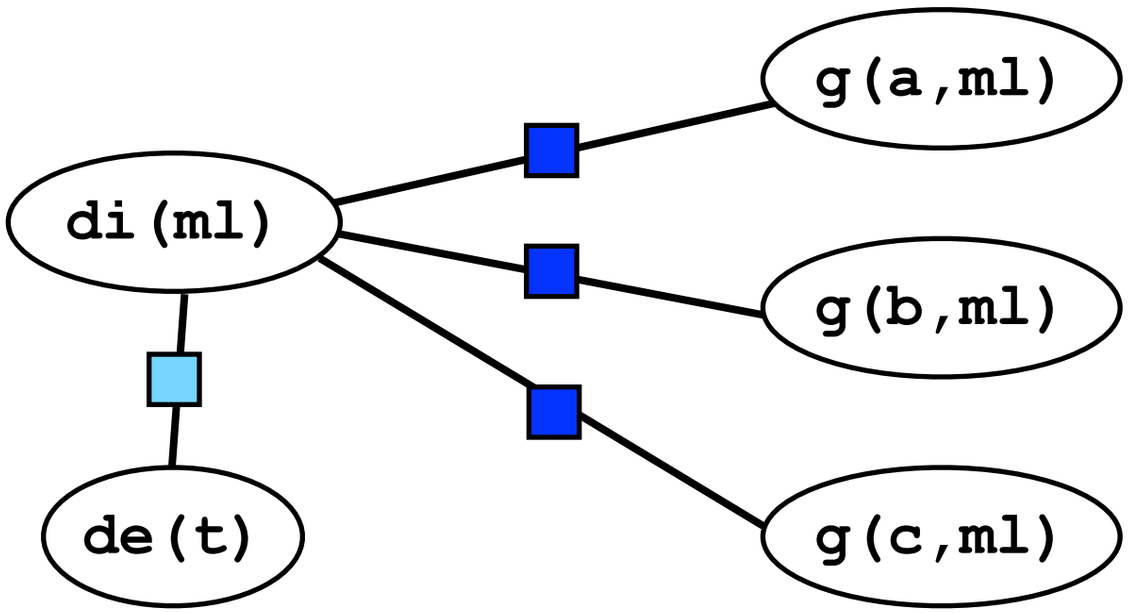

g(a,ml)
di(ml)
g(b,ml)
de(t)
g(c,ml)

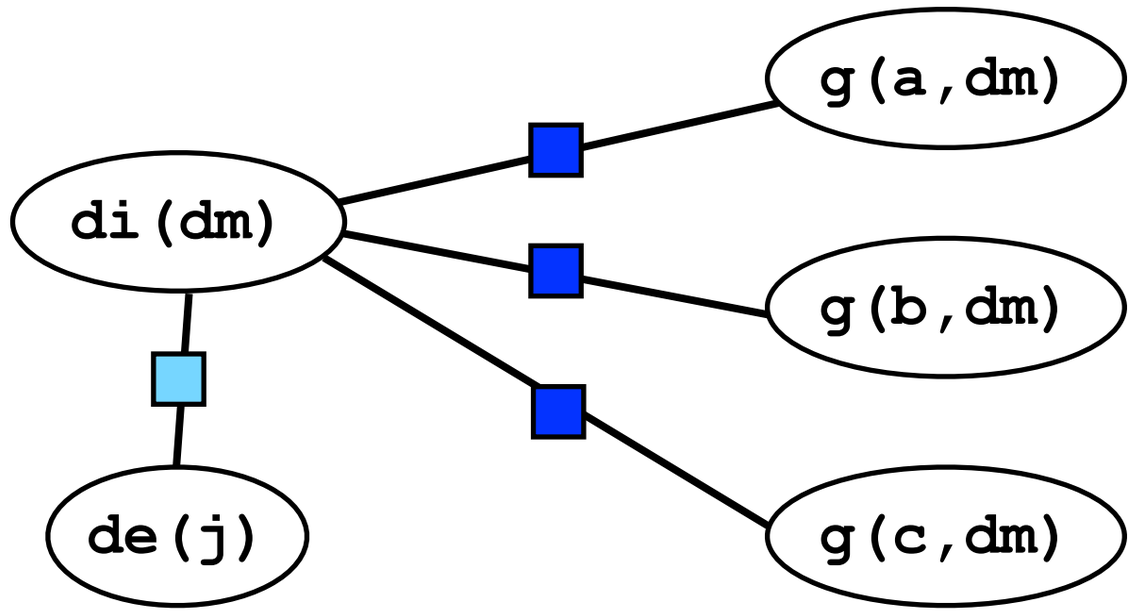

g(a,dm)
di(dm)
g(b,dm)
de(j)
g(c,dm)

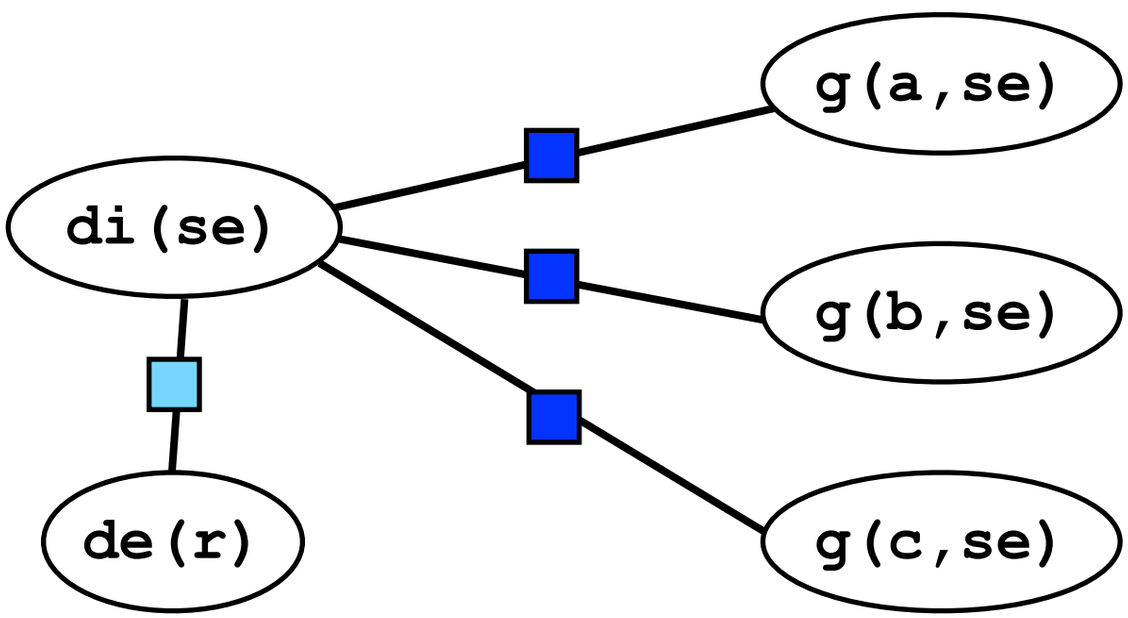

g(a,se)
di(se)
g(b,se)
de(r)
g(c,se)



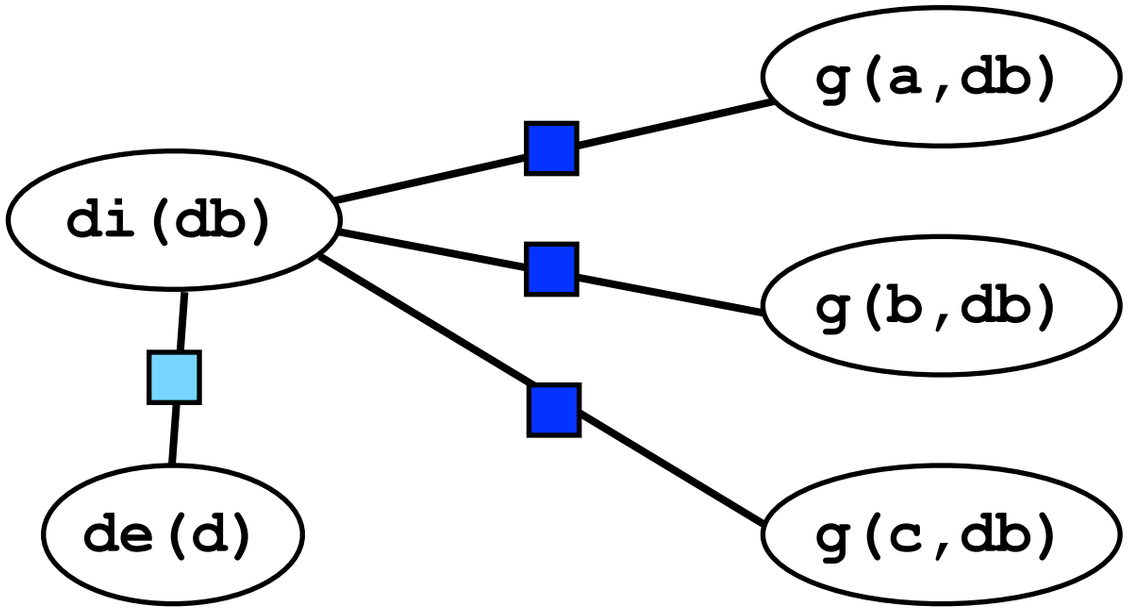


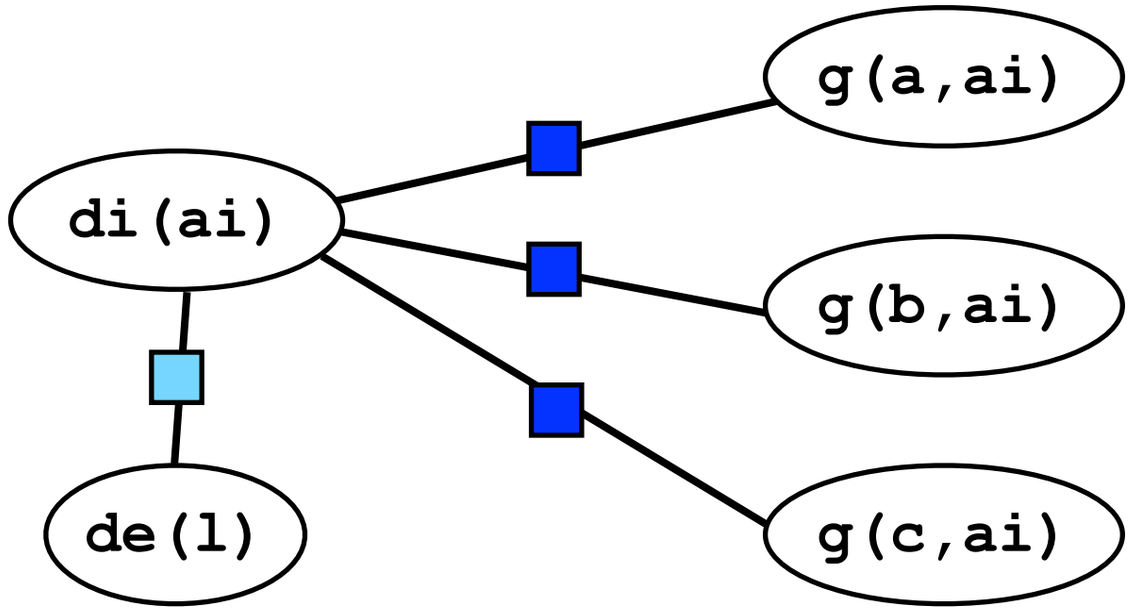


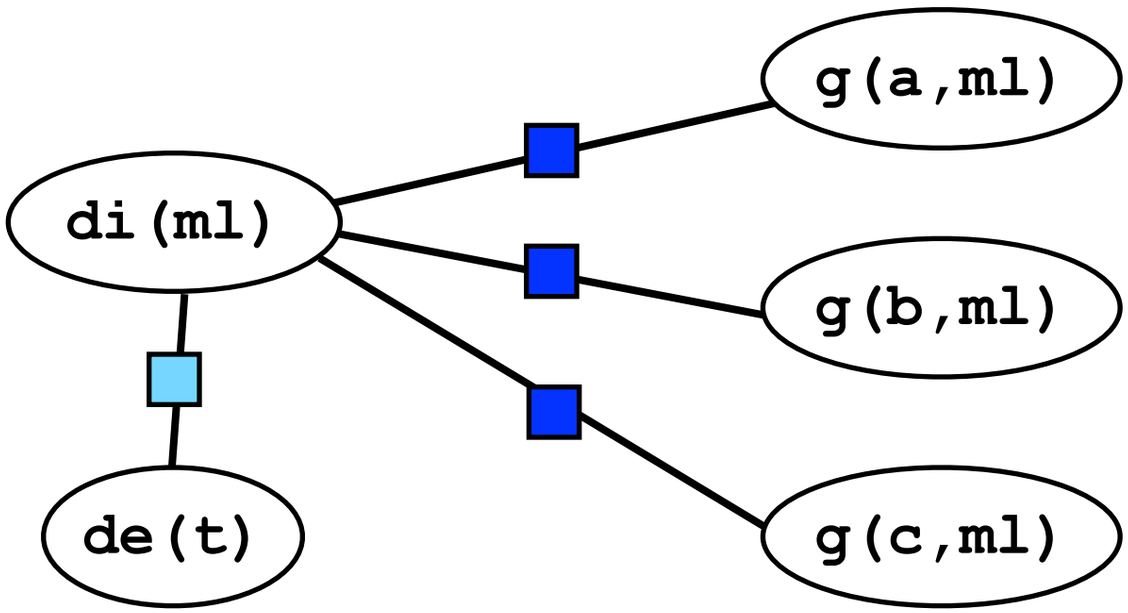


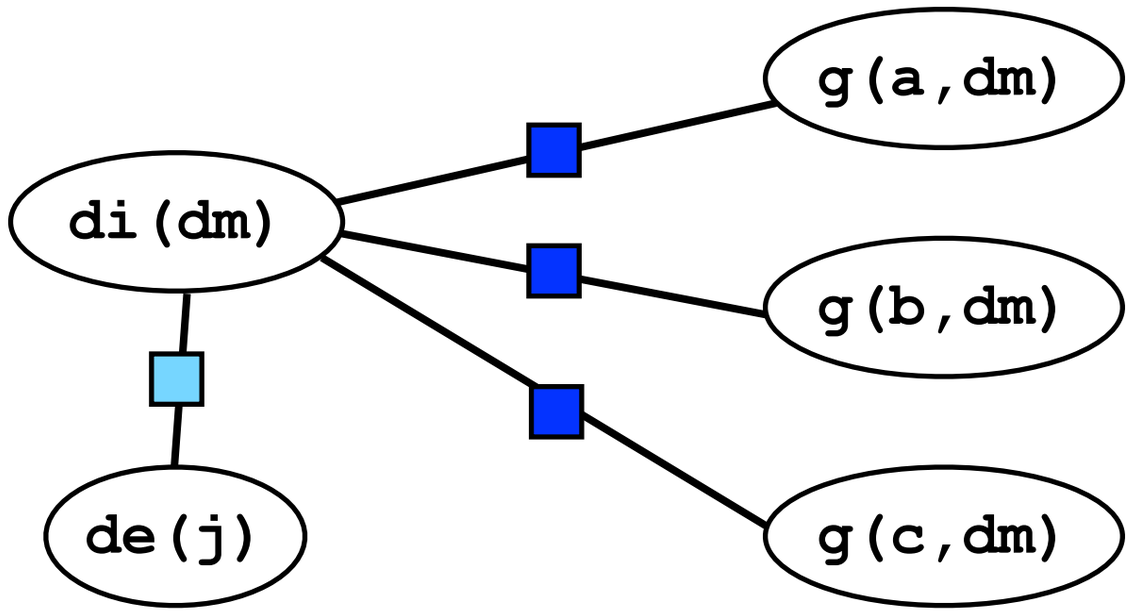


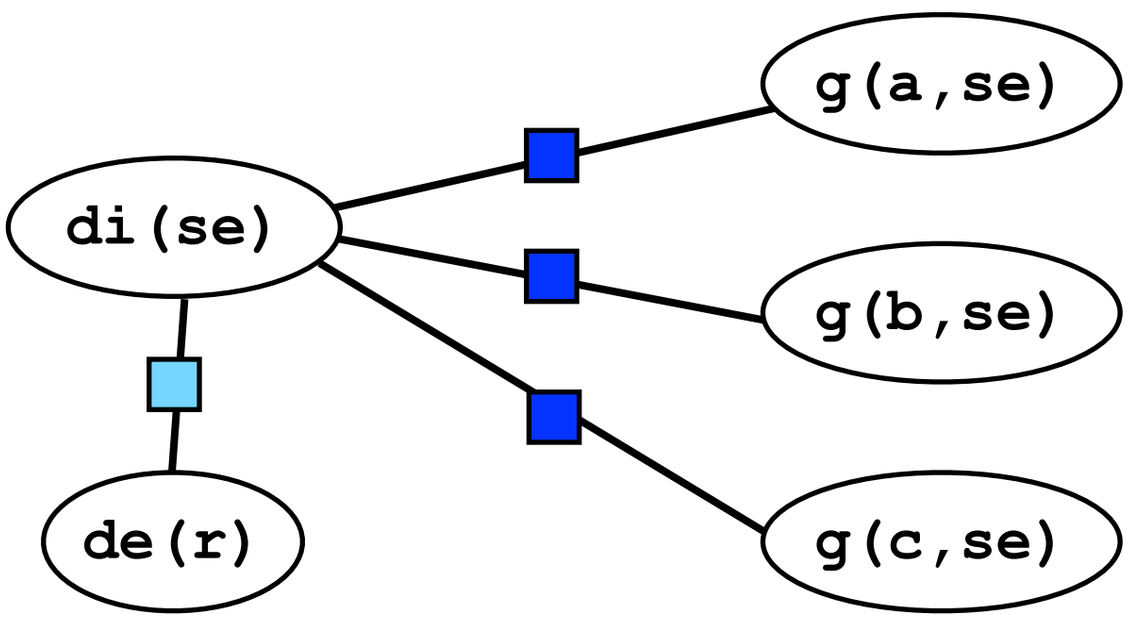
compute probability distribution over grades for every student-course pair





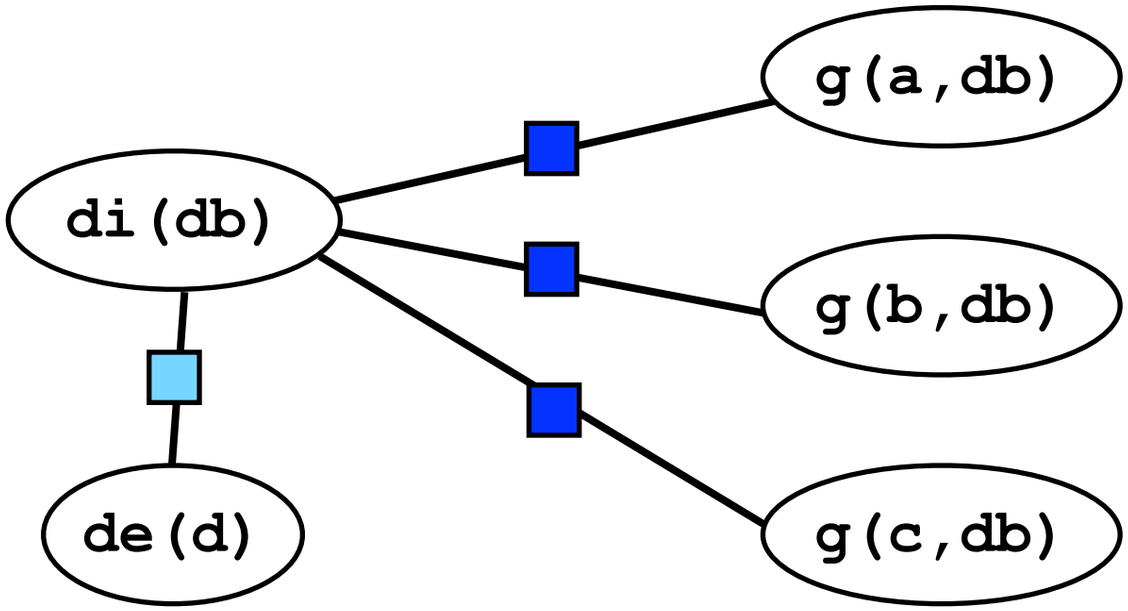


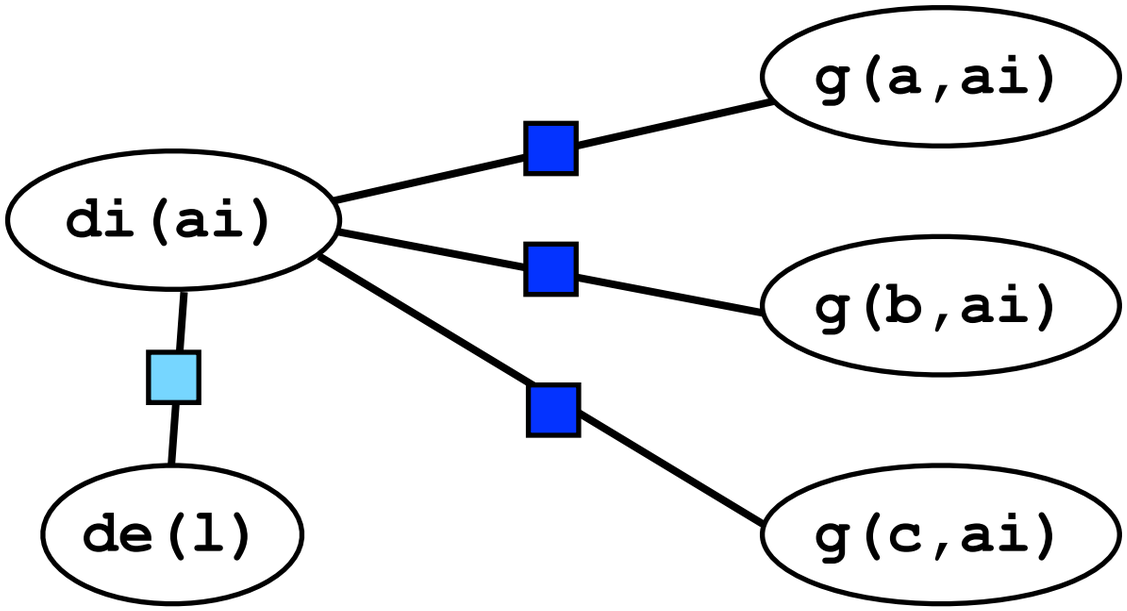


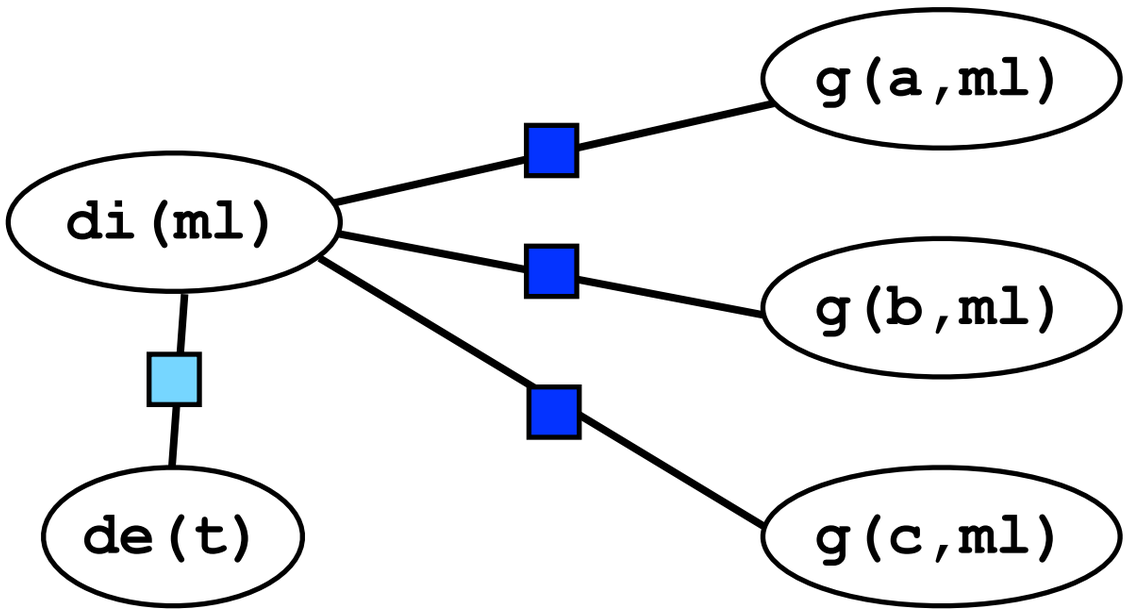


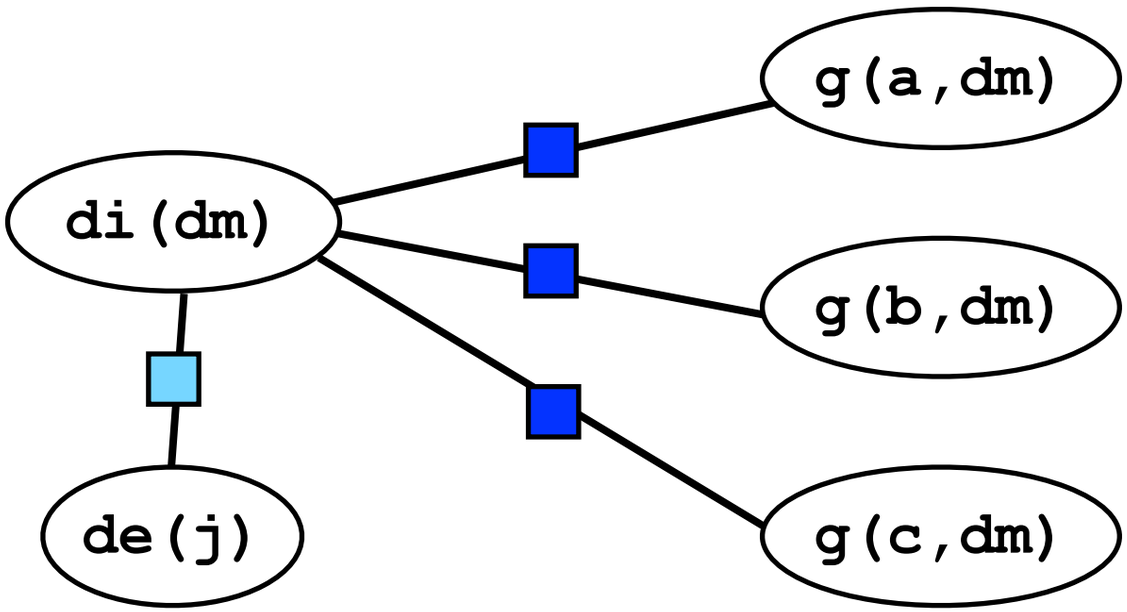
compute probability distribution over grades for every student-course pair

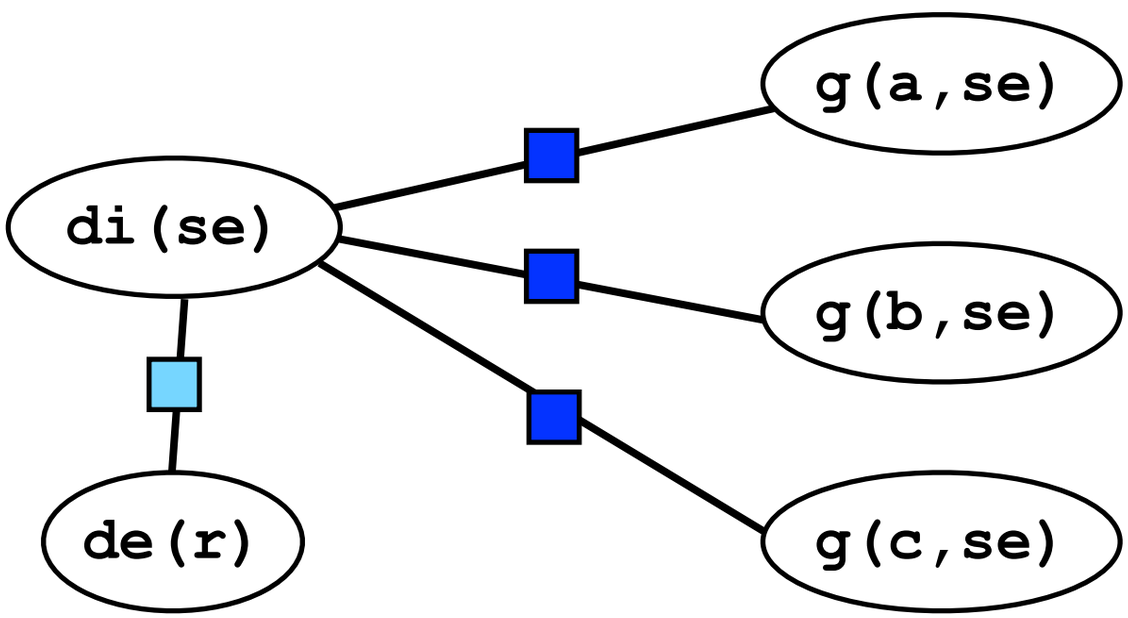
same computation for each pair!



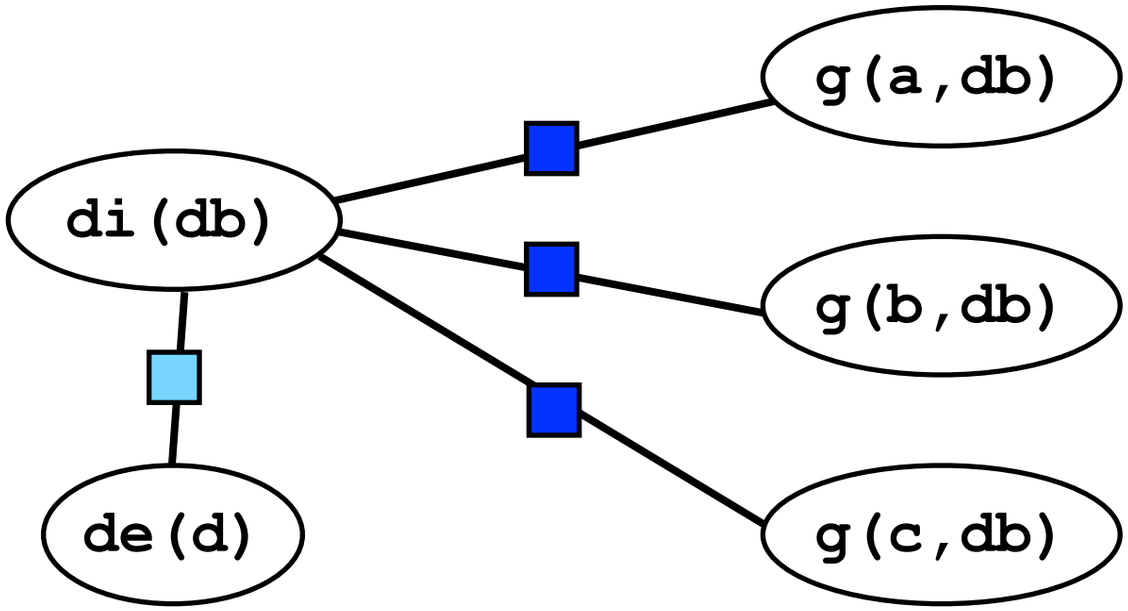


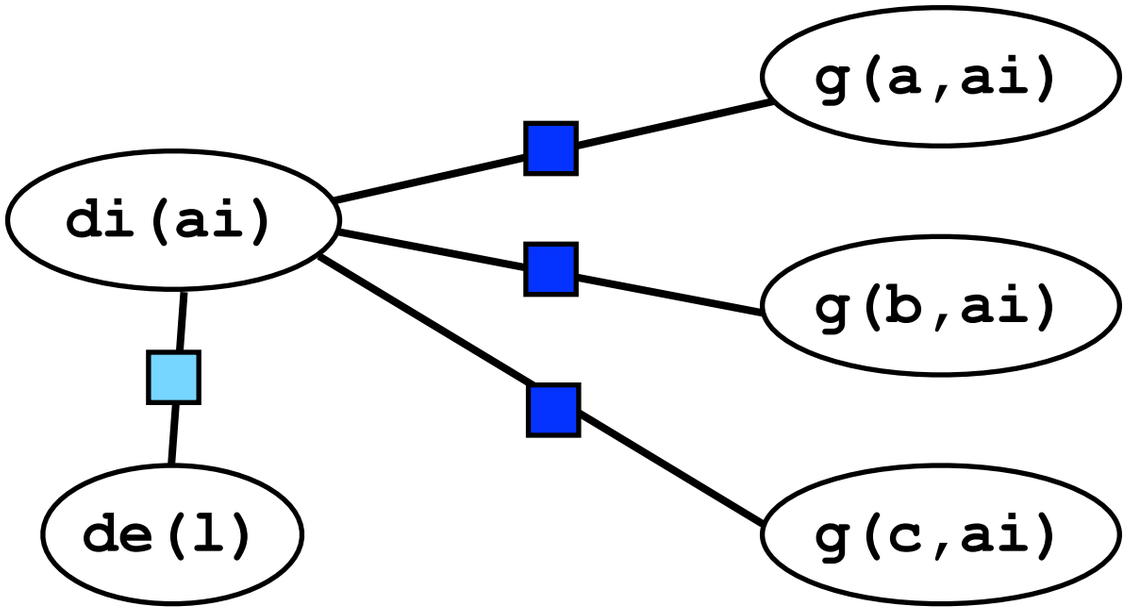


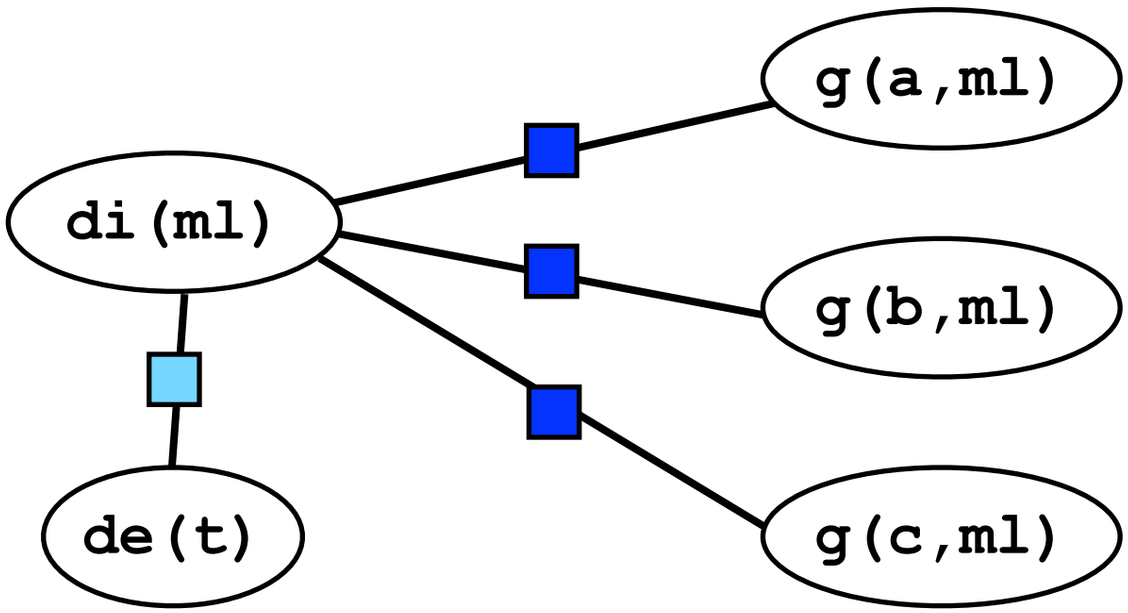


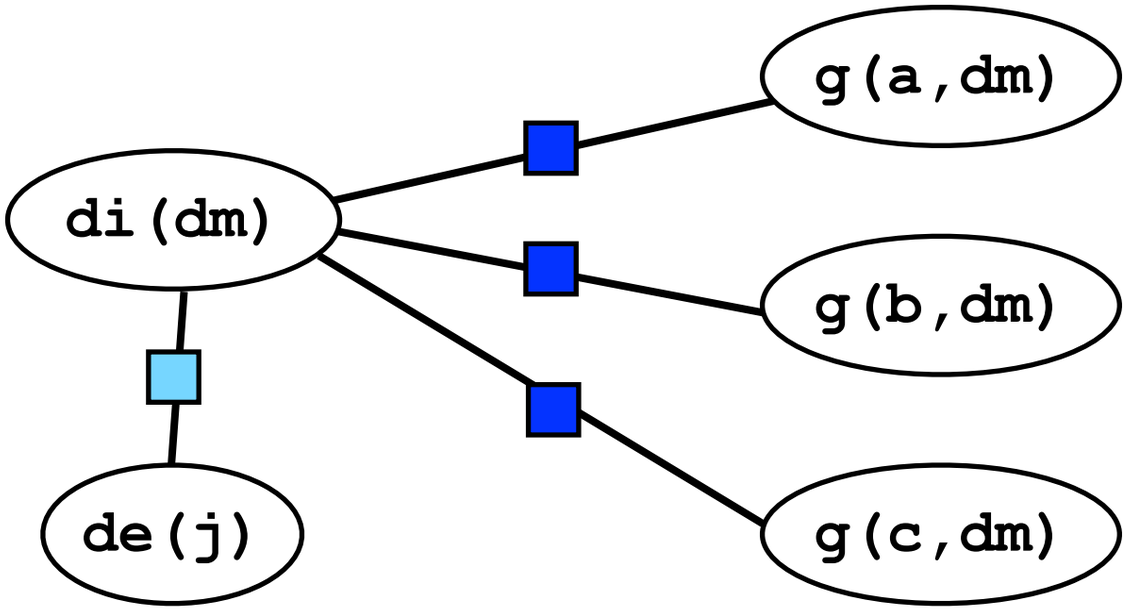


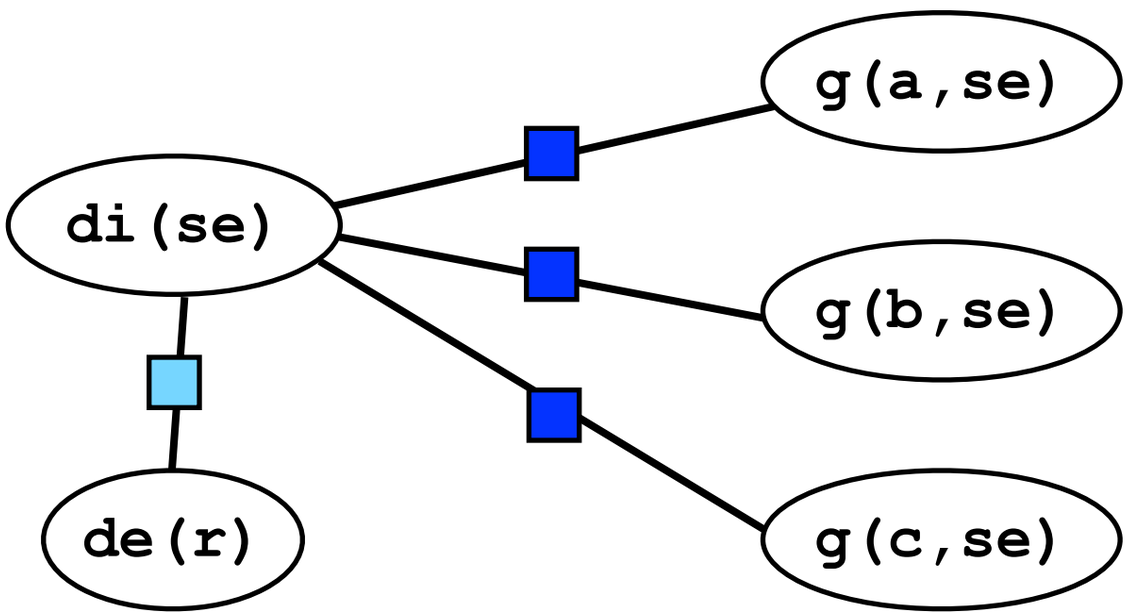
what is the probability that some professor is demanding?











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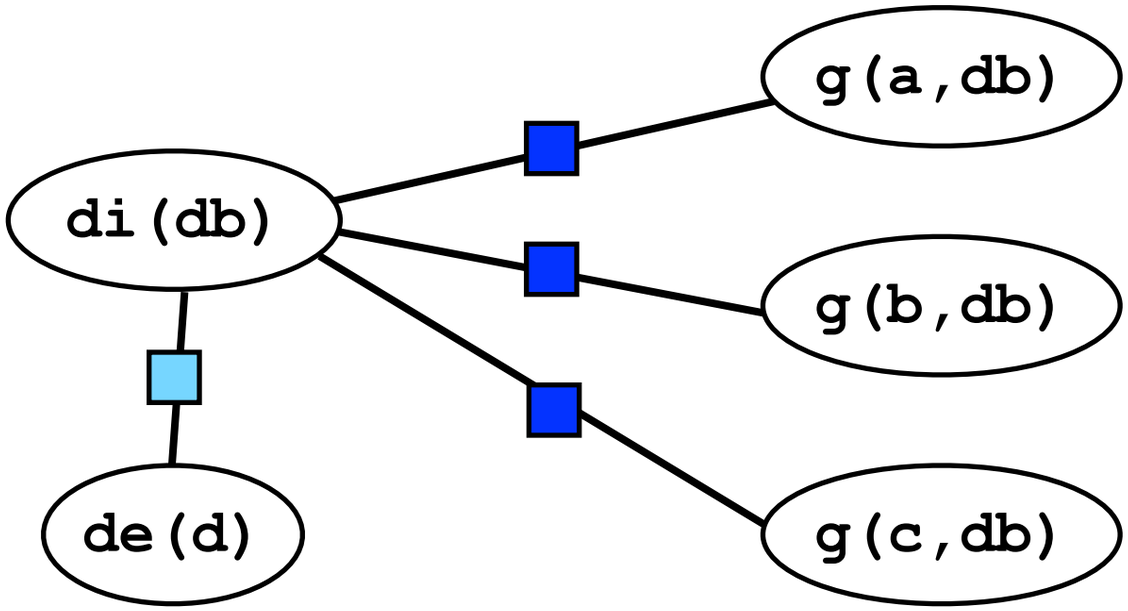

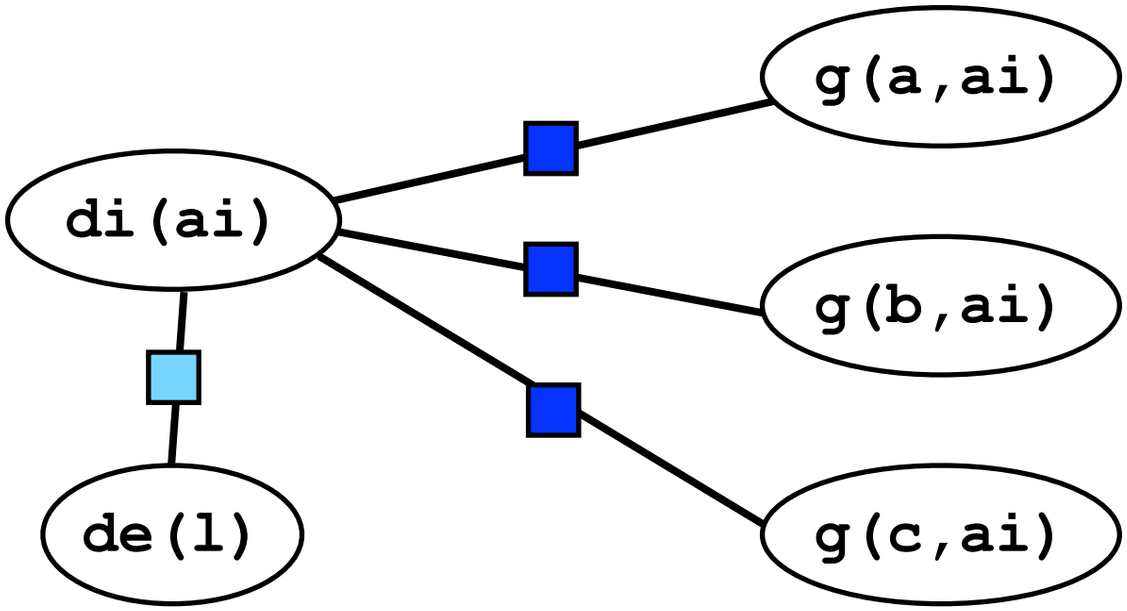

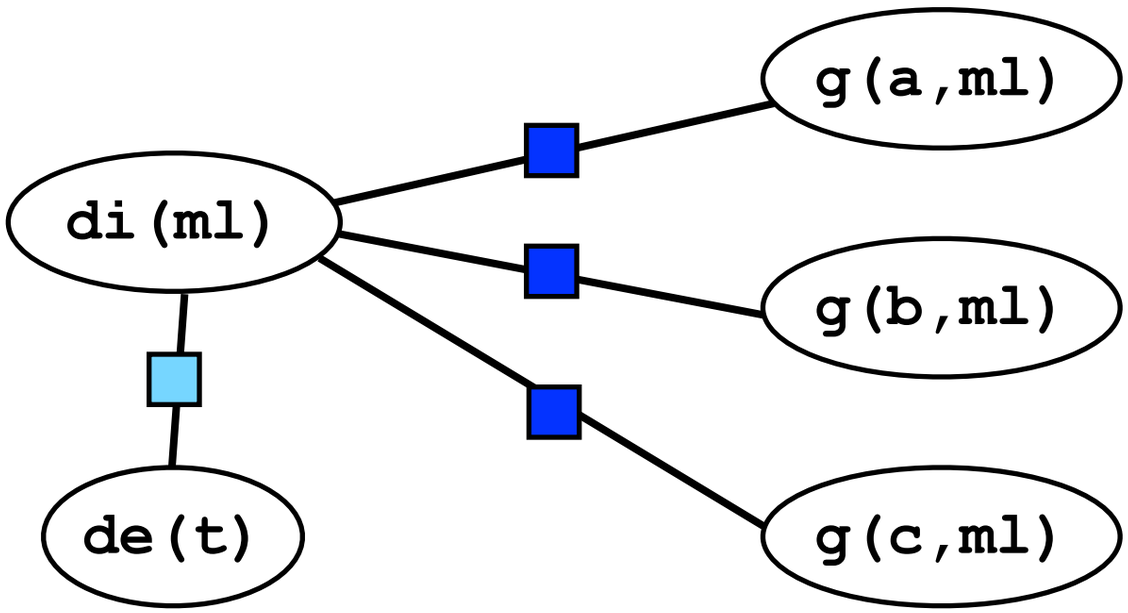

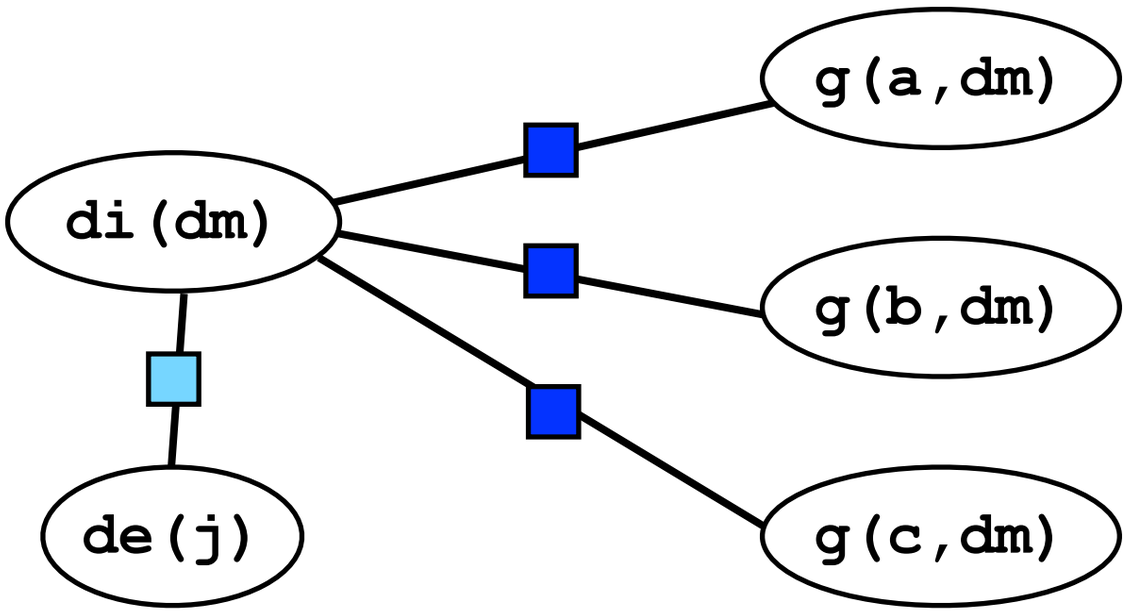

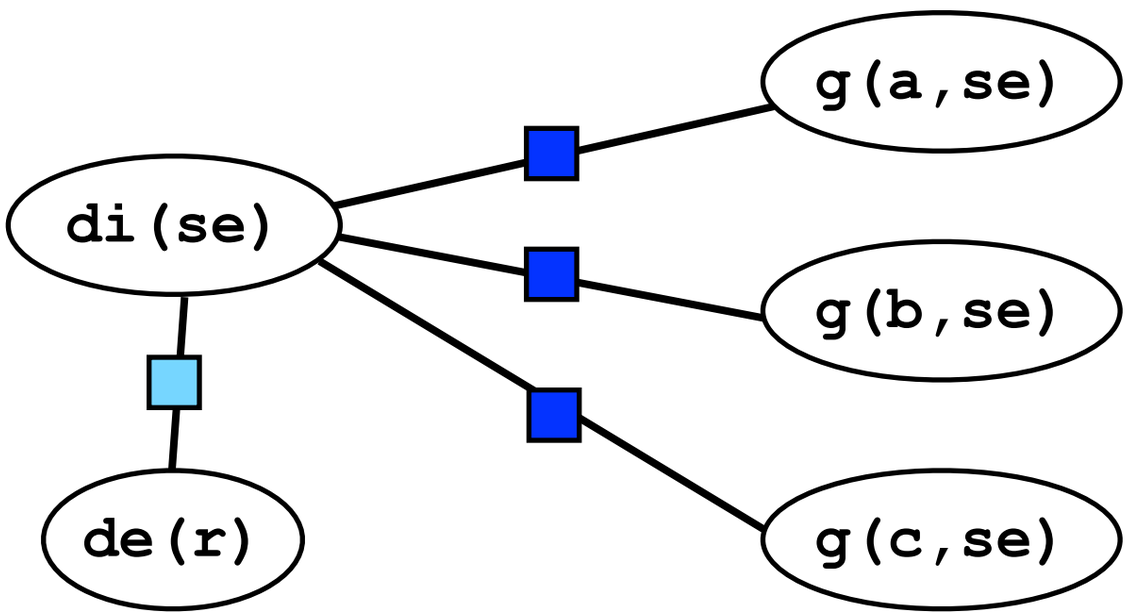

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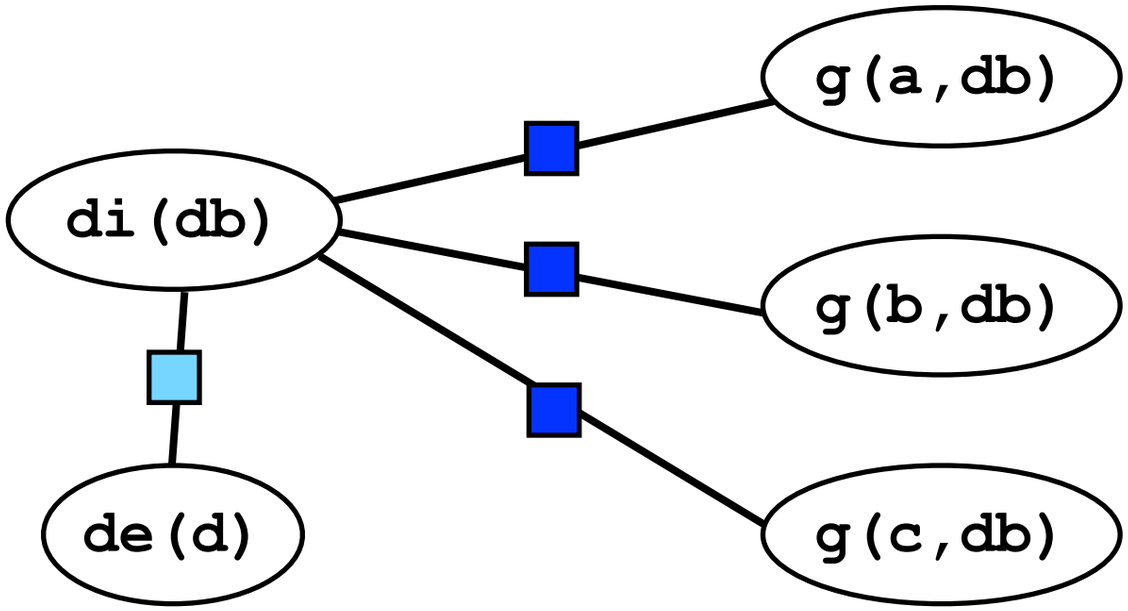
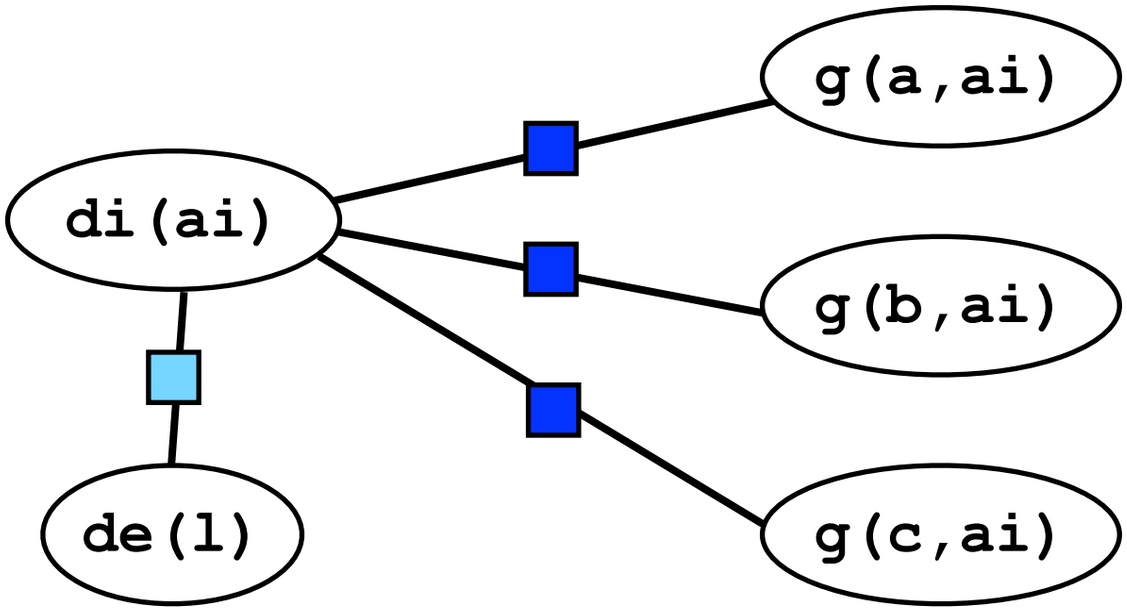
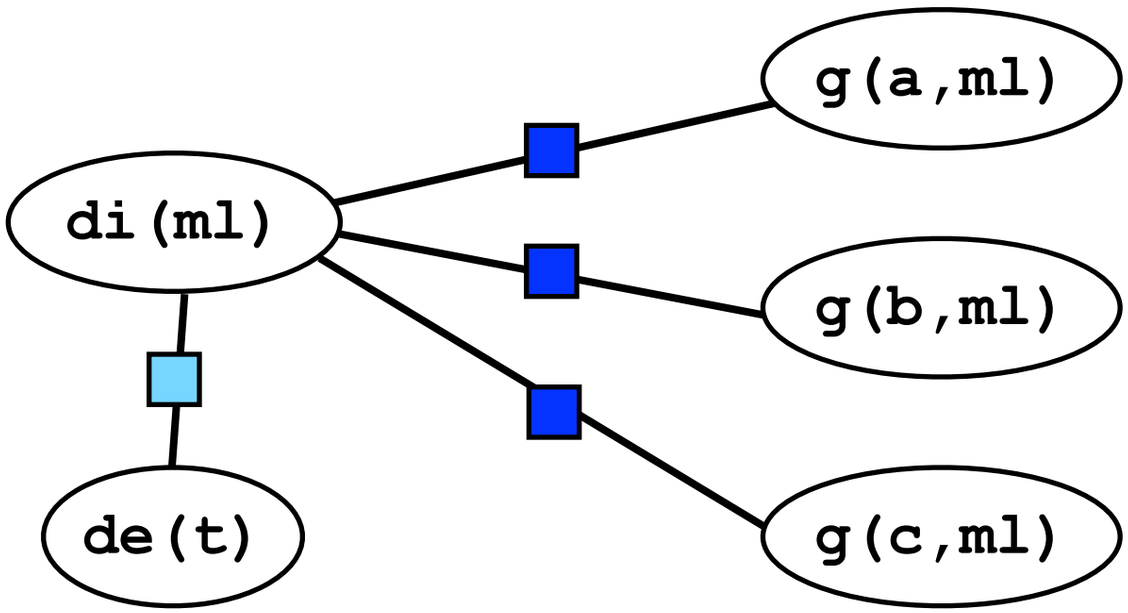
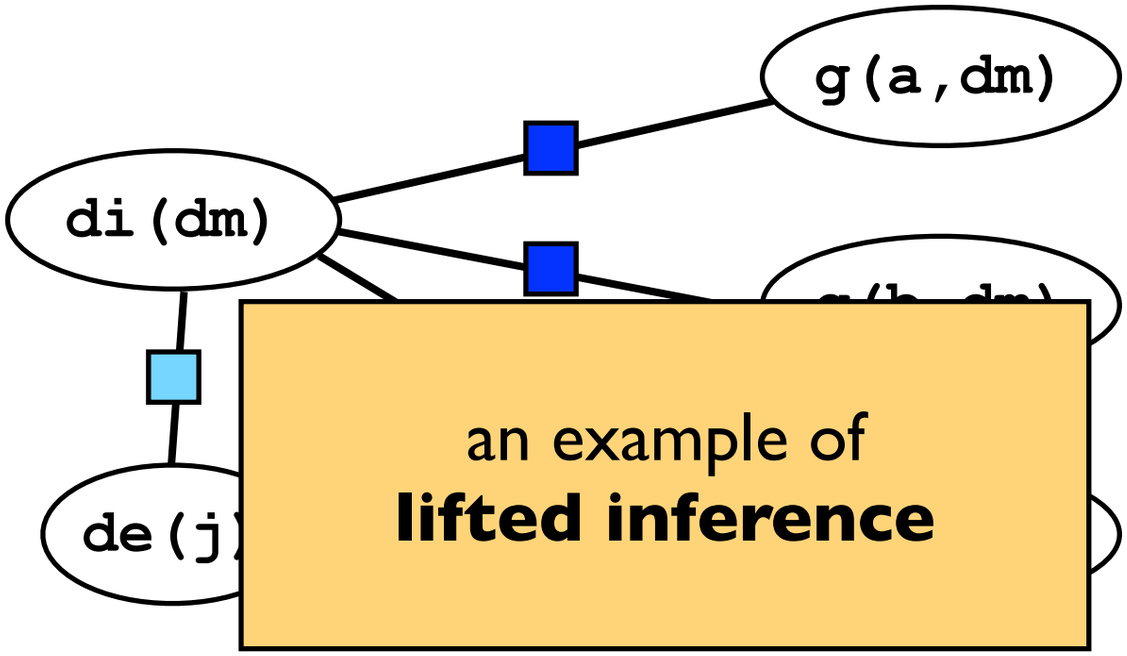
$$1 - \prod_{\mathtt{P}} (1 - P(\mathtt{de(P)})) = 1 - (1 - P(\mathtt{de(someP)}))^{\#\mathtt{P}}$$

$$= 1 - (1 - P(\mathtt{de(r)}))^5$$

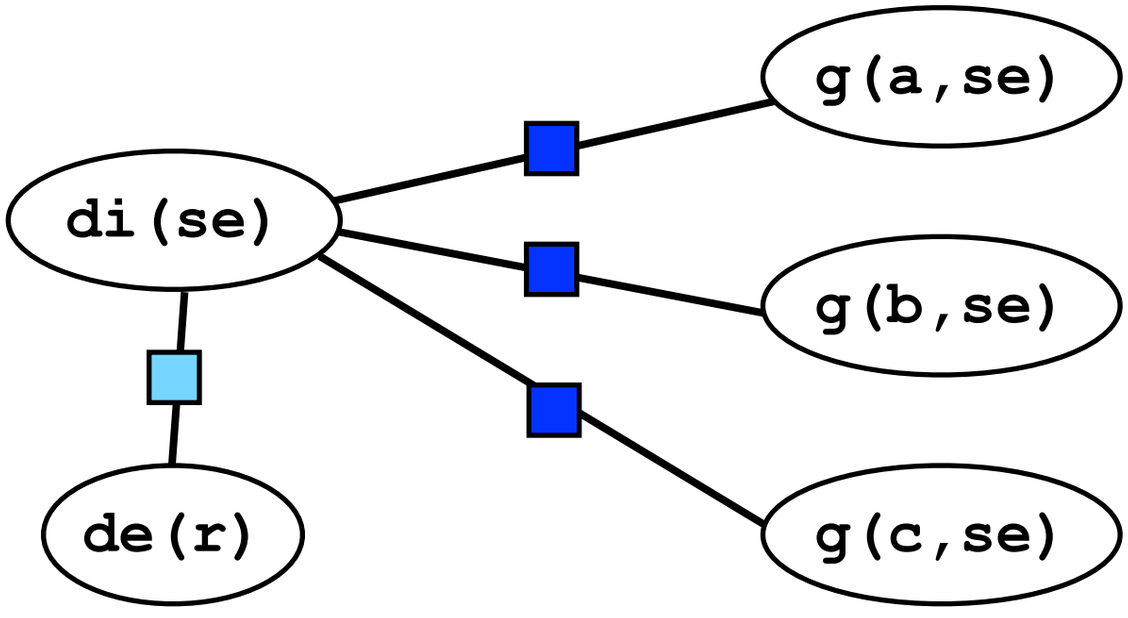








an example of
lifted inference



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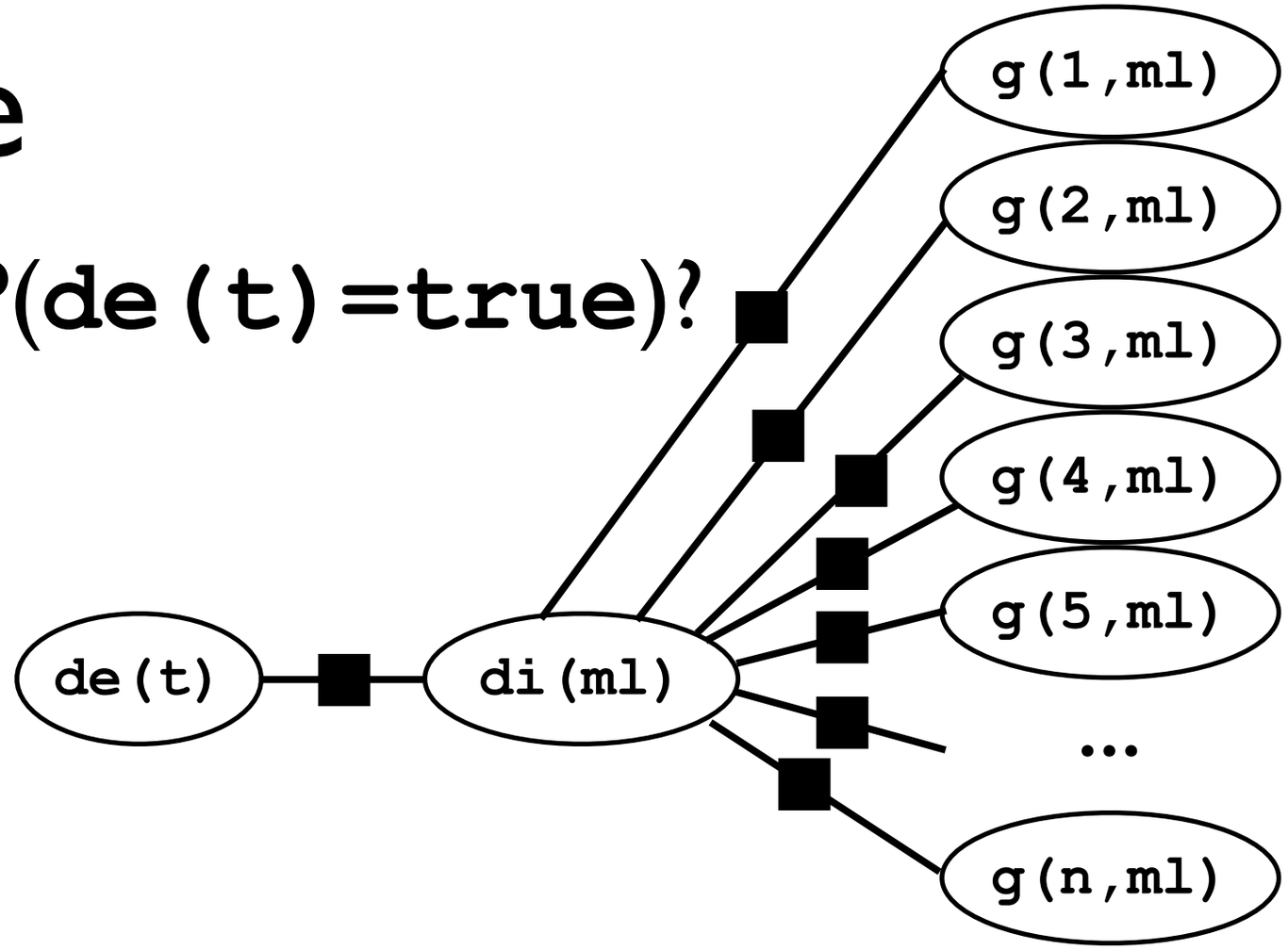
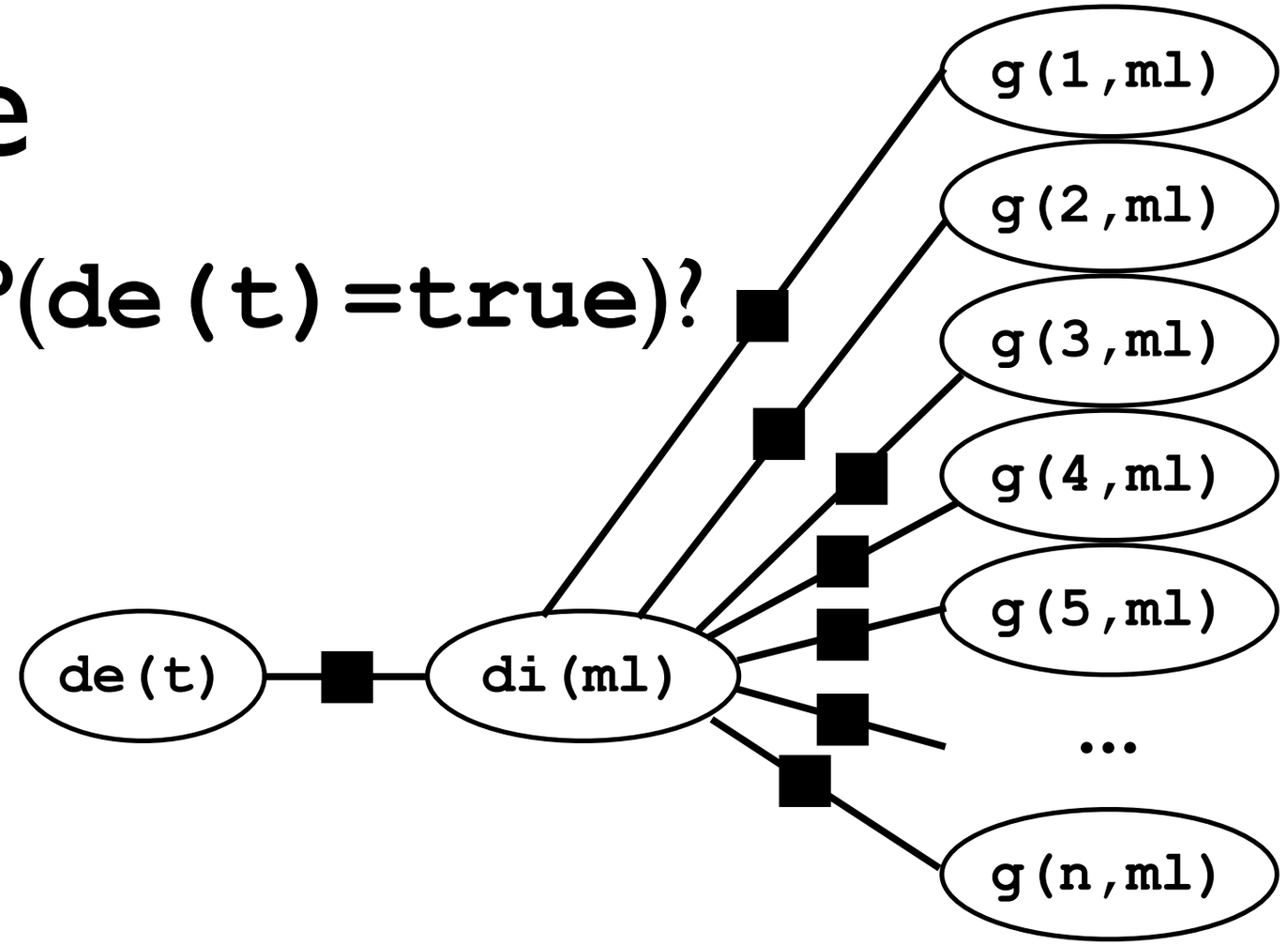


Another example

sum $\phi_2(\text{true}, d) \prod_{i=1..n} \phi_1(g_i, d)$

for all combinations of difficulty d and students' grades (g_1,...,g_n)



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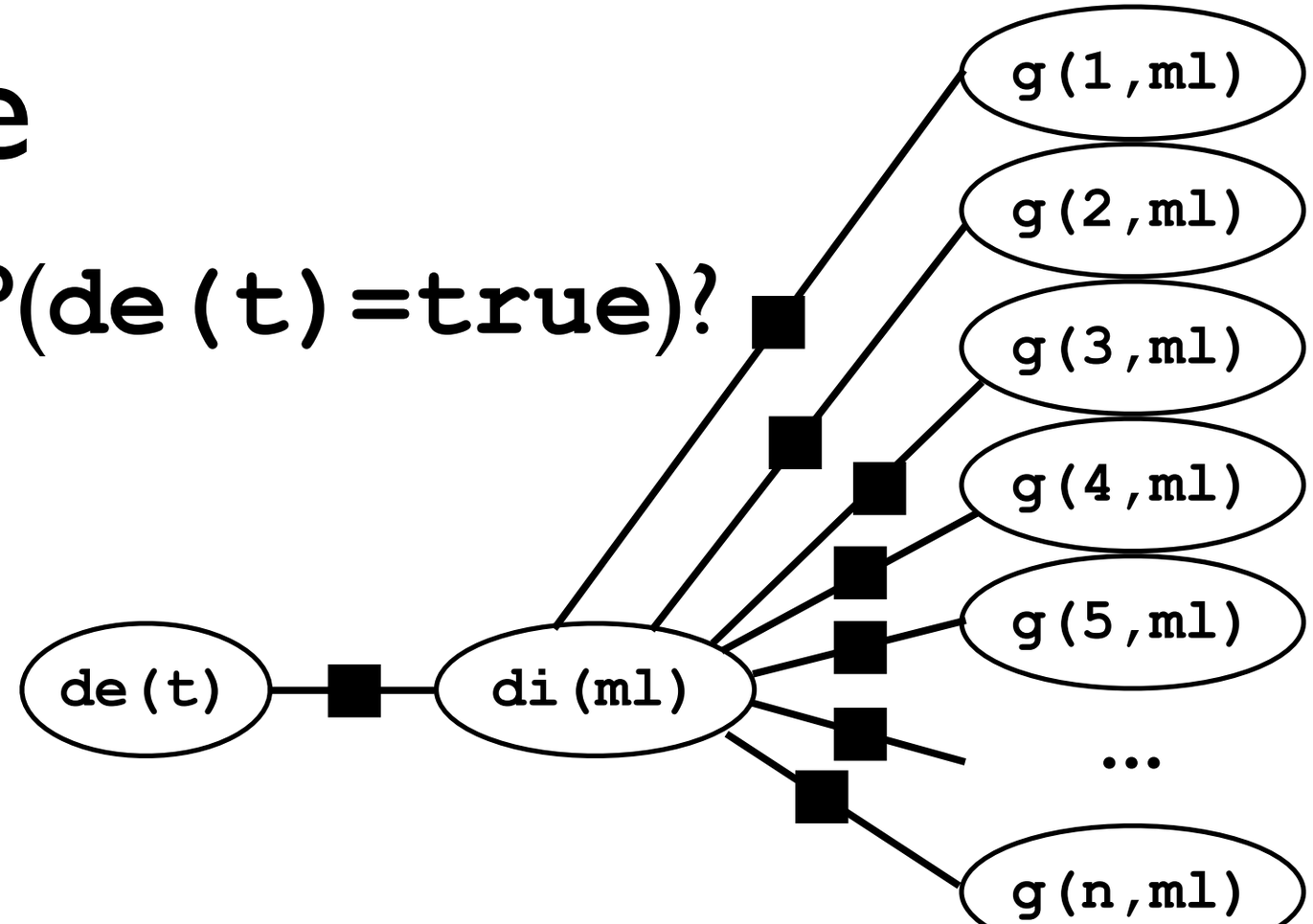

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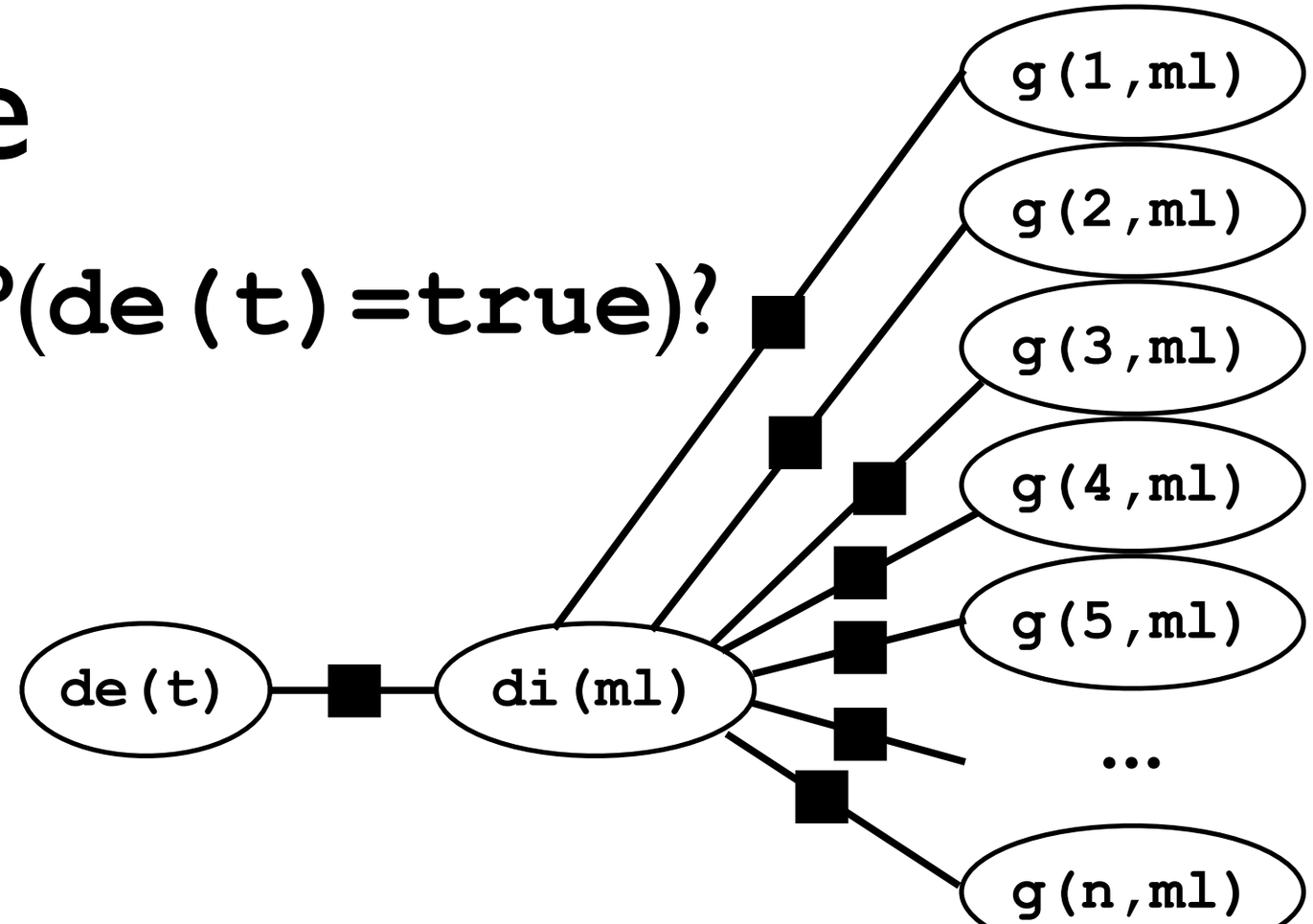

but: identity of students doesn't matter!
sufficient to count how often each grade m_1,...,m_k appears

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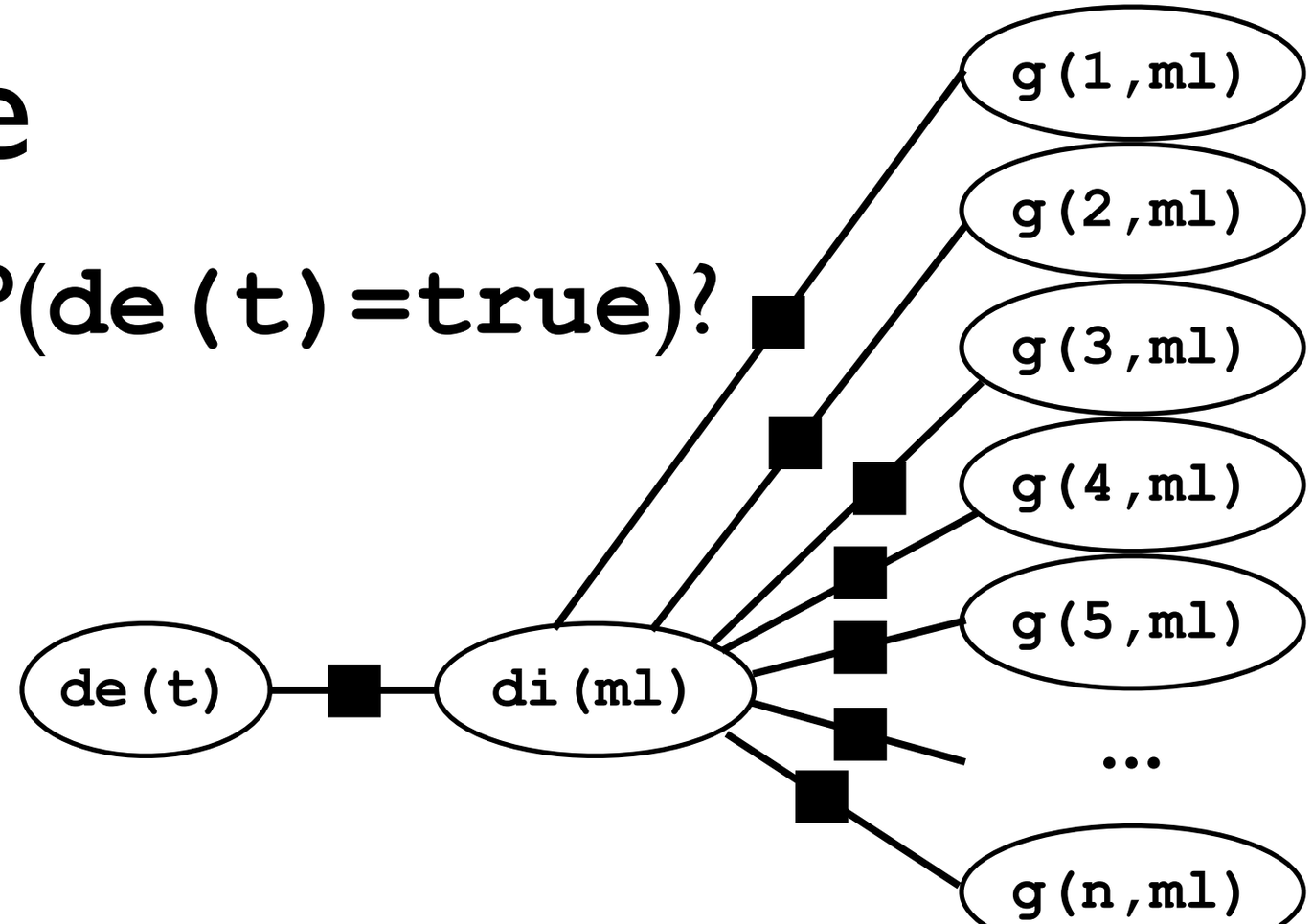

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sum $\phi_2(\text{true}, d) \prod_{i=1..k} \phi_1(m_i, d)^{\#m_i}$ instead

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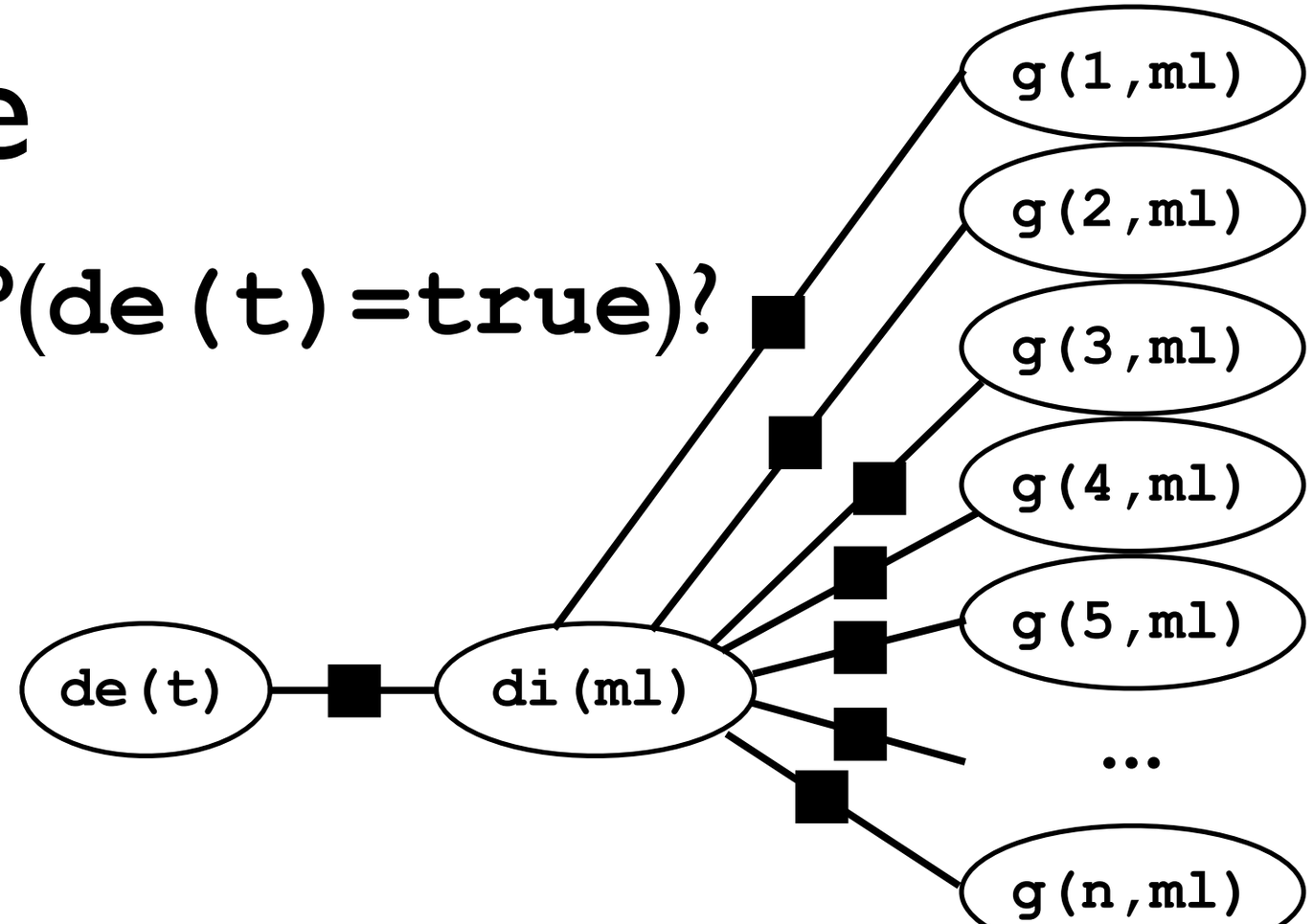

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sum $\phi_2(\text{true}, d) \prod_{i=1..k} \phi_1(m_i, d)^{\#m_i}$ instead

e.g., k=3, n=15: >14M grade vectors vs 136 count vectors

Lifted Inference

- exploiting symmetries & repeated structure
- reasoning on first order level as much as possible
- aiming at independence from number of objects
- approximation: grouping similar computations
- very active research area
- one example: weighted first order model counting

Parameter Learning

- **data** = interpretations on ground level
- **estimate** parameters on first-order level taking all instances of same par-RV together

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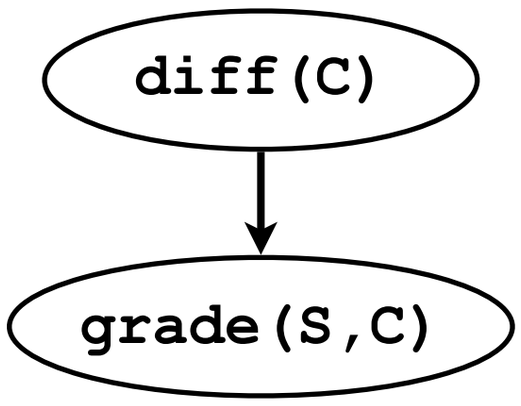


```
diff(ml)=t      diff(se)=f      diff(ai)=t
diff(db)=f      diff(ds)=t      grade(f,ai)=l
grade(a,ml)=h   grade(b,se)=l   grade(e,ai)=h
grade(b,ml)=l   grade(c,se)=l   grade(g,ai)=h
grade(a,db)=l   grade(d,ds)=h   grade(d,ai)=h
grade(c,db)=h   grade(g,ds)=l   grade(a,ai)=l
```

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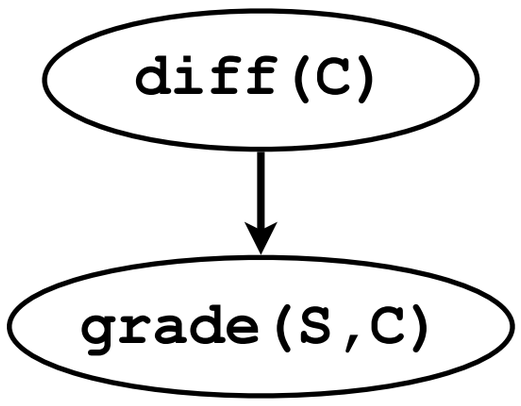


```
diff(ml)=t
diff(db)=f
grade(a,ml)=h
grade(b,ml)=l
grade(a,db)=l
grade(c,db)=h
```

```
diff(se)=f
diff(ds)=t
grade(b,se)=l
grade(c,se)=l
grade(d,ds)=h
grade(g,ds)=l
```

```
diff(ai)=t
grade(f,ai)=l
grade(e,ai)=h
grade(g,ai)=h
grade(d,ai)=h
grade(a,ai)=l
```

$$P(\text{grade}=h \mid \text{diff}=t) =$$

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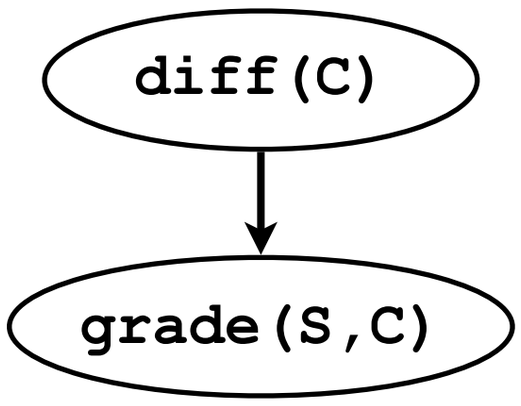


```
diff(ml)=t
diff(db)=f
grade(a,ml)=h
grade(b,ml)=l
grade(a,db)=l
grade(c,db)=h
```

```
diff(se)=f
diff(ds)=t
grade(b,se)=l
grade(c,se)=l
grade(d,ds)=h
grade(g,ds)=l
```

```
diff(ai)=t
grade(f,ai)=l
grade(e,ai)=h
grade(g,ai)=h
grade(d,ai)=h
grade(a,ai)=l
```

$$P(\text{grade}=h \mid \text{diff}=t) = 5/9$$

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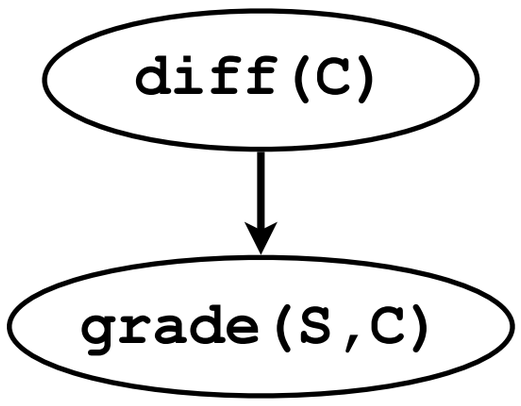


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grade(d,ds)=h
grade(g,ds)=l

diff(ai)=t
grade(f,ai)=l
grade(e,ai)=h
grade(g,ai)=h
grade(d,ai)=h
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```

$P(grade=h|diff=t) = 5/9$

$P(grade=h|diff=f) =$

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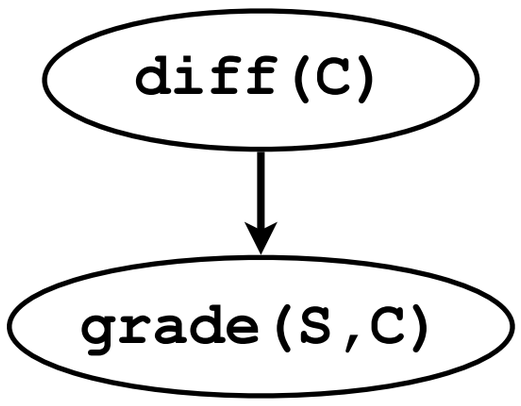


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```

$$P(grade=h|diff=t) = 5/9$$
$$P(grade=h|diff=f) = 1/4$$

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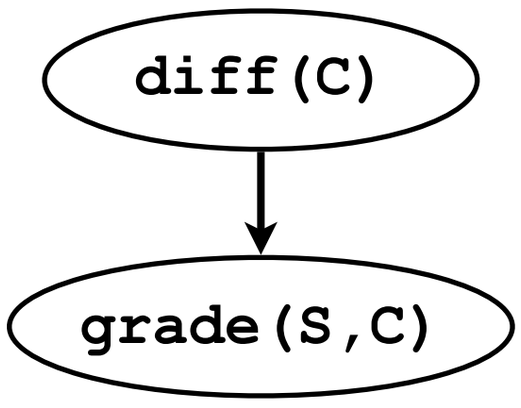
- **data** = interpretations on ground level
- **estimate** parameters on first-order level taking all instances of same par-RV together




```
diff(ml)=t      diff(se)=f      diff(ai)=t
diff(db)=f      diff(ds)=t      grade(f,ai)=l
grade(a,ml)=h   grade(b,se)=l   grade(e,ai)=h
grade(b,ml)=l   grade(c,se)=l   grade(g,ai)=h
grade(a,db)=l   grade(d,ds)=h   grade(d,ai)=h
grade(c,db)=h   grade(g,ds)=l   grade(a,ai)=l
```

P(grade=h|diff=t)= 5/9
P(grade=h|diff=f)= 1/4

P(diff=t)=3/5

Structure Learning

- often based on search over candidate structures of specific form
- close connection to both inductive logic programming and structure learning in graphical models

Languages to define par-factor graphs

- relational Markov networks (RMNs) [Taskar et al 2002]
- Markov logic networks (MLNs) [Richardson & Domingos 2006]
- probabilistic soft logic (PSL) [Broecheler et al 2010]
- FACTORIE [McCallum et al 2009]
- Bayesian logic programs (BLPs) [Kersting & De Raedt 2001]
- relational Bayesian networks (RBNs) [Jaeger 2002]
- logical Bayesian networks (LBNs) [Fierens et al 2005]
- probabilistic relational models (PRMs) [Koller & Pfeffer 1998]
- Bayesian logic (BLOG) [Milch et al 2005]
- CLP(BN) [Santos Costa et al 2008]
- probabilistic programming languages such as ProbLog, PRISM, Church, ...
- and many more ...

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constraint logic programming
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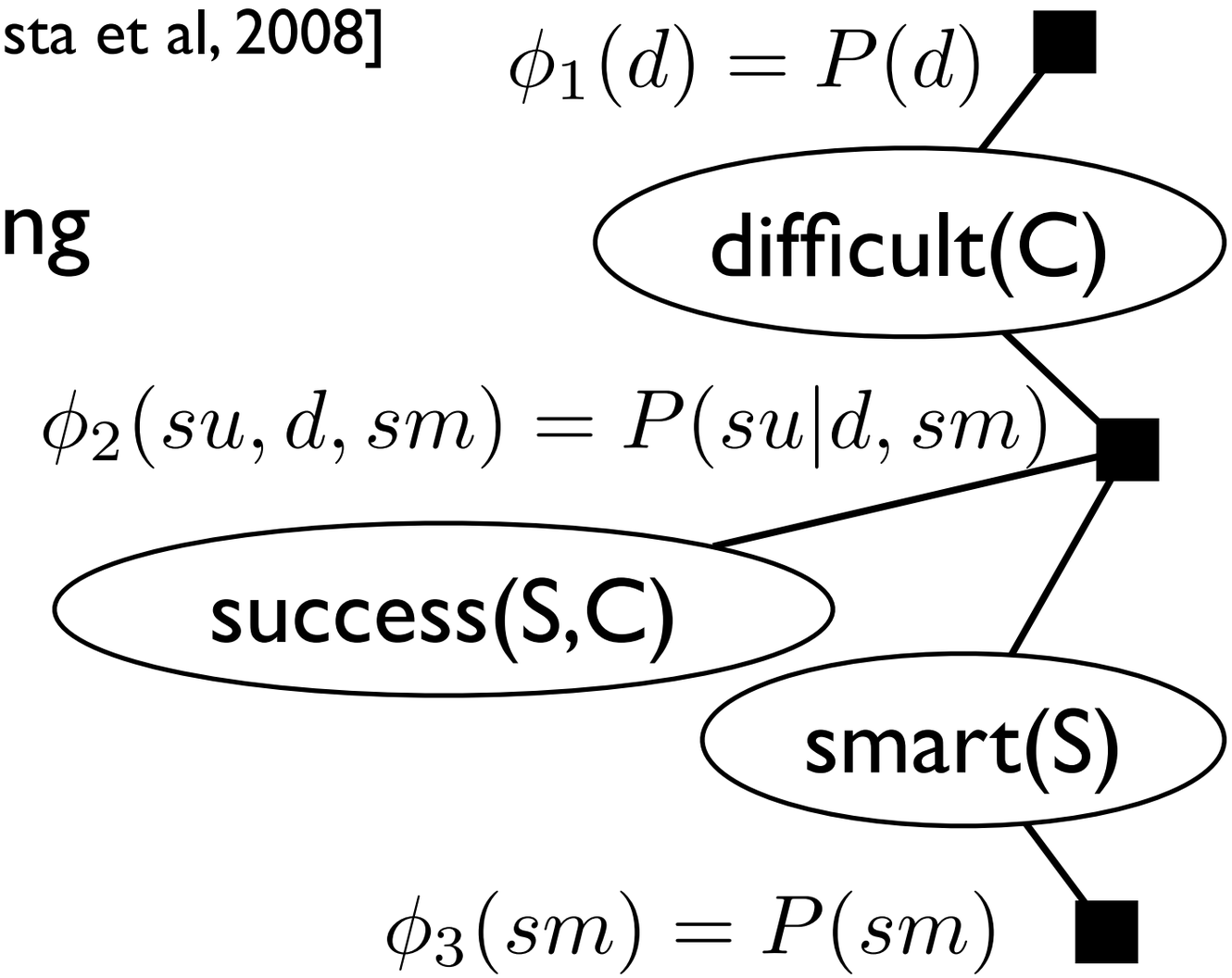

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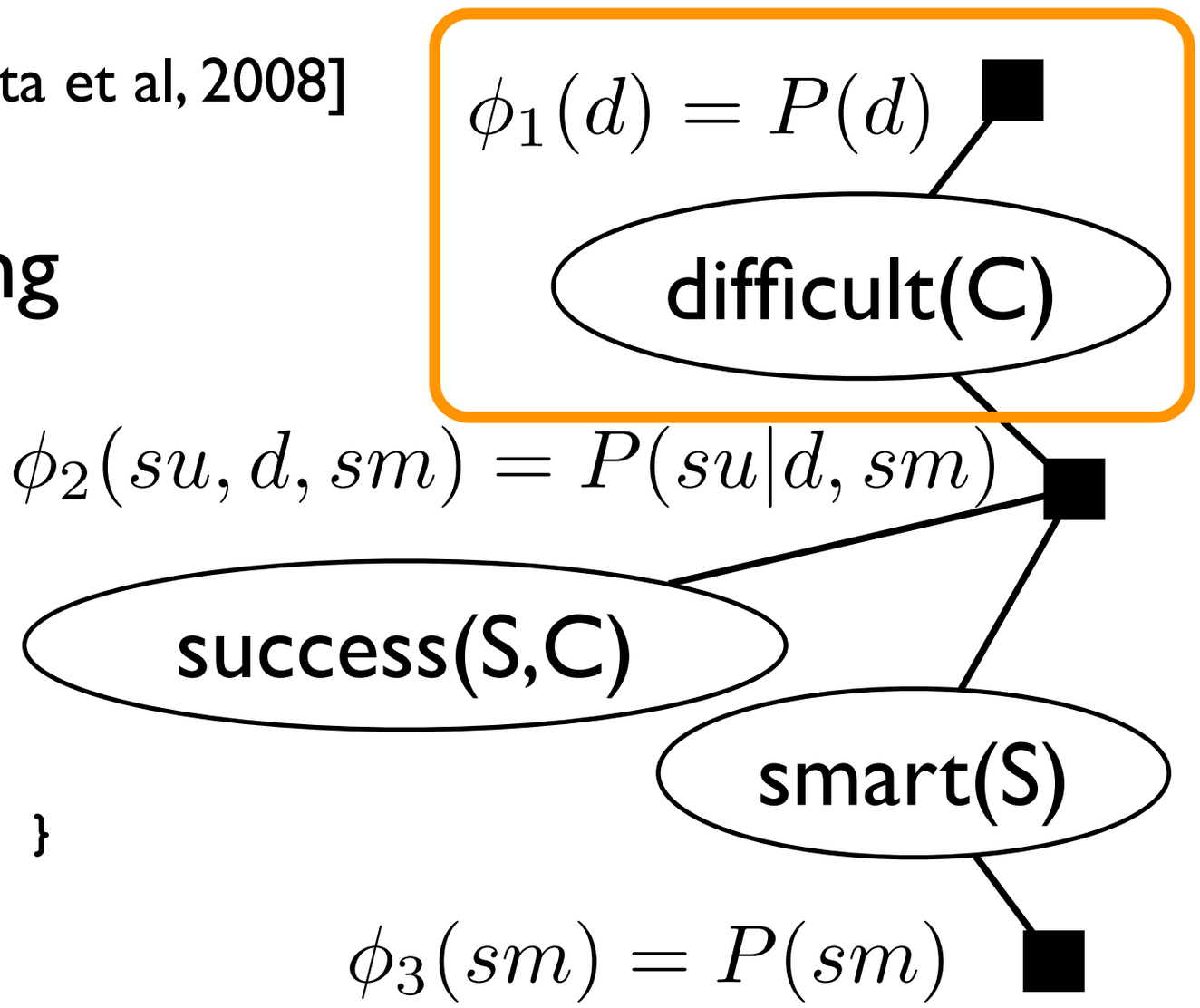


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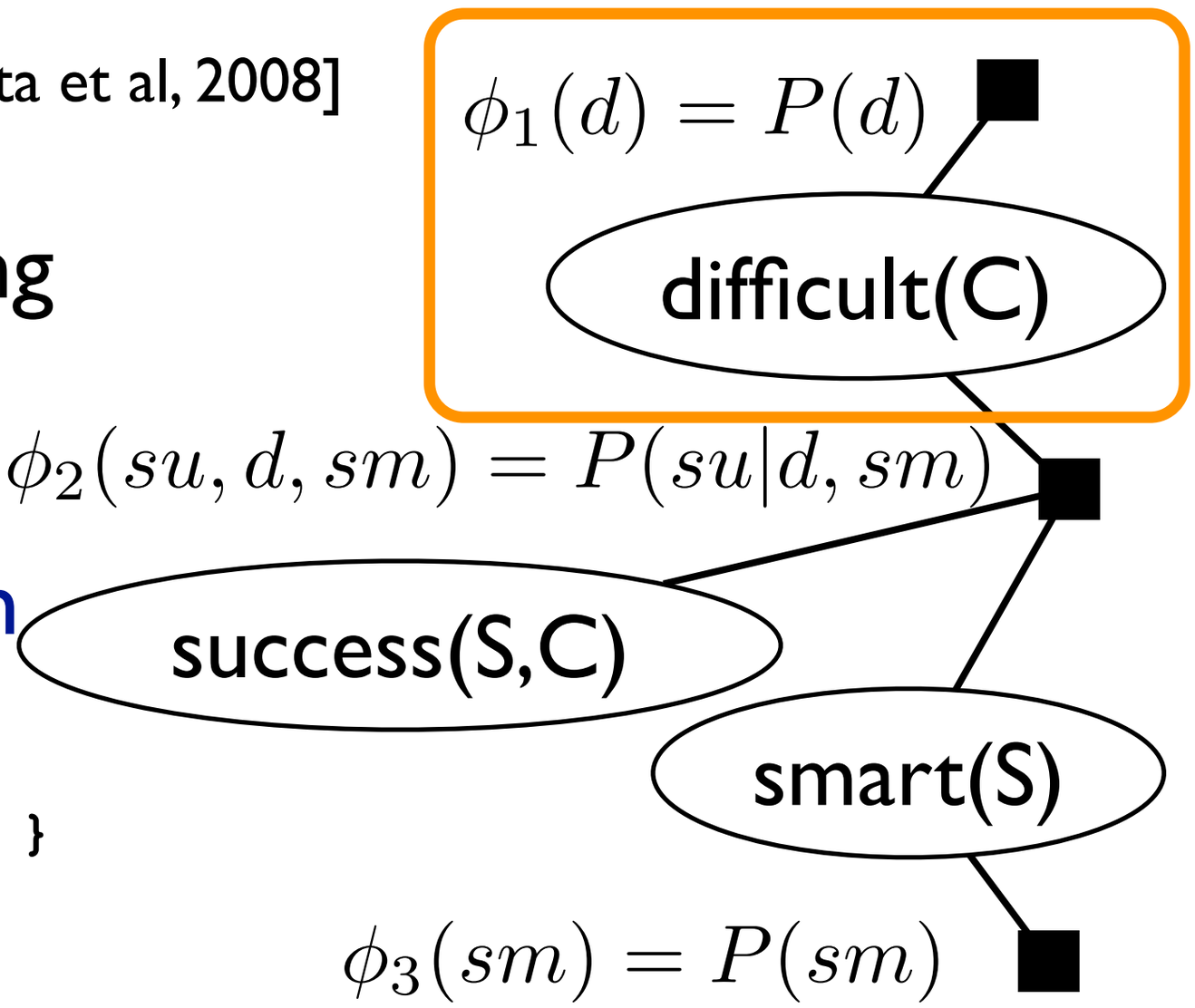

par-RV: unique skolem term

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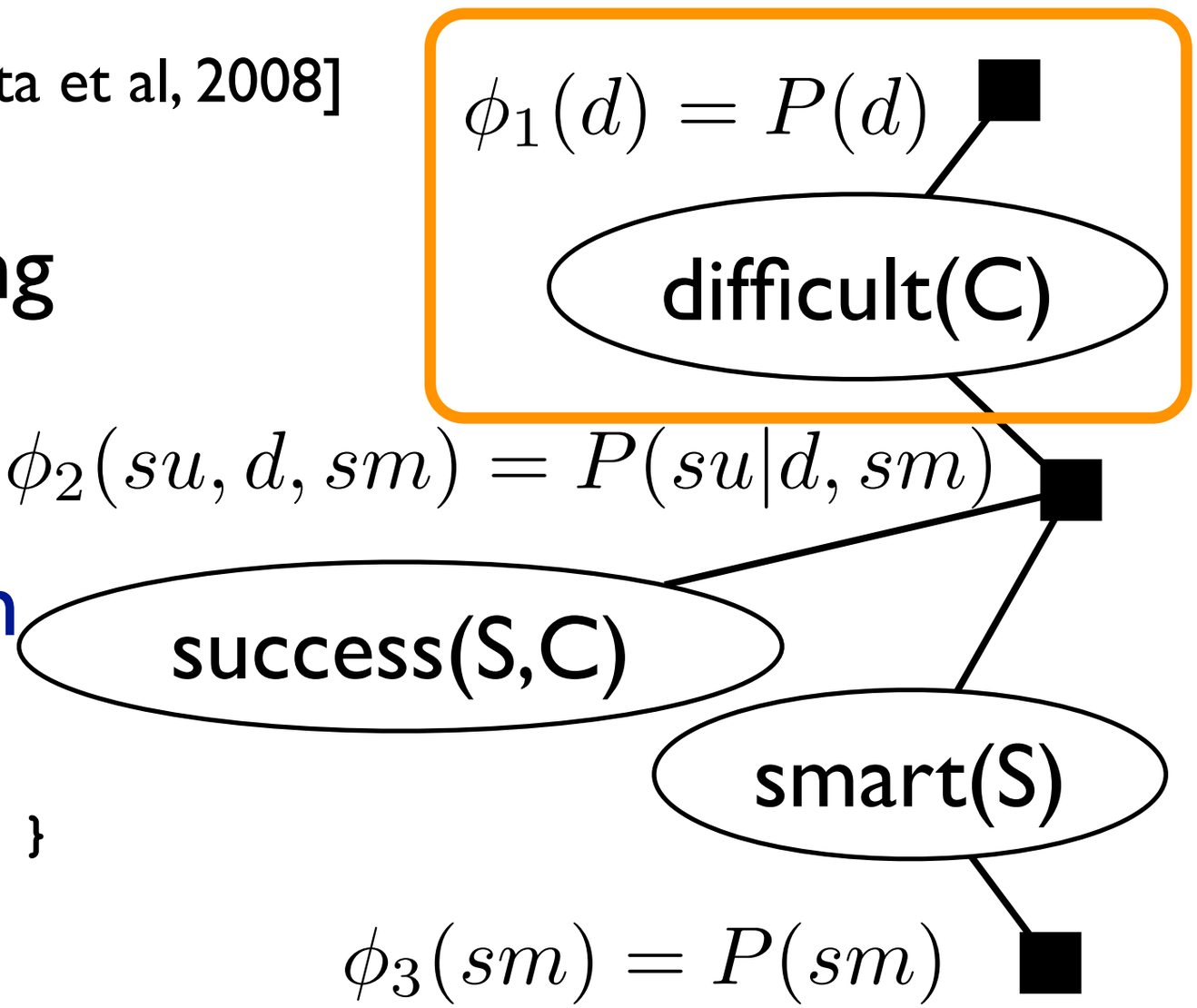

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possible values

CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming
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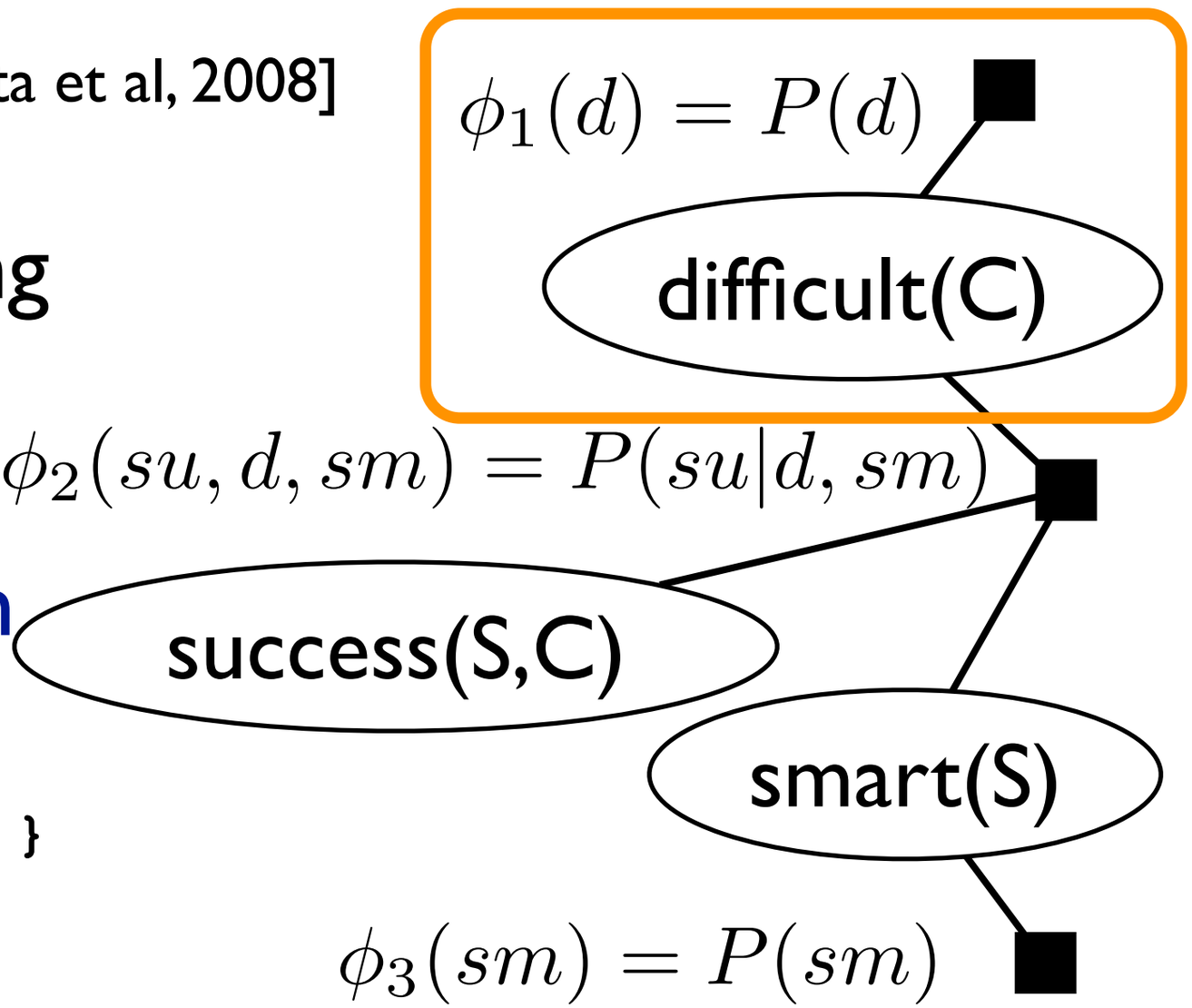

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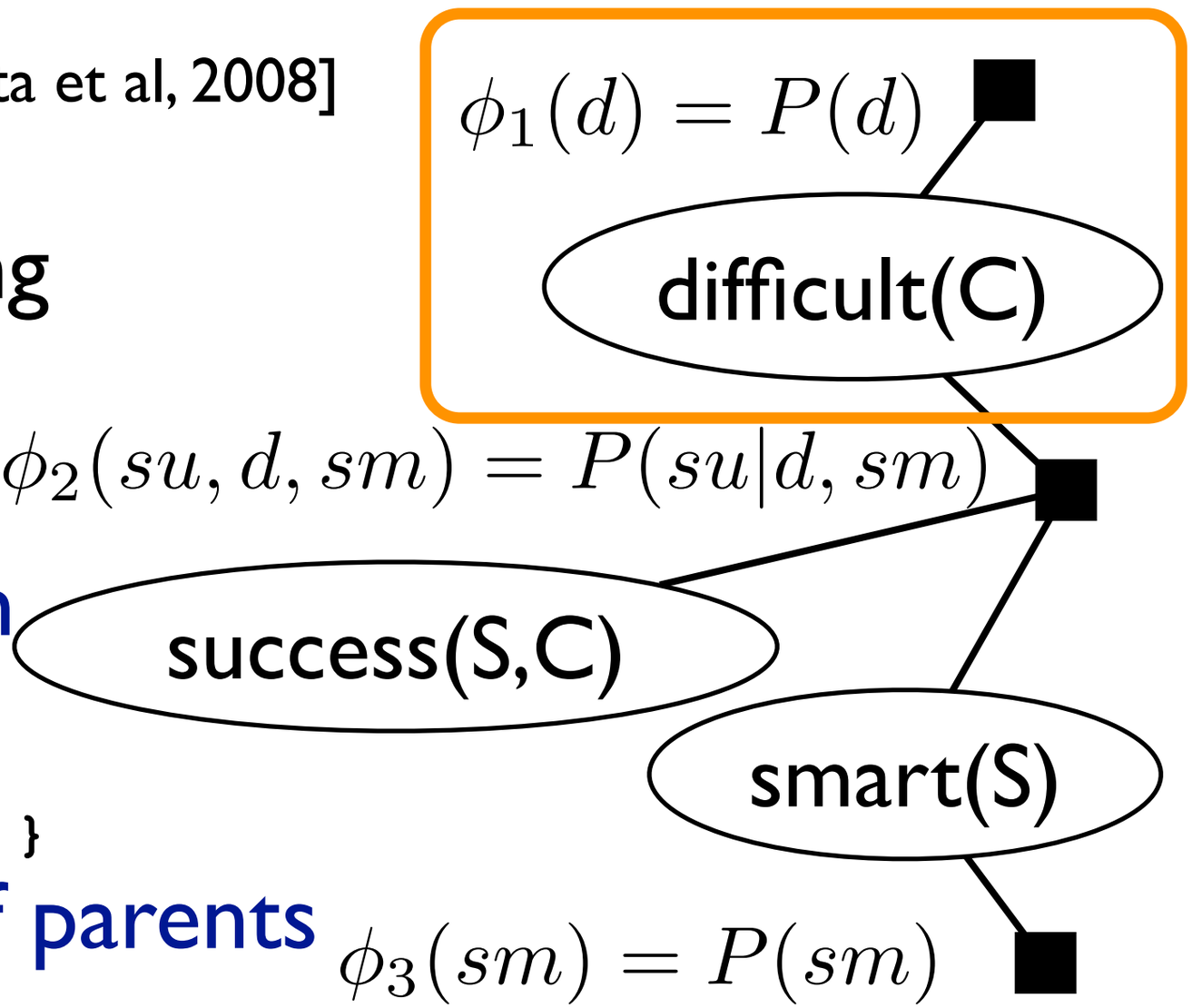

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difficult(Course,Diff) :-
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possible values

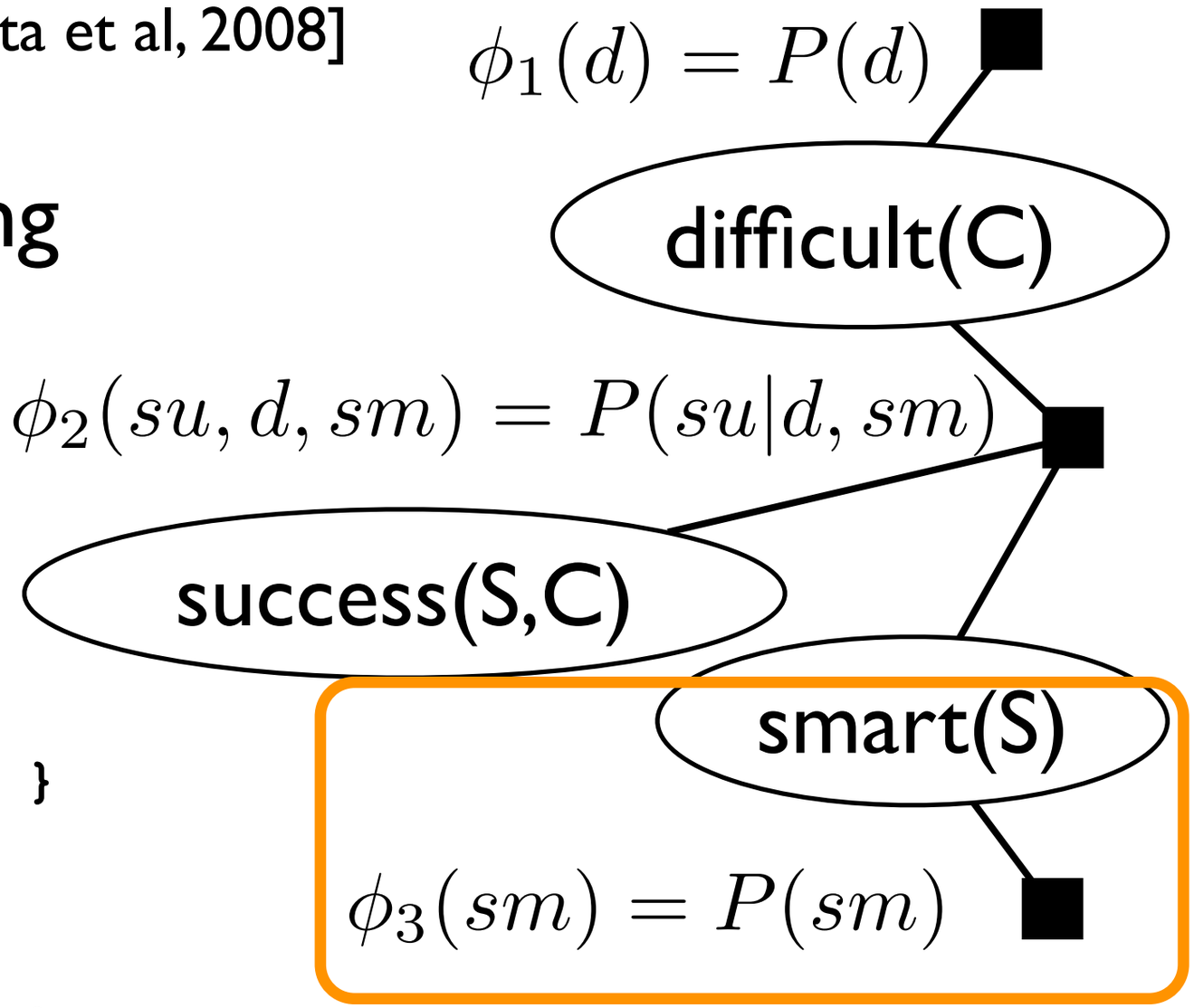
their probabilities

list of parents

CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming for Bayesian networks

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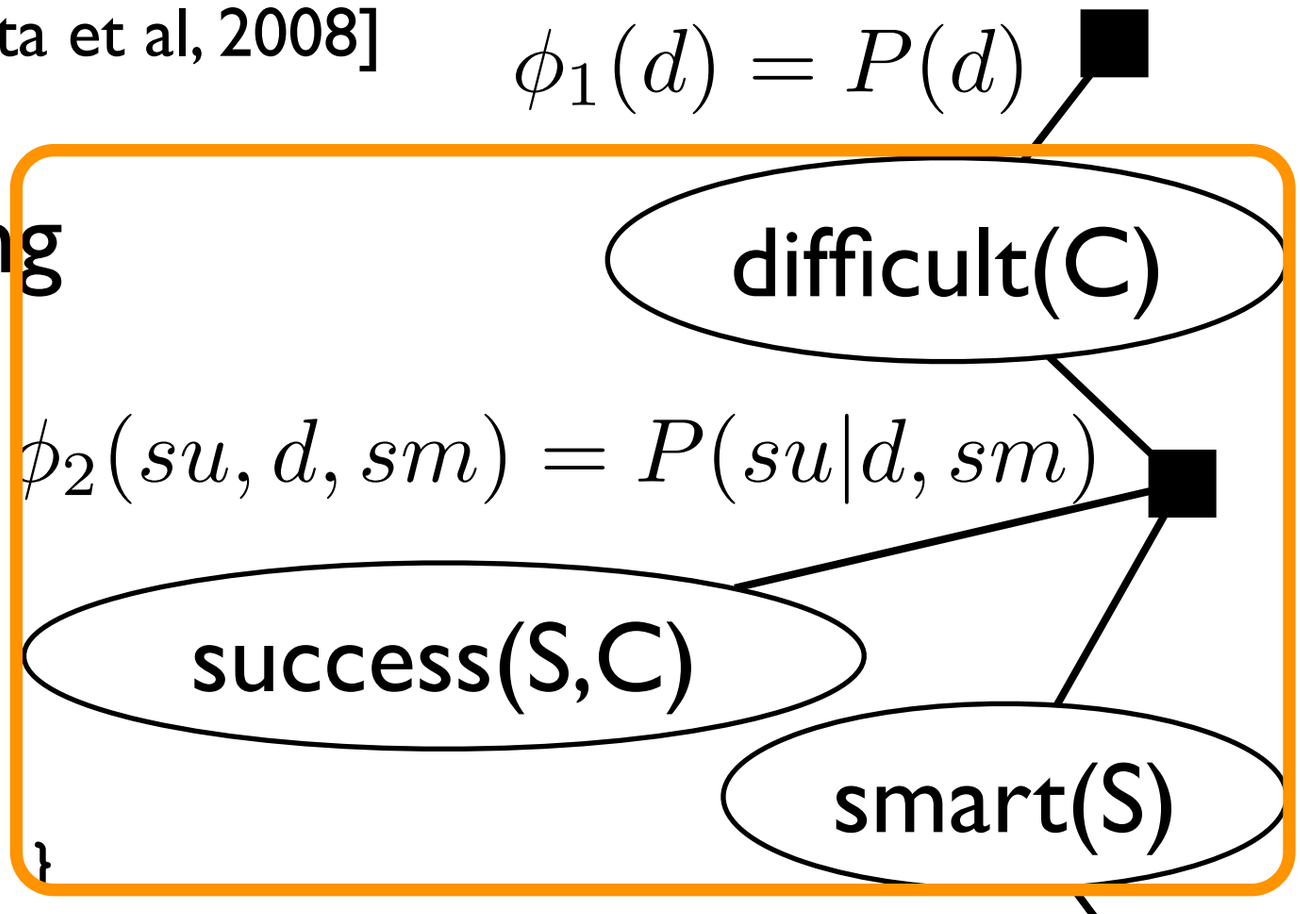


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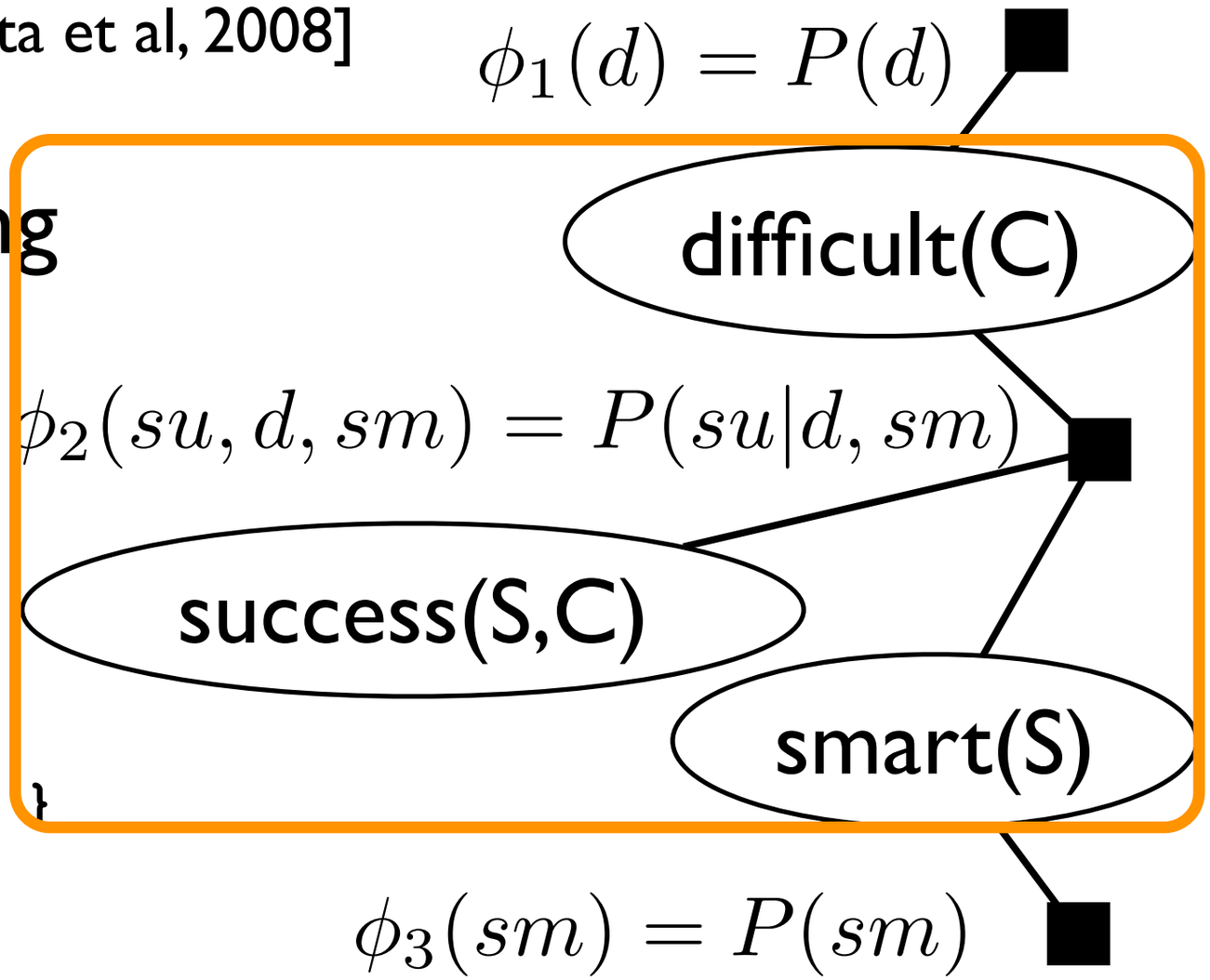


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Prolog call constrains grounding

CLP(BN)

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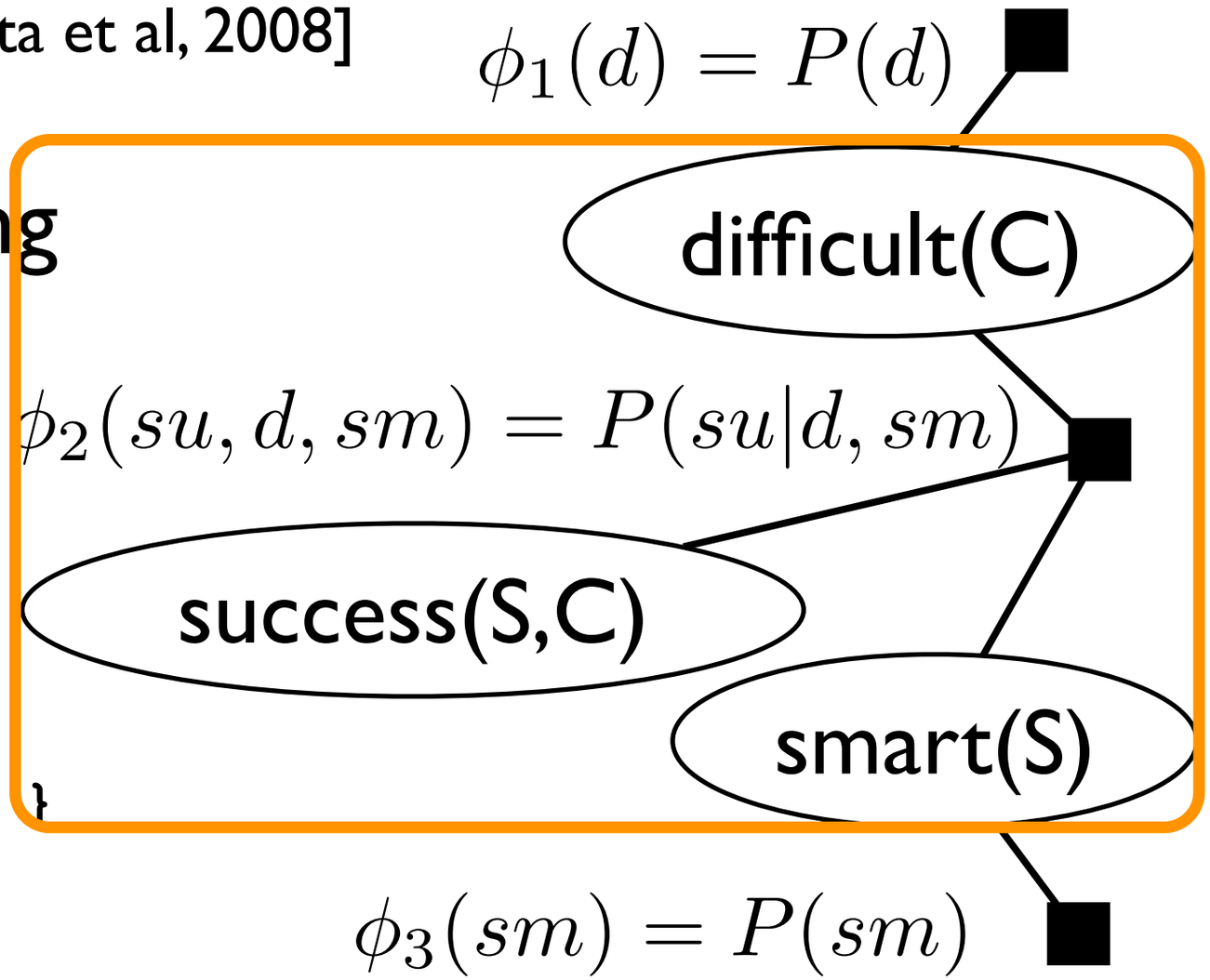
constraint logic programming for Bayesian networks

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get parents

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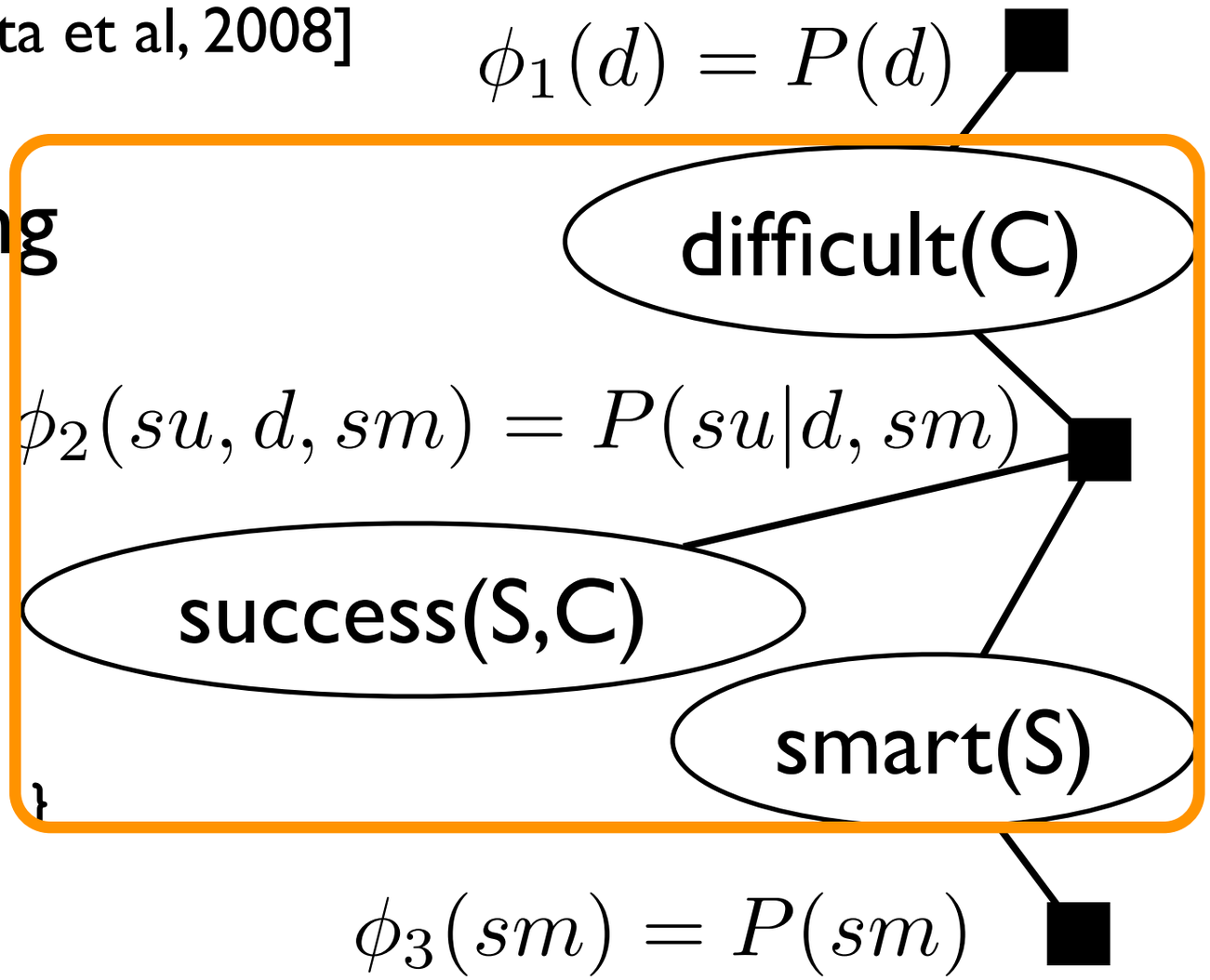


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```

get parents

P(su|d=t,sm=t)

CLP(BN)

[Santos Costa et al, 2008]

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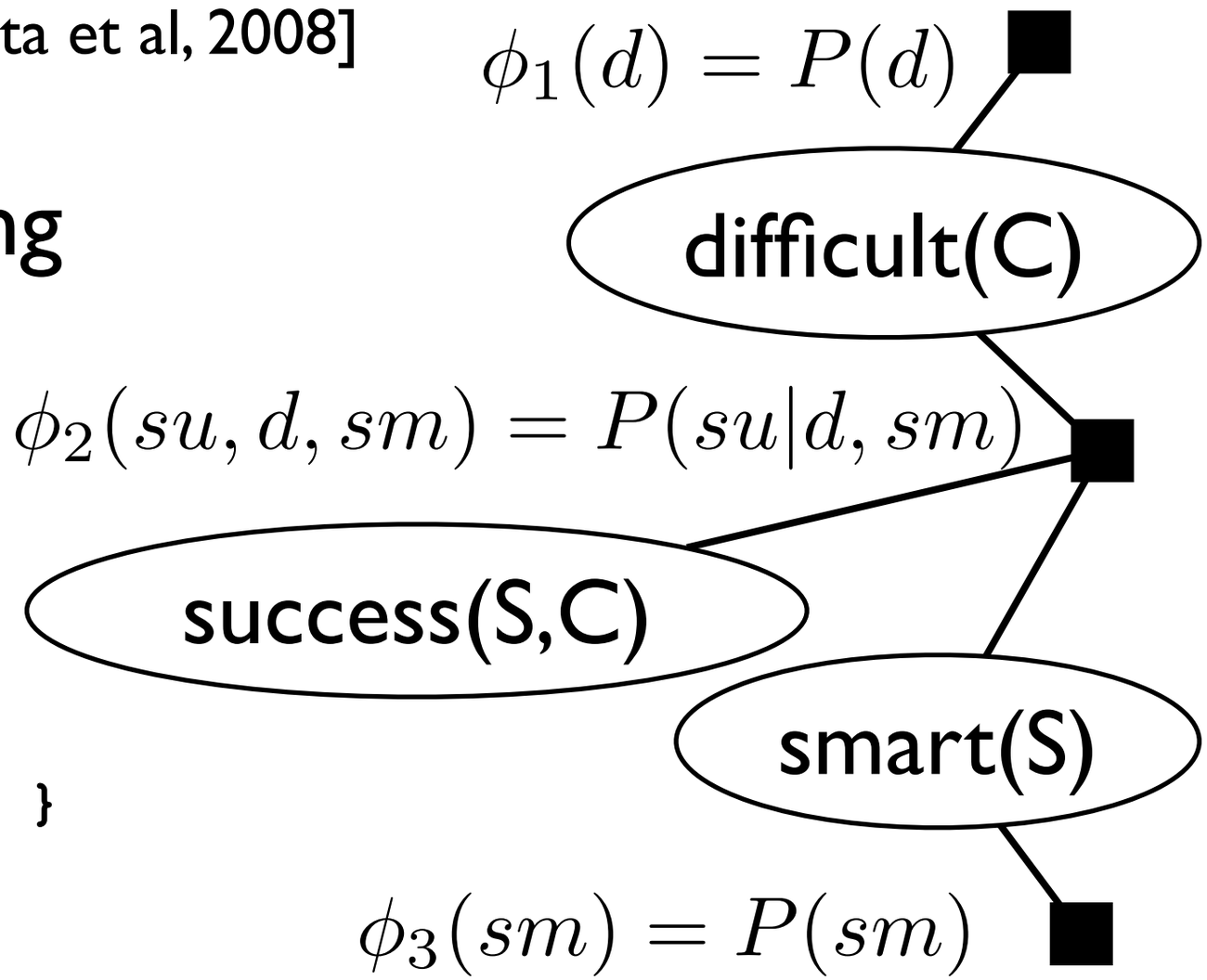


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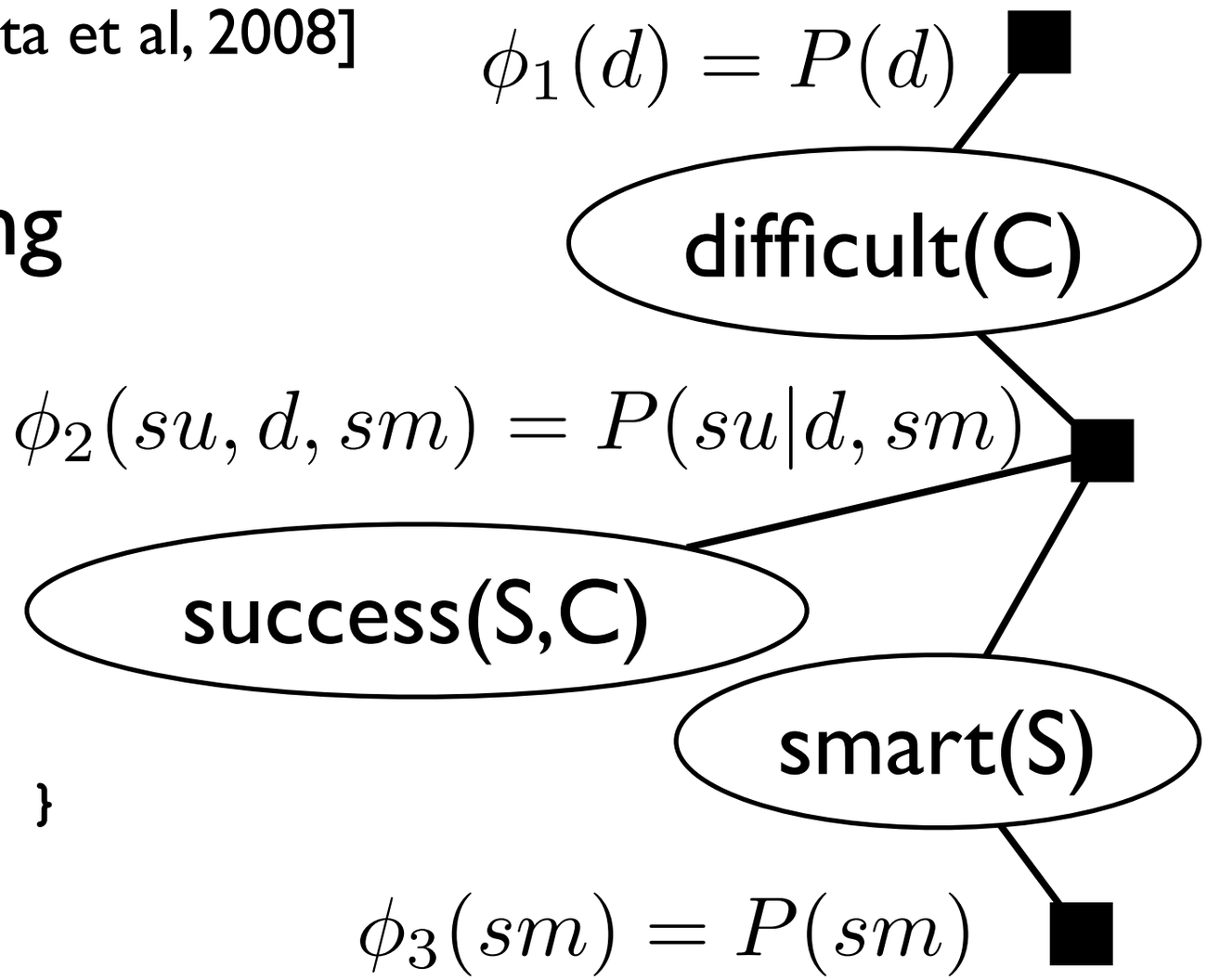


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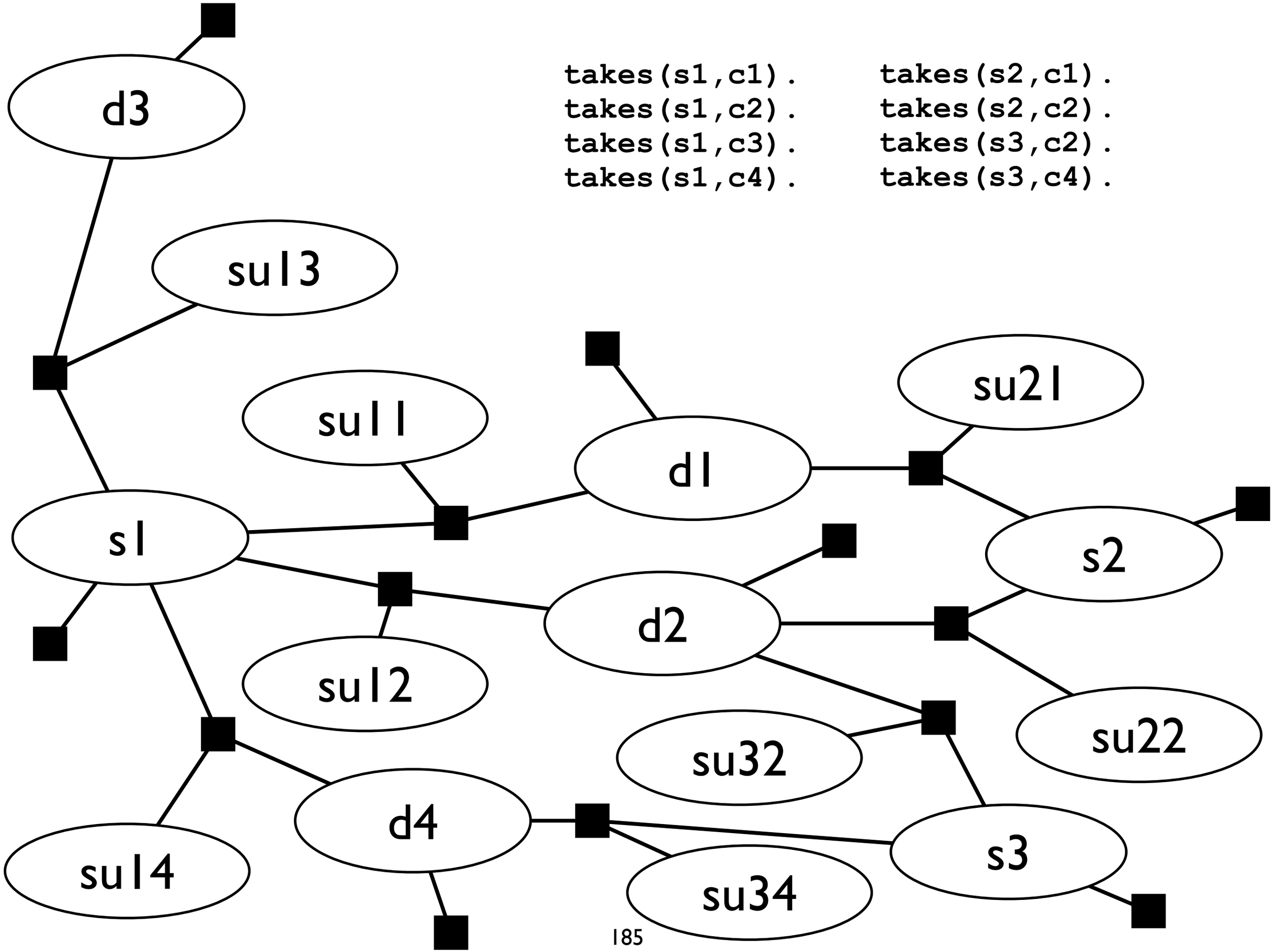

in ProbLog?

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```

```
0.6::difficult(C) :- takes(_,C).
0.7::smart(S) :- takes(S,_).
0.85::success(S,C) <- takes(S,C),difficult(C), smart(S).
0.10::success(S,C) <- takes(S,C),difficult(C), \+smart(S).
0.98::success(S,C) <- takes(S,C),\+difficult(C), smart(S).
0.45::success(S,C) <- takes(S,C),\+difficult(C), \+smart(S).
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Inference Example

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?-success(s2,c1,S).
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D=d(c1) with p(...,[])

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    { Success = su(Student,Course)
      with p([t,f],[0.85,0.1,0.98,0.45
                    0.15,0.9,0.02,0.55],[D,Sm])}
```

```
takes(s1,c1).
takes(s1,c2).
takes(s2,c1).
takes(s2,c2).
takes(s3,c2).
takes(s3,c4).
```

```
?-success(s2,c1,S).
```

Success=su(s2,c1) with p(...,[D,Sm])

Sm=sm(s2) with p(...,[])

D=d(c1) with p(...,[])

Inference Example

```
difficult(Course,Diff) :-
    { Diff = d(Course)
      with p([t,f],[0.6,0.4],[]) }

smart(Student,Smart) :-
    { Smart = sm(Student)
      with p([t,f],[0.7,0.3],[]) }

success(Student,Course,Success) :-
    takes(Student,Course),
    difficult(Course,D),
    smart(Student,Sm),
    { Success = su(Student,Course)
      with p([t,f],[0.85,0.1,0.98,0.45
                    0.15,0.9,0.02,0.55],[D,Sm])}
```

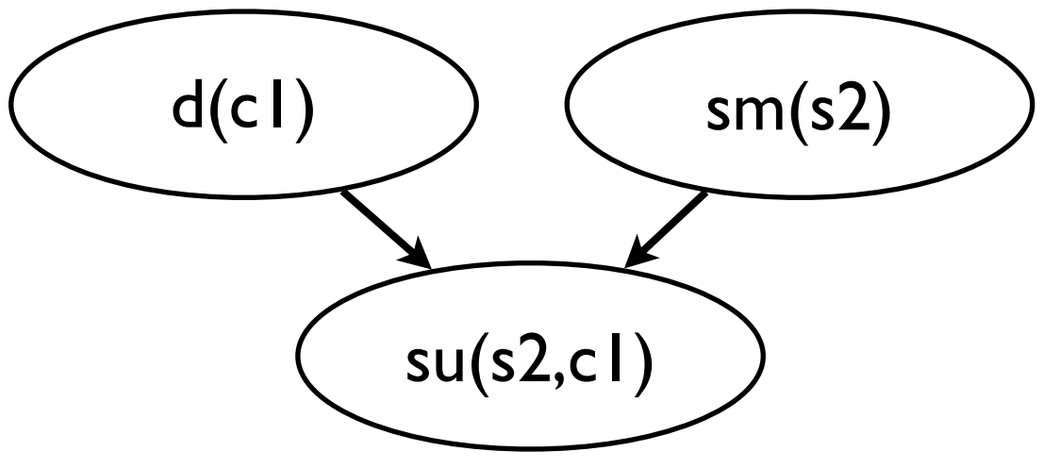
```
takes(s1,c1).
takes(s1,c2).
takes(s2,c1).
takes(s2,c2).
takes(s3,c2).
takes(s3,c4).
```

```
?-success(s2,c1,S).
```

Success=su(s2,c1) with p(...,[D,Sm])

Sm=sm(s2) with p(...,[])

D=d(c1) with p(...,[])



CLP(BN) Summary

- Templating Bayesian networks via constraint logic programming
- Knowledge-based model construction (KBMC):
  - construct relevant ground BN by backward reasoning, adding constraints to constraint store
  - run any propositional inference technique

Lifted Graphical Models Summary

- graphical model + relational templating language
- par-factor graphs as unifying framework
- aggregation / combining rules
- grounding vs lifted inference
- many different languages
  - CLP(BN): directed models via constraint LP

Advanced Topics

- parameter estimation
- complexity of querying
- lifted graphical models and KBMC

Roadmap

- Modeling
- Reasoning
- Language extensions
- Advanced topics

... with some detours on the way

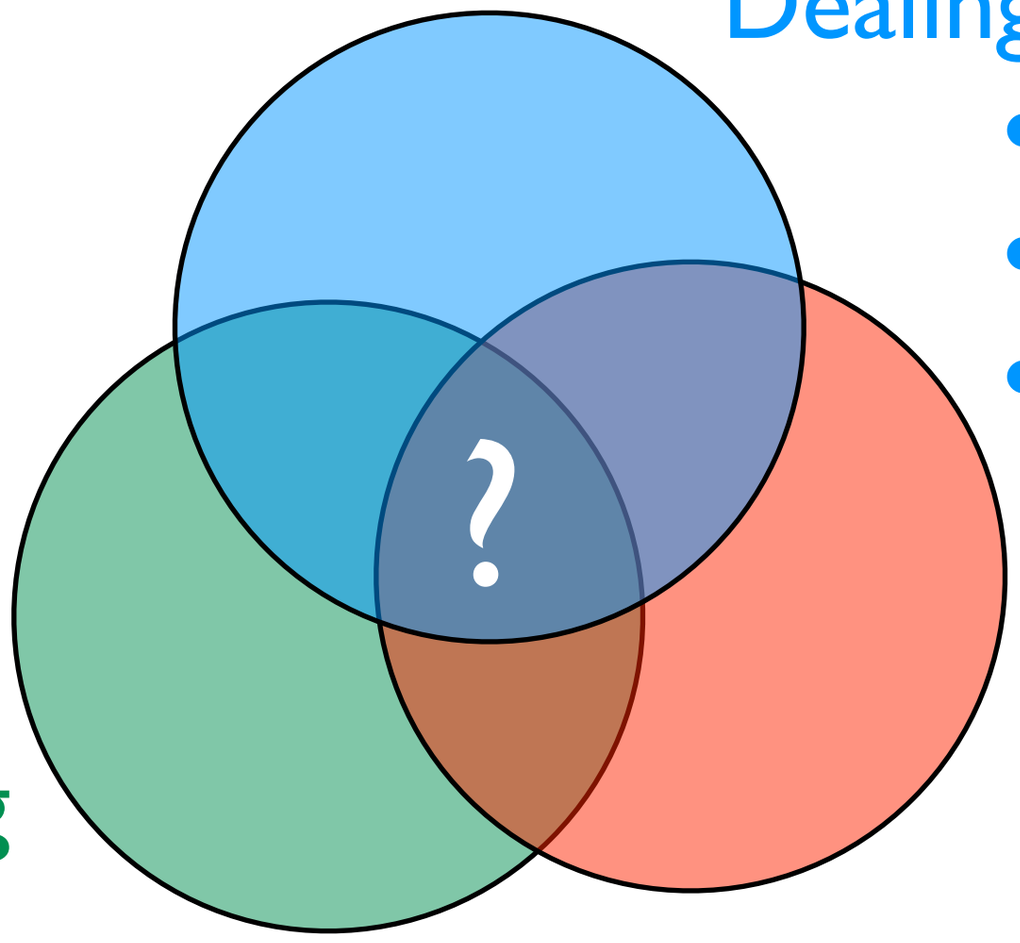
A key question in AI:

Dealing with uncertainty

- probability theory
- graphical models
- ...

Reasoning with relational data

- logic
- databases
- programming
- ...

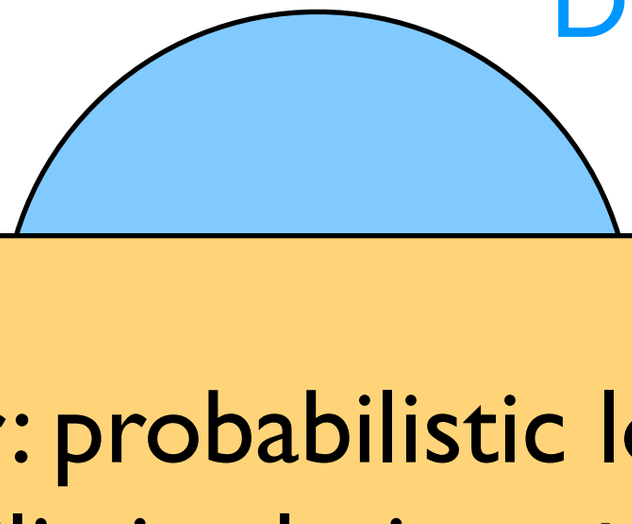


Learning

- parameters
- structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

A key question in AI:



Dealing with uncertainty
- probability theory

Reas
relat
- log
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- pr
- ...

dels

- Our answer: probabilistic logic programming
  = probabilistic choices + logic program
- Many languages, systems, applications, ...
- ... and much more to do!

structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

PLP Systems

- **PRISM** http://sato-www.cs.titech.ac.jp/prism/
- **ProbLog2** http://dtai.cs.kuleuven.be/problog/
- Yap Prolog http://www.dcc.fc.up.pt/~vsc/Yap/ includes
  - **ProbLog1**
  - **cplint** https://sites.google.com/a/unife.it/ml/cplint
  - **CLP(BN)**
  - **LP2**
- **PITA** in XSB Prolog http://xsb.sourceforge.net/
- **AILog2** http://artint.info/code/ailog/ailog2.html
- **SLPs** http://stoics.org.uk/~nicos/sware/pepl
- **contdist** http://www.cs.sunysb.edu/~cram/contdist/
- **DC** https://code.google.com/p/distributional-clauses
- **WFOMC** http://dtai.cs.kuleuven.be/ml/systems/wfomc

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